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RESEARCH ON RELATIONSHIP BETWEEN ECONOMICAL PROFIT AND ENVIRONMENTAL POLLUTION OF IMPERFECT PRODUCTION INVENTORY CONTROL PROBLEM

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ABSTRACT. This article presents a new model to optimize a production inventory model and prevent the environmental pollution. For this purpose, various policies have been implemented to reduce the polluting gases such as reliability development by modern technology on production system, rework on imperfective item and incorporate extra objective function to calculate the amount of the polluting gases. Here, we have considered a multi-objective production control inventory problem, the first objective function is for economical profit in maximizing form and second objective function for calculating the amount of the polluting gases in minimizing form. To evaluate the performance of our model, we give some illustrative examples using the Mathematica 9.0 software.

Keywords: Multi-objective inventory control problem, Imperfect Production, Rework, Reliability, Pontryagin's Maximum Principle.

1. INTRODUCTION

The inquiry about the relationship between economical profit and environmental pollution has always been an apple of discord. As to analyze this we can say corporate executives sometimes act as if making profits today, under tighter environmental law, is stealing cheese from a mousetrap no matter how deadly the consequences of industrial decisions on environment are. Proponents of the mousetrap position argue that corporate manager can either control pollution or maximize profit but that former can be accomplished only at the expense of the letter. From the investor's perspective this in turn implies that he can either invest in a profitable company or a good company (which protects its environment) but no company is likely to be both. In order to bring economical profit and environmental pollution control in the same track, we are trying to present some suggestive measures in this research paper.

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From the year 1970, the relationship between profit of production inventory and he amount of produced polluting gases throughout the production has received growing appreciation all over world. Already in some developed countries the said relationship has been extensively investigated and analyzed.[[1],[2] [3],[4]]. However, there has been relatively little research on the relationship for developing countries. The emission of key industrial pollutants and average per capita income from developed countries do not always correspond to the truth of developing countries. Therefore, considering environmental impacts of industrial decisions play an important role in preserving our environment. The first and most important step in this endeavor is to analyze products impacts on environment with a holistic approach. This holism includes the analysis of the products' life cycle from the very beginning up to the very end of it. Using this approach, ecological impacts of every little decision in various product stages such as product conceptualization, design, raw materials processing, manufacturing, assembly, warehousing, packaging, transportation, reusing and refurbishing is measured and considered in designing the product and the required operations.

During the production industrial environmental polluting gases are produced as well as the gases are produced from the disposal of defective items, etc. like ozone, CFC, HCFC, HFC, carbon dioxide, methane, sulfur-dioxide, nitrous oxide which are known as Industrial Solid Waste (ISW) in production industry. The said gases have been an important contributor to greenhouse gas (GHG) emissions and these green house gases are considered the principal components contributing to global warming. The companies agreed to invest a large amount of money to manage the ISW that reduces the said GHG in environment. Also, modern technologies are implemented by the industry to increase the reliability of the production system and decrease the GHG emission in the environment. Andrade et. al.[5] has managed the hazardous waste in the printing industry. Braschel and Posch [6] have done a review of system boundaries of GHG emission inventories in waste management. Recently, several researchers like Fawole et. al. [7], Villamizar and Brown [1], Kader et. al. [8] have been developed the pollution control model.

Most of the researchers assume that in the basic production inventory model all produced items are perfect. But in long-run production it is almost impossible In long-run process throughout the production imperfect quality items may be produced. The production of imperfect items is increased with time and decreased with reliability parameters. Rework is a process using materials(imperfective items) into new products to prevent waste of potentially useful materials, reduce the consumption of fresh raw materials, reduce energy usage, reduce air pollution in production inventory system. So, in this process, defective items are reworked at a cost to make the products as new as perfect one keeping in mind the spotted image of trademark of the manufacturing scheme. Lin et. al.[9] developed a model on optimal replenishment policy for imperfect production inventory model with rework and backlogging. Next, Chiu et. al.[10] generalized the optimization of the finite production rate model with scrap, rework and stochastic machine breakdown. Recently, Sana[11], Pal et. al.[12], Selcuk and Agrali[13], Hazari et. al. [14], Maity [15], Sarkar and Saren [16], Paul et. al. [17] have been developed an imperfect production inventory models with/ without rework.

In this study, we established a multi-objective production control problem to make the relationship between economical profit and the environmental pollution. For this reason, the polluting gases are controlled through several processes like technological development, rework, ISW management, green plantation etc.In this research paper, we are first time formulating a multi-objective production control inventory problem with environmental consciousness. The first objective for economical profit in maximizing form and second objective for pollution gases in minimizing form. production cost is dependent on raw material cost, development cost and wear-tear cost which is taken in this research area. The multi-objective problem is converted into a single objective problem by weight sum method (cf. [18]) and using optimal control theory(cf. [19]), we solved the optimal control problem and obtain the required optimal production. Then using Mathematica 9.0 software, we solved the differential equation to obtain the optimal stock. The model is illustrated through numerical examples. Using the said software, sensitivity analysis for different parameters and results of objective functions are presented in tabular and graphical form.

The paper is organized as follows: In section 2, we discuss the literature review about this paper. Section 3 for assumptions and notation. Also in section 3 , we formulate the mathematical model and discuss the solution method. We have discussed a brief description of the numerical results and figures of the numerical experiments in section 4. In section 5 we give a way how to use the model practically. This paper is concluded in section 6.

2. LITERATURE REVIEW

Most of the classical inventory models generally deal with a single-objective function. But in real world situations, multi-objective inventory models for two or more objectives are required to be maximized and minimized and $\cdots$/ minimized and minimized and $\cdots$ / maximized and maximized and $\cdots$ by the companies or the retailers simultaneously over a given set of decision variables . This leads to the idea of a multi-objective mathematical programming problem.

Liu et al.[20] have developed a model on multi-objective programming to control the water pollution. Zhang and Huang [21] have developed a multi-objective problem to control the GHG gas emission Recently Woinaroschy [22] has used a multi-objective optimal problem for biodiesel sustainable production. Very recent many researchers have been developed the multi-objective inventory models [[23],[24], [25]].

Observing the environmental economics the real economy is in which we all live and work as an open system. This means that in order to function, the economy must remove the properties (raw material and fuel) from the environment, process these properties, and dispose large amount of chemically transmuted properties back into the environment. The process starts with the abstraction of properties, which can be exhaustible (fixed in overall quantity) or renewable (resource grows through time). When the process gets end then it leaves the disposal of transmuted properties which can pollute the environment. The costs of a pollution free society would be very high. The other life-threatening is to live in a society where there is no pollution control. The real world is somewhere in between these two extremes, i.e., it is necessary to achieve a balance between the social costs and social benefits of reducing pollution.

Xuemei et al.[4] developed a model on the relationship between the economic growth and environmental pollution. Recently, Zhang and Huang [21] have showed that how to control the emission of the GHG. Liu et al.[26] also have developed a model to control both local air pollutant and CO_2. Also many papers have been developed on pollution control [[3], [1], [7] etc.].

Most of the basic assumption of the classical Inventory model is that quality of all manufacturing products are perfect. But the assumption is not true in practical. In a production system, it is quite natural that a machine cannot produce all the items perfect during a whole production period.

Khouja and Mehrez [27] have developed an economic production lot size model with variable production rate and imperfect quality. Later Sana[28] have represented a production-inventory model in an imperfect production. Recently many papers have been developed on imperfect production [[29] etc.]

Rework process is necessary to convert those imperfective into finished goods.Rework is a process using materials(imperfective items) into new products to prevent waste of potentially useful materials, reduces the consumption of fresh raw materials, reduces energy usage, reduces air pollution in production inventory system.

Lin et. al.[9] developed a model on optimal replenishment policy for imperfect production inventory model with rework and backlogging. Next, Chiu et. al.[10] generalized the optimization of the finite production rate model with scrap, rework and stochastic machine breakdown. Later, Cardenas-Barron et al.[30] made an improved solution to replenishment lot size problem with discontinuous issuing policy and rework. After that Cardenas-Barron et al.[31] improved a solution to replenishment policy for the EMQ model with rework.

Using Pontryagin's Maximal Principles ([19]), many researchers have been optimized the single objective function [([32] etc.]. First time, in this paper we have solved the multi-objective production inventory problem by the said principles where the first objective function is for economical profit in maximizing form and second objective is for calculating the amount of polluting gases in minimizing form.

3. OPTIMAL CONTROL FRAMEWORK

The mathematical model will be developed by using the following assumptions and notations.

3.1. ASSUMPTIONS AND NOTATIONS

For a reliability dependent imperfect production inventory control model, following assumptions and notations are used.

3.1.1. ASSUMPTIONS

(1) Production rate is function of time which is taken as control variable,
(2) Imperfective rate is depended on reliability,
(3) This is a single period inventory model with finite time horizon $[0, T]$,
(4) Imperfect units occur only when the item is effectively produced and there is repair of imperfect units over the period $[0, T]$,
(5) Imperfect units are partially or fully reworked at a constant rate over the period $[0, T]$,
(6) Unit production cost is produced-quantity dependent,
(7) Unit pollution cost due to the GHG emission and IWM policies in the production industry is constant,
(8) Operating cost is also constant,
(9) The salvage value price of the finish stock is constant, (x) Shortage is not allowed,
(10) The inventory level is assumed to be continuous function of time.

3.1.2. NOTATIONS

T : time length of the cycle,
$U(t)$: production rate at time t which is a control variable,
$X(t)$: inventory level at time t,
$D(t)$: dynamic demand rate where d_0 is initial demand which is constant,
V : constant advertisement rate,
q : catchabiliy parameter,
r : reliability parameter,
δ : imperfective parameter,
$\delta\, e^{(1-r)t}$: reliability dependent imperfective rate,
γ : constant rate of rework,
r_{min} : minimum value of the reliability parameter,
r_{max} : maximum value of the reliability parameter,
L : the costs like labour and energy,

T : the cost of technology, resource and design complexity for production when $R = R_{min}$,
$C_d(r) = M + N\, e^{k\frac{(r-r_{min})}{(r_{max}-r)}}$: development cost dependent on reliability parameter,
C_m : raw material cost,
σ : wear-tear cost,
$C_p(r,t)$: unit production cost which depend on production rate, raw material cost, development cost and wear tear cost,
C_{op} : operating cost ,
C_{pl} : pollution cost,
C_{rw} : rework cost per unit defective item ,
A_{plg} : pollution gas per unit production,
A_{rwg} : re-work gas per unit production,
A_{wg} : wastage gas per unit production,
C_h : holding cost per unit item ,
C_v : advertisement cost per unit advertisement,
P_s : selling price per unit items sold ,
P_{s_1} : salvage value price of finish stock at time T,
$\lambda_i(t)$: adjoint function at time t, $i = 1, 2$.

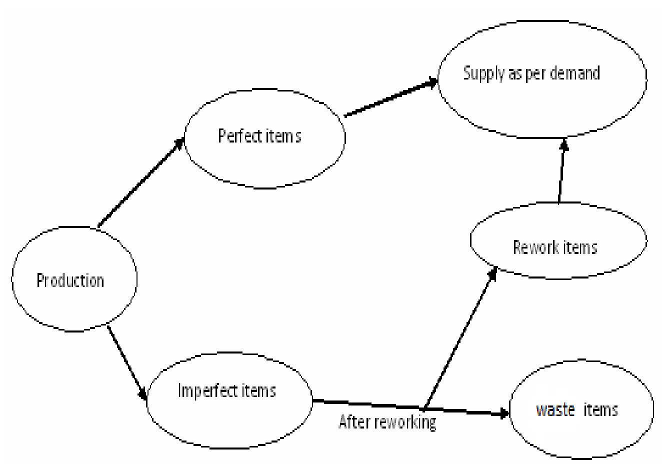

Fig. 1. Imperfect Production Model

3.2. PROPOSED RELIABILITY DEPENDENT IMPERFECT PRODUCTION INVENTORY AND POLLUTION CONTROL MODEL

According to the market research, it is observed that in recent times advertisement rate of a product has a vital role to create a center of attention on sales for many retail products. The advertisement through media using Inventive behavior to exhibit of product is one of the best tool for the promotion of products to mass customers. Now-a-days, it is seen to that demand of a product completely relies on catchability of the advertisement. The more impactful it is, the more the demand. In this paper,we assume that the demand function as

$$D(t) = d_0 + q\frac{V}{1+V}t \tag{1}$$

In this model, we consider a single-item imperfect production-inventory control model with rework which depends on reliability of the product and shown in figure-3. Here, the production rate of the items are at a variable rate $U(t)$ of which $\delta e^{(1-r)t}$ (where $\delta e^{(1-r)t} < 1$) is reliability dependent imperfective rate. The reliability r decreases the imperfective rate. The defective item after rework is treated as disposal part. The demand rate $D(t)$ of customers meet from the inventory.

Therefore, the differential equation for stock level $X(t)$ representing above system during a fixed time-horizon, T is

$$\dot{X}(t) = \left(1-(1-\gamma)\delta e^{(1-r)t}\right)U(t) - D(t) \tag{2}$$

where (.)denotes differentiation. Now we consider the unit production cost as a function of produced-quantity, raw material cost, development cost and wear-tear cost. So unit production cost is given by

$$C_p(r,t) = \left(C_m + \frac{C_d(r)}{U(t)} + \sigma U(t)\right) \tag{3}$$

The final stock $X(T)$ is selling with salvage value price P_{s_1} where $P_{s_1} < P_s$ and the corresponding marginal revenue is $P_{s_1}X(T)$. Then the profit function is

$$\begin{aligned} J_1 &= \int_0^T \Big[P_sD(t) - C_hX(t) - C_mU(t) - C_d(r) - \sigma U(t)^2 \\ &- C_{rw}\gamma\delta\, e^{(1-r)t}U(t) - C_{op}U(t) - C_{pl}U(t) - C_ve^qV\Big]dt \\ &+ Ps_1X(T) \end{aligned}$$

At the time of production there is emanated a certain amount of gas in the production house which acts as pollutant and creates pollution. For this reason we introduce another objective function to calculate the amount of the created gas for an item is produced. We try to minimize that objective function. Now A_{plg} unit gas emitted per unit production and $A_{plg}U(t)$ unit gas is emitted for total production. A_{rwg} unit gas emitted for rework per unit rework. But after the rework the whole thing cannot be turned out usable but resulting wastage. Now A_{wg} unit gas emitted for wastage per unit wastage and $C_{wg}(1-\gamma)\delta e^{(1-R)t}U(t)$is emitted gas for wastage. Then the amount of polluting gas is emitted after the whole production is

$$J_2 \quad = \quad \int_0^T \Big[A_{plg}U(t) + A_{rwg}\delta\, e^{(1-r)t}U(t) + A_{wg}(1-\gamma)\delta\, e^{(1-r)t}U(t)\Big]dt$$

This is the realistic multi-objective inventory problem in which the profit function is maximized and the amount of polluting gas function is minimized. i.e. The problem is

$$\begin{aligned}
\text{Maximize } J_1 \quad &= \quad \int_0^T \Big[P_s D(t) - C_h X(t) - C_m U(t) - C_d(r) - \sigma U(t)^2 \\
&- \quad C_{rw}\gamma\delta\, e^{(1-r)t}U(t) - C_{op}U(t) - C_{pl}U(t) - C_v e^q V\Big]dt \\
&+ \quad Ps_1\, X(T) \qquad\qquad (4) \\
\text{Minimize } J_2 \quad &= \quad \int_0^T \Big[A_{plg}U(t) + A_{rwg}\delta\, e^{(1-r)t}U(t) + A_{wg}(1-\gamma)\delta\, e^{(1-r)t}U(t)\Big]dt \\
\text{subject to} \quad & \quad (2)
\end{aligned}$$

Follow the researchers like Maity and Maiti (cf. [18],[34]), we convert the above multi-objective problem in a single objective problem by weight sum method as $J = w_1 J_1 + w_2(-J_2)$ where $w_1 + w_2 = 1$, $w_1, w_2 \geq 0$.

3.3. OPTIMAL CONTROL POLICY

The above control problem is solved using Pontryagin's maximum principle([19]). Using this principle, we construct the Hamiltonian function as

$$\begin{aligned}
H &= w_1\Big(P_sD(t) - C_hX(t) - C_mU(t) - C_d(r) - \sigma U(t)^2 \\
&- C_{rw}\gamma\delta\, e^{(1-r)t}U(t) - C_{op}U(t) - C_{pl}U(t) - C_v e^q V\Big) \\
&- w_2\Big(A_{plg}U(t) + A_{rwg}\delta\, e^{(1-r)t}U(t) + A_{wg}(1-\gamma)\delta\, e^{(1-r)t}U(t)\Big) \\
&+ \lambda(t)\dot{X}(t) \\
&= w_1\Big(P_sD(t) - C_hX(t) - C_mU(t) - C_d(r) - \sigma U(t)^2 \\
&- C_{rw}\gamma\delta\, e^{(1-r)t}U(t) - C_{op}U(t) - C_{pl}U(t) - C_v e^q V\Big) \\
&- w_2\Big(A_{plg}U(t) + A_{rwg}\delta\, e^{(1-r)t}U(t) + A_{wg}(1-\gamma)\delta\, e^{(1-r)t}U(t)\Big) \\
&+ \lambda(t)[(1-(1-\gamma)\delta e^{(1-r)t})U(t) - D(t)] \qquad (5)
\end{aligned}$$

where adjoint variable $\lambda(t)$ is given by

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial X(t)} \qquad (6)$$

Solving the above differential equations (5)-(6) with boundary condition $\lambda(T) = P_{s_1}$, we get

$$\lambda(t) = P_{s_1} - w_1C_h(T-t) \qquad (7)$$

Putting the value of $\lambda(t)$ in (5), we get, the Hamiltonian of the system

$$\begin{aligned}
H &= w_1\Big(P_sD(t) - C_hX(t) - C_mU(t) - C_d(r) - \sigma U(t)^2 \\
&- C_{rw}\gamma\delta\, e^{(1-r)t}U(t) - C_{op}U(t) - C_{pl}U(t) - C_v e^q V\Big) \\
&- w_2\Big(A_{plg}U(t) + A_{rwg}\delta\, e^{(1-r)t}U(t) + A_{wg}(1-\gamma)\delta\, e^{(1-r)t}U(t)\Big) \\
&+ (P_{s_1} - w_1C_h(T-t))[(1-(1-\gamma)\delta e^{(1-r)t})U(t) - D(t)] \qquad (8)
\end{aligned}$$

Therefore, our objective is to find out the optimal path of $U^*(t)$ such that H is maximum *i.e.* J is maximum.

Differentiating Hamiltonian function H with respect to $U(t)$, we have

$$\begin{aligned}\frac{\partial H}{\partial U(t)} &= (P_{s_1} - w_1C_h(T-t))(1-(1-\gamma)\delta e^{(1-r)t})\\ &- w_1(2\sigma U(t) + C_m + C_{op} + C_{pl} + C_{rw}\gamma\delta e^{(1-r)t})\\ &- w_2(A_{plg} + A_{rwg}\delta e^{(1-r)t} + A_{wg}(1-\gamma)\delta e^{(1-r)t})\end{aligned}$$

For finding the optimal path $U^*(t)$,we know that

$$\frac{\partial H}{\partial U(t)} = 0 \tag{9}$$

i.e.

$$\begin{aligned}&\Big(P_{s_1} - w_1C_h(T-t)\Big)\Big(1-(1-\gamma)\delta e^{(1-r)t}\Big)\\ &- w_1\Big(2\sigma U(t) + C_m + C_{op} + C_{pl} + C_{rw}\gamma\delta e^{(1-r)t}\Big)\\ &-w_2\Big(A_{plg} + A_{rwg}\delta e^{(1-r)t} + A_{wg}(1-\gamma)\delta e^{(1-r)t}\Big) = 0\end{aligned}$$

implies that,

$$\begin{aligned}U^*(t) &= \sqrt{\frac{1}{2\alpha w_1}}\Bigg[\Big(P_{s_1} - w_1C_h(T-t)\Big)\Big(1-(1-\gamma)\delta e^{(1-r)t}\Big)\\ &- w_1\Big(2\sigma U(t) + C_m + C_{op} + C_{pl} + C_{rw}\gamma\delta e^{(1-r)t}\Big)\\ &- w_2\Big(A_{plg} + A_{rwg}\delta e^{(1-r)t} + A_{wg}(1-\gamma)\delta e^{(1-r)t}\Big)\Bigg].\end{aligned} \tag{10}$$

Using the above optimal production rate given in (10) in the equation (2), we obtain the optimal inventory level $X^*(t)$ in the interval [0,T] which is given by

$$\begin{aligned}X^*(T) &= \frac{1}{2\sigma w_1}\Bigg[At - \frac{Be^{(1-r)t}}{(1-r)} + \frac{Ce^{2(1-r)t}}{2(1-r)} - D\Big(\frac{te^{(1-r)t}}{(1-r)} - \frac{e^{(1-r)t}}{(1-r)^2}\Big)\\ &+ E\Big(\frac{te^{2(1-r)t}}{2(1-r)} - \frac{e^{2(1-r)t}}{4(1-r)^2}\Big) + w_1C_h\frac{t^2}{2} + \frac{B}{(1-r)} - \frac{C}{2(1-r)}\\ &- \frac{D}{(1-r)^2} + \frac{E}{4(1-r)^2}\Bigg] - (d_0t + \frac{qV}{1+V}\frac{t^2}{2}),\end{aligned}$$

where

$$A = P_{s_1} - w_1 C_h T - w_1(C_m + C_{op} + C_{pl}) - w_2 A_{plg}$$
$$B = (P_{s_1} - w_1 C_h T - w_1(C_m + C_{op} + C_{pl}) - w_2 A_{plg})\delta(1-\gamma)$$
$$+P_{s_1}\delta(1-\gamma) + w_2(\delta A_{rwg} + \delta A_{wg}(1-\gamma)) + w_1 C_{rw}\delta\gamma - w_1 C_h T\delta(1-\gamma)$$
$$C = \delta(1-\gamma)(P_{s_1}\delta(1-\gamma) + w_2(\delta A_{rwg} + \delta A_{wg}(1-\gamma)) - w_1 C_h T\delta(1-\gamma))$$
$$D = 2w_1 C_h \delta(1-\gamma)$$
$$E = w_1 C_h \delta^2(1-\gamma)^2$$

4. NUMERICAL ILLUSTRATION

To illustrate the above reliability dependent production inventory model numerically, we consider the following input datas given in Table−1. The demand rate, optimum production rate and corresponding stock are shown in figure −2. Optimal values of demand, production and stock level at time t are given in Table−2. Also the profit and amount of polluting gases at time t are shown in figure−3.

4.1. INPUT DATA

The input datas for inventory parameters are given in Table- 1.

Table 1. Input Data for inventory parameters (PAMES)

PAMES	values	PAMES	values	PAMES	values	PAMES	values
w_1	0.85	w_2	0.15	C_h	1	C_m	1.4
C_{pl}	0.5	C_{rw}	1.0	A_{plg}	0.55	A_{rwg}	0.25
L	230	M	10	P_s	75	P_{s_1}	60
T	28	r_{min}	0.45	r_{max}	0.9	δ	0.04
d_0	4.5	q	0.6	V	20	C_V	5
C_{op}	0.45	A_{wg}	0.1	k	0.6	γ	0.9
σ	0.4						

4.2. OPTIMUM RESULT

For the above crisp input data, the optimum production rate $U^*(t)$, optimum stock level $X^*(t)$ and demand rate $D(t)$ are shown in figure−2 respectively. Also optimum reliability, optimum profit and optimum gas emissions are $r^* = 0.82$, ${J_1}^* = 15471.40\$$ and ${J_2}^* = 1296.5$ *litre* respectively. The development cost and profit function, gases emission with respect to reliability are shown in figures−4,−5,−6 respectively. Also the profit function and gases emission with respect to rework is shown in figure−8. Marginal stock with respect to different parameters like reliability,defectiveness of the products, rework on imperfect products, holding cost are shown in figures−12,−13,−14,−15 respectively.

Table 2. Optimal values of demand, production and stock level at time t

t	0	2	4	6	8	10	12	14	16	18	20	22
$D(t)$	4.5	5.6	6.8	7.9	9.1	10.2	11.3	12.5	13.6	14.8	15.9	17.1
$U(t)$	84.7	84.6	84.3	84	83.4	82.8	81.7	80.2	78.1	74.9	70.5	64.1
$X(t)$	0	155.9	303.9	443.5	574.5	696.0	807.4	907.3	993.9	1065.1	1117.6	1147.4

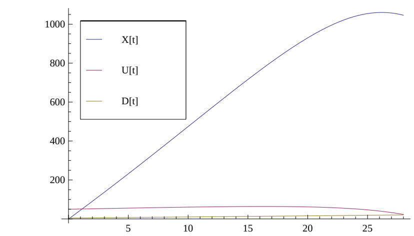

Fig.2. Optimum production rate, optimum stock rate and demand rate versus time

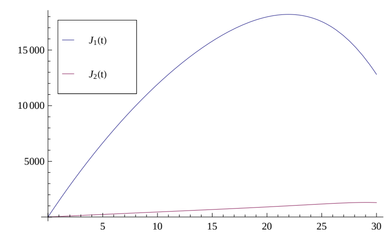

Fig.3. Profit function and gases emission versus time

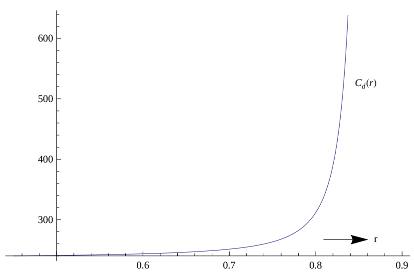

Fig.4. Development cost versus reliability

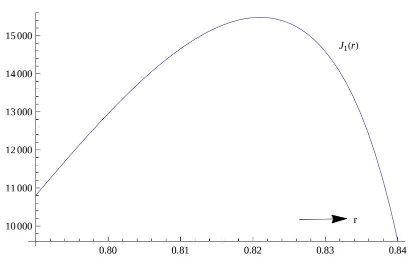

Fig.5. Profit function versus reliability

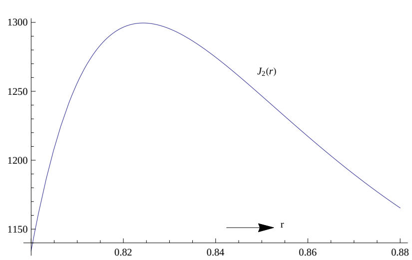

Fig.6. Gases emission versus reliability

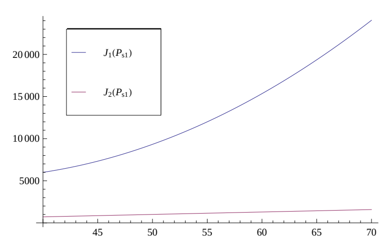

Fig.7. Profit function and gases emission versus marginal selling price

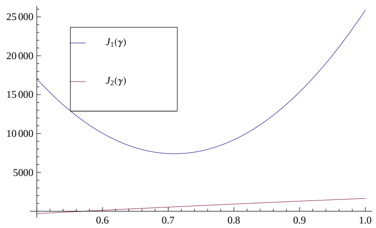

Fig.8. Profit function and gases emission versus rework

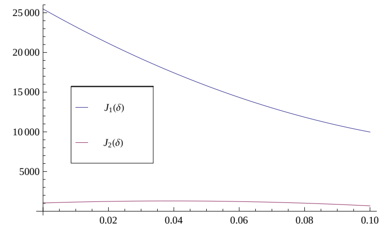

Fig.9. Profit function and gases emission versus defectiveness

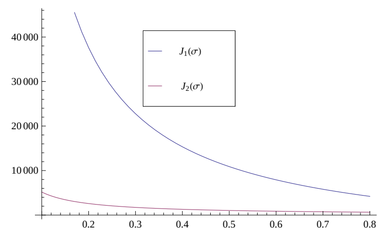

Fig.10. Profit function and gases emission versus wear-tear cost

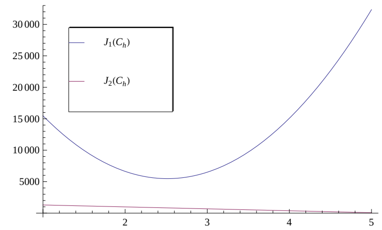

Fig.11. Profit function and gases emission versus holding cost

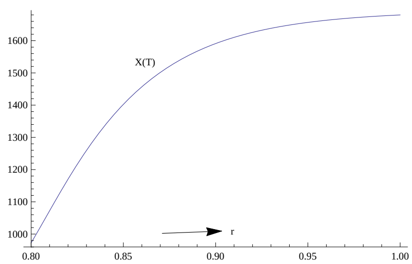

Fig.12. Marginal stock versus reliability

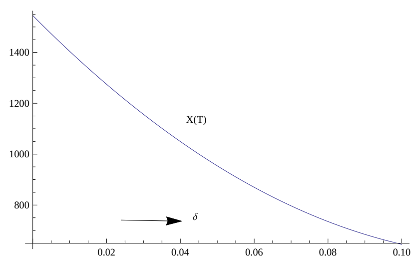

Fig.13. Marginal stock versus defectiveness

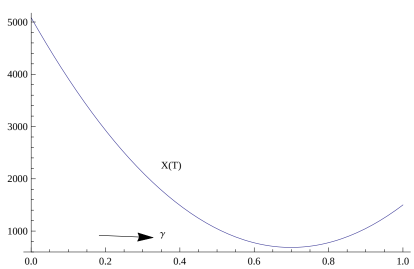

Fig.14. Marginal stock versus rework

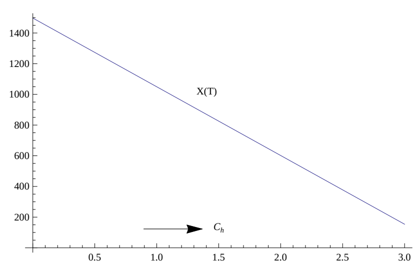

Fig.15. Marginal stock versus holding cost

4.3. DISCUSSION AND SENSITIVITY ANALYSIS

The sensitivity analysis for marginal selling price P_{s_1}, rework γ are given in tables−3 and −4. The table−5 is represented the sensitivity analysis of defective parameter. From the result in table−7, we observed that the development cost is increased for increased value of reliability and near the maximum value of reliability, development cost is very high shown in figure−4, so the profit shown in figure−5 is maximum to the reliability value $r = 0.82$.

From the results in Table−3, we observed that the profit is increased for increased value of marginal selling price P_{s_1} because of the increased value of marginal stock $X(T)$ and also the emission of the polluting gases are increased. It is shown in figure−7.

From the results in Table−4, we observed that the rework has positive impact to the profit function but the negative effect to the emission of the polluting gases. When we will done maximum rework then the profit is maximum and the amount of emission of the polluting gases are also maximum which is shown in figure−8.

From the results in Table−5, we observed that when the defective parameter δ is increased then the profit is decreased. The emission of polluting gases are also increased but after certain time it is also decreased which is shown in figure−9. The stock level is decreased for increased value of δ.

From the results in Table−6, we see that when the wear-tear cost is increased then the profit of the model is decreased as well as the the amount of polluting gases are also decreased. It is shown in figure−10.

From the results in Table−7, we observed that when the reliability is increased the profit is also increased and the emission of the gases are decreased (shown in figures−5 and −6). But after some time suddenly the marginal revenue is decreased and for this reason the profit is also decreased.

And also from the Table−8, we get for the increased holding cost the profit is decreased as well as the amount of polluting gases are also decreased which is shown in figure−11.

Based on the above results of this study suggest that in the lessons of economic profit and environmental issues, the industries should be given more attention on improvement of the quality of products, quality of machines. In the early stage of the economic growth have greatest impacts of development of the quality of products, quality of machines on the environment. Overall, this study suggests that the economy towards less polluting activities tend to be given to environmental concerns in different phases of economic growth reduces or abolishes the pace of economic growth. It also imports and production apparatus, improvement of factory and industry, the factors of pollutants into the environment are getting more attention.

Table 3. Sensitivity analysis for marginal cost P_{s_1}

$P_{s_1}(\$)$	45	50	55	60	66 70	
$X^*(T)$	558.9	722.5	885.9	1049.5	1213.04	1376.6
$J1^*(\$)$	7410.1	9423	12111	15471	19505.2	1585.2
$J2^*(litre)$	863.4	1007.8	1152.1	1296.5	1440.9	1585.2

Table 4. Sensitivity analysis for γ

γ	0.6	0.65	0.7	0.75	0.8	0.85	0.9
$X^*(T)$	775.7	708.95	687.15	710.3	778.42	891.5	1049.51
$J1^*(\$)$	10085.5	8245.58	7500.69	7850.82	9295.99	11836.2	15471.4
$J2^*(litre)$	123.6	330.1	532.3	729.9	923.3	1112.1	1296.54

Table 5. Sensitivity analysis for δ

δ	0	0.02	0.04	0.06	0.08	0.1
$X^*(T)$	1544.9	1274.6	1049.5	869.7	735	645.7
$J1^*(\$)$	25632.3	20245.3	15471.4	11310.8	7763.3	4828.9
$J2^*(litre)$	1042.2	1235.9	1296.5	1223.9	1018.3	679.4

Table 6. Sensitivity analysis for wear-tear cost σ

σ	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$X^*(T)$	2449	1516	1049.5	769.6	583	449.7	349.7
$J1^*(\$)$	37870.9	22937.9	15471.4	10991.5	8004.9	5871.6	4271.7
$J2^*(litre)$	2593	1728.7	1296.5	1037.2	864.4	740.9	648.3

Table 7. Sensitivity analysis for reliability r

R	0.79	0.80	0.81	0.82	0.83	0.84
$c_d(\$)$	293.9	311.7	340.2	390.4	489.7	724.0
$X^*(T)$	784.2	863.4	957.1	1049.5	1133.4	1206.3
$J1^*(\$)$	10806.8	12953.4	14656.8	15471.4	14582.6	9600.8
$J2^*(litre)$	860.7	1134.2	1256.1	1296.5	1295.4	1274.7

Table 8. Sensitivity analysis for holding cost C_h

σ	0.5	1	1.5	2	2.5	3
$X^*(T)$	1273.4	1049.5	825.6	601.6	377.7	153.8
$J1^*(\$)$	23151.7	15471.4	9966.6	6637.2	5483.1	6504.5
$J2^*(litre)$	1449.9	1296.5	1143.1	989.6	836.2	682.8

5. Practical Implementation

The relationship between economic growth and environmental pollution have been extensively investigated and analyzed in some developed countries. For this purpose, many industries have been already implemented high level technology eco-efficiency. At present, recycling economy, rework of imperfective goods, green supply chain models have been taken in almost every industry of developed / developing countries. The government of many countries have implemented the environmental legislation at par with technologically developed countries to force the industry to invest in green technologies. For this reason, we have developed mathematically a model to make the relationship between economic growth and environmental pollution for a imperfect production inventory control problem including rework and reliability. The figure-6 is shown that the maximum reliability is reduce the pollution gas. So the model is applicable in any production industry to reach our goal and save the environment.

6. Conclusion

In this research paper, first time we have developed the relationship between economical profit and environmental pollution of imperfect production inventory control problem. To establish the said relationship, various policies have been implemented to reduce the amount of polluting gases such as reliability development by modern technology on production system, rework on imperfective item and extra objective function to calculate the amount of polluting gases. In this model we observe that the reliability has positive effect to the profit function up to certain time and after this it has negative effect to the said profit function. At the same time, the amount of the polluting gases (i.e the second objective function) are monotonically decreasing with respect to the reliability of the production system. So the decision maker of each industry should take to implement the reliability of the production system in different way. The following suggestions and recommendations are proposed, based on the study, as an action plan for the decision/policy makers in government and industry to control the pollution in environment:

1. The government should put in place required environmental legislation at par with technologically developed countries to force the industry to invest in green technologies.
2. The government should build and upgrade necessary infrastructure to enforce the environmental legislation effectively.
3. The Government should also ensure the uniform environmental legislation in all states/regions of the country to stop companies from shifting the dirty manufacturing to places with lax environmental legislation.

The purpose of this paper is to determine for a dynamic constant advertisement dependent demand over a given period, a cost effective production plan and a maintenance policy with environmental requirements and regulations.

The investigation can be extended for future research work:
I. Multi-item optimal control problems where limits on t are imprecise/ uncertain.
II. Development of some optimal production- recycling/recovery models in inventory system that can reduce the environmental pollution, save the crisis of natural resources and raw materials in imprecise environment.
III. Development of some production reliability models in production inventory model in imprecise environment.

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