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Electromagnetic field of Surface waves propagation in fiber-reinforced generalized thermoelastic medium

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ABSTRACT. The objective of this paper is to investigate the surface waves in fibre-reinforced anisotropic elastic medium subjected to magnetic and thermal fields. We introduce the coupled theory (CD), Lord-Shulman (LS) theory and Green-Lindsay (GL) theory to study the influence of magnetic field on 2D problem of a fibre-reinforced thermoelastic. The analytical expressions for displacement components and force stress are obtained in the physical domain by using the harmonic vibrations. The wave velocity equations have been obtained in different cases. Numerical results for the temperature, displacement, and thermal stress components are given and illustrated graphically in the presence and absence of the magnetic field of the material medium. A comparison is also made between the three theories in the case of presence and absence of fiber-reinforced parameters.

Keywords: Lord-Shulman, Green-Lindsay, Fibre-reinforced, Surface waves, thermoelastic, magnetic field

1. INTRODUCTION

In the postwar years, we have seen a rapid development of themoelasticity stimulated by various engineering sciences. Most of investigations were done under the assumption of the temperature-independent material properties, which limited the applicability of the solutions obtained to certain ranges of temperature.

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At high temperature, the material characteristics, such as the modulus of elasticity, the Poisson'sratio, and the coefficient of thermal conductivity, are no longer constants. In recent years, due to the progress in various fields of science and technology, taking into consideration the real behaviour of the material characteristics becomes an actual necessity. In some investigations, they have been taken as functions of coordinates.

A reinforced concrete member should be designed for all conditions of stresses that may occur and in accordance with principles of mechanics. The characteristic property of a reinforced concrete member is that its components, namely concrete and steel, act together as a single unit as long as they remain in the elastic condition i.e. the two components are bounded together so that there can be no relative displacement between them. In the case of an elastic solid reinforced by a series of parallel fibers, it is usual to assume trans- verse isotropy. In the linear case, the associated constitutive relations, relating infinitesimal stress and strain components have five material constants. In the last three decades, the analysis of stress and deformation of fiber-reinforced composite materials has been an important research area of solid mechanics. The wave's propagation in a reinforced media plays a very interesting role in civil engineering and geophysics. The studies of propagation, reflection and transmission of waves are of great interest to seismologists. Such studies help them to obtain knowledge about the rock structures as well as their elastic properties and at the same time information regarding minerals and fluids present inside the earth. The idea of introducing a continuous self reinforcement at every point of an elastic solid was given by Belfied et al. [1]. The model was later applied to the rotation of a tube by Verma and Rana [2]. Verma [3] has also discussed the magneto elastic shear waves in self-reinforced bodies. Sengupta and Nath [4] discussed the problem of the surface waves in fibre-reinforced anisotropic elastic media. Singh [5] showed that, for wave propagation in fibre-reinforced anisotropic media, this decoupling can not be achieved by the introduction of the displacement potential. Hashin and Rosen [6] gave the elastic moduli for fibre-reinforced materials. The problem of reflection of plane waves at the free surface of a fibre-reinforced elastic half-space was discussed by Singh and Singh [7]. Chattopadhyay and Choudhury [8] have discussed the problem of propagation, reflection and transmission of magneto elastic shear waves in a self-reinforced medium. The reflection and transmission of plane SH wave through a self-reinforced elastic layer sandwiched between two homogeneous viscoelastic solid half-spaces has been studied by Chaudhary et al. [9] . Chattopadhyay and Chaudhary [10] studied the propagation of magneto-elastic shear waves in an infinite self-reinforced plate. Pradhan et al. [11] studies the dispersion of Loves waves in a self-reinforced layer over an elastic non-homogenous half space. The propagation of plane waves in fibre-reinforced media is discussed by Chattopadhyay et al. [12]. The theory of couple thermo-elasticity was extended by Lord and Shulman [13] and Green and Lindsay [14] by including the thermal relaxation time in constitutive relations. These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermoelasticity.

mMore realistic elastic model since earth, moon and other plants have angular velocity.
Othman and Song [15] showed the effect of initial stress, thermoelastic parameter and thermal boundary condition upon the reflection amplitude ratios. The problem of magneto-elastic transverse surfaces waves in self-reinforced elastic solid was studied by Verma et al. [16]. The problem of wave propagation in thermally conducting linear fibre-reinforced composite materials was discussed by Singh [17].

Othman and Lotfy [18] studied two-dimensional problem of generalized magneto-thermoelasticity under the effect of temperature dependent properties. Othman et al. [19] studied transient disturbance in a half-space under generalized magneto-thermoelasticity with moving internal heat source. Othman and Lotfy [20] studied the plane waves in generalizedthermo-microstretch elastic half-space by using a general model of the equations of generalized thermo-microstretch for a homogeneous isotropic elastic half space. Othman and Lotfy [21] studied the generalized thermo-microstretch elastic medium with temperature dependent properties for different theories. This study included some discussion on the free-surface phenomenon in a rotating half-space. Results concerning slowness surfaces, energy flux, reflected waves, and generalized Rayleigh waves have been obtained. Later on, several authors[22–25] studied plane waves in rotating thermoelastic and magneto– thermoelastic media in the context of generalized theories. The normal mode analysis was used to obtain the exact expression for the temperature distribution, thermal stresses, and the displacement components.

Recently, Abd-Alla and Abo-Dahab [26] investigated the rotation and initial stress effects on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity. Many authors [27-32] discusses the dual phase lag model on magneto-thermoelasticity and fibre reinforced infinite non-homogeneous solid having a spherical cavity.

The present investigation, we shall formulate the fiber-reinforced two-dimensional problem under the effect of magnetic field and solving for the considered variables leading to particular cases such as Rayleigh waves and Stoneley waves. The displacement, temperature and stress are obtained in the physical domain by using the harmonic vibrations. The distributions of the considered variables are represented graphically. A comparison is carried out between the temperature, stresses and displacements as calculated from the generalized thermoelasticity LS, GL and coupled theory in the presence and absence of magnetic field. A comparison also is made between the three theories in the presence and absence of fiber-reinforced.

2. Formulation of the problem and basic equations

We consider the problem of a thermoelastic half space $(x \geq 0)$. A magnetic field with constant intensity $\mathbf{H} = (0, 0, H_0)$, acting parallel to the boundary plane (taken as the direction of the z-axis). The surface of the half space is subjected to a thermal shock which is a function of y and t. Thus, all quantities considered will be functions of the time variable t, and of the coordinates x and y.

Let $\coprod_1$ and $\coprod_2$ be two fibers-reinforced elastic semi-infinite solid media. They are perfectly welded in contact to prevent any relative motion or sliding before and after the disturbances and that the continuity of displacement, stress etc. hold good across the common boundary surface. Further the mechanical properties of $\coprod_1$ are different from those of $\coprod_2$. These media extend to an infinite great distance from the origin and are separated by a plane horizontal boundary and $\coprod_2$ is to be taken above $\coprod_1$. These two assumptions conclude that the wave is a surface wave and all partial derivatives with respect to z are zero.

Further let us assume that u, v are the components of displacements at any point (x,y,z) at any time t. We begin our consideration with linearized of electro-dynamics slowly moving medium

$$\mathbf{J} = \operatorname{curl} \mathbf{h} - \varepsilon_0 \dot{\mathbf{E}}, \tag{1}$$

$$\operatorname{curl} \mathbf{E} = -\mu_0 \dot{\mathbf{h}}, \tag{2}$$

$$\mathbf{E} = -\mu_0 (\dot{\mathbf{u}} \times \mathbf{H}), \tag{3}$$

$$\nabla . \mathbf{h} = 0 \tag{4}$$

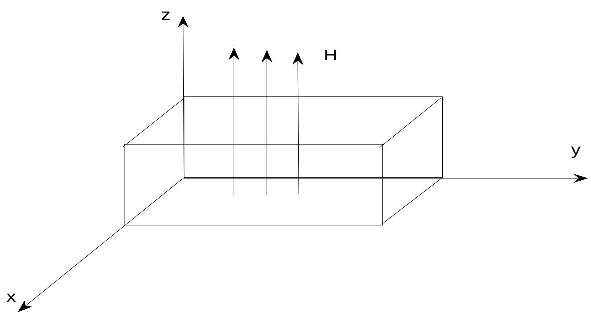

Fig. a. Schematic of the problem

where, μ_0 is the magnetic permeability, ε_0 is electric permeability, $\dot{\mathbf{u}}$ is the particle velocity of the medium, and the small effect of temperature gradient on $\mathbf{J}$ is also ignored.

These equations are supplemented by the constitutive equations for a fibre-reinforced linearly thermoelastic medium with respect to the reinforcement direction $\mathbf{a}$ are

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta ij + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) +$$

$$\beta a_k a_m e_{km} a_i a_j - \gamma_{ij}(1 + \nu_0 \frac{\partial}{\partial t})(T - T_0)\delta_{ij}. \tag{5}$$

where, σ_{ij} are components of stress, e_{ij} are the components of strain, λ and μ_1 are elastic constants, α, β, $(\mu_L - \mu_T), \gamma_{ij}$ are reinforcement parameters, and $\mathbf{a} \equiv (a_1, a_2, a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fibre-direction as $\underline{a} \equiv (1, 0, 0)$. The strains can be expressed in terms of the displacement u_i as

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{6}$$

For plane strain deformation in the xy-plane equation (5) then yields to

$$\sigma_{xx} = A_{11}u_{,x} + A_{12}\text{v}_{,y} - \gamma_{11}(1+\nu_0 \frac{\partial}{\partial t})(T - T_0), \tag{7}$$

$$\sigma_{yy} = A_{22}\text{v}_{,y} + A_{12}u_{,x} - \gamma_{22}(1+\nu_0 \frac{\partial}{\partial t})(T - T_0), \tag{8}$$

$$\sigma_{zz} = A_{12}u_{,x} + \lambda\text{v}_{,y} - \gamma_{33}(1+\nu_0 \frac{\partial}{\partial t})(T - T_0), \tag{9}$$

$$\sigma_{xy} = \mu_L(u_{,y} + v_{,x}), \qquad \sigma_{zx} = \sigma_{zy} = 0 \tag{10}$$

where

$$A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta, \qquad A_{12} = \alpha + \lambda,$$

$$A_{22} = \lambda + 2\mu_T. \tag{11}$$

The equation of motion, taking into consideration the Lorentz force guess

$$\rho\,\ddot{u}_i = \sigma_{ij,j} + \mu_0(\mathbf{J} \times \mathbf{H}) \qquad i, j = 1, 2, 3 \tag{12}$$

The dynamic displacement vector is actually measured from a steady state deformed position and the deformation is supposed to be small. Due to the application of the initial magnetic field **H,** there results an induced magnetic field $\mathbf{h} = (0, 0, h)$ and an induced electric field **E,** as well as the simplified equations of electro-dynamics of slowly moving medium for a homogeneous, thermal and electrically conducting elastic solid.

The heat conduction equation in general formula is

$$kT_{,ii} = \rho C_E (n_1 + \tau_0 \frac{\partial}{\partial t})\dot{T} + \gamma T_0 (n_1 + n_0 \tau_0 \frac{\partial}{\partial t})\dot{e} \tag{13}$$

where, k is thermal conductivity, ρ is density, C_E is the specific heat at constant strain, T is the temperature above reference temperature T_0, n_1 and n_0 are parameters. In the above equations a dot denotes differentiation with respect to time, and comma followed by a subscript denotes partial differentiation with respect to the corresponding coordinates. Expressing the components of the vector $\mathbf{J} = (J_1, J_2, J_3)$ in terms of displacement, where

$J_1 = H_0(-\dfrac{\partial e}{\partial y} + \mu_0\,\varepsilon_0\,\ddot{v})$, $\quad J_2 = H_0(\dfrac{\partial e}{\partial x} - \mu_0\,\varepsilon_0\,\ddot{u})$ and $J_3 = 0$, by eliminating the quantities $\mathbf{h}$ and $\mathbf{E}$ from equation (1).

3. Solution of the problem

Using the summation convection. From (7)-(10) we note that the third equation of motion in (12) is identically satisfied and first two equation become

$$\rho\left(\frac{\partial^2 u}{\partial t^2}\right) = A_{11}\frac{\partial^2 u}{\partial x^2} + B_2\frac{\partial^2 \mathrm{v}}{\partial x \partial y} + B_1\frac{\partial^2 u}{\partial y^2} - \gamma(1+\nu_0\frac{\partial}{\partial t})\frac{\partial T}{\partial x} - \mu_0 H_0\frac{\partial h}{\partial x} - \varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}, \tag{14}$$

$$\rho\left(\frac{\partial^2 \mathrm{v}}{\partial t^2}\right) = A_{22}\frac{\partial^2 \mathrm{v}}{\partial y^2} + B_2\frac{\partial^2 u}{\partial x\,\partial y} + B_1\frac{\partial^2 \mathrm{v}}{\partial x^2} - \gamma(1+\nu_0\frac{\partial}{\partial t})\frac{\partial T}{\partial y}\,\mu_0 H_0\frac{\partial h}{\partial y} - \varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 \mathrm{v}}{\partial t^2} \tag{15}$$

where, $\quad B_1 = \mu_L, \qquad\qquad B_2 = \alpha + \lambda + \mu_L.$

For convenience, the following non-dimensional variables are used:

$x' = c_1\eta x, \qquad y' = c_1\eta\, y, \qquad u' = c_1\eta\, u, \qquad v' = c_1\eta\, v,$
$t' = c_1^2\eta\, t,$

$$\theta=\gamma\frac{(T-T_0)}{(\lambda+2\mu_T)},\quad \sigma'_{ij}=\frac{\sigma_{ij}}{\mu_T},\quad \tau'_0=c_1^2\eta\tau_0,\quad h'=\frac{h}{H_0}.\ i,j=1,2 \quad (16)$$

where, $\eta=\dfrac{\rho C_E}{K}$, $c_1^2=\dfrac{\lambda+2\mu_T}{\rho}$.

In terms of non-dimensional quantities defined in Eq. (16), the above governing equations reduce to (dropping the dashed for convenience)

$$\alpha_1\frac{\partial^2 u}{\partial t^2}=h_{11}\frac{\partial^2 u}{\partial x^2}+h_2\frac{\partial^2 v}{\partial x\partial y}+h_1\frac{\partial^2 u}{\partial y^2}-(1+\nu_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial x}-h_0\frac{\partial h}{\partial x},$$

(17) $$\alpha_1\frac{\partial^2 v}{\partial t^2}=h_{22}\frac{\partial^2 v}{\partial y^2}+h_2\frac{\partial^2 u}{\partial x\,\partial y}+h_1\frac{\partial^2 v}{\partial x^2}-(1+\nu_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial y}-h_0\frac{\partial h}{\partial y}$$

(18)
where

$$(h_{11},\ h_{22},\ h_1,h_2,\ h_0)=\frac{(A_{11},\ A_{22},\ B_1,\ B_2,\ \mu_0 H_0^2)}{(\lambda+2\mu_T)},$$

$$\alpha_1=1+\frac{\varepsilon_0\,\mu_0^2 H_0^2}{\rho},$$

$$\left(L_{11},L_{22},L_1,L_2\right)=\left(h_{11},h_{22},h_1,h_2\right)+h_0,\qquad \varepsilon=\frac{\gamma^2 T_0}{\rho C_E\left(\lambda+2\mu_T\right)}.$$

Taking $h=-H_0 e$ and using Eq. (13) and substituting into Eqs. (17) and (18) we obtain the following wave equation for $\amalg_1$ satisfied by u and v as we get

$$\alpha_1\frac{\partial^2 u}{\partial t^2}=L_{11}\frac{\partial^2 u}{\partial x^2}+L_2\frac{\partial^2 \mathrm{v}}{\partial x\partial y}+h_1\frac{\partial^2 u}{\partial y^2}-(1+\nu_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial x}, \qquad (19)$$

$$\alpha_1\frac{\partial^2 \mathrm{v}}{\partial t^2}=L_{22}\frac{\partial^2 \mathrm{v}}{\partial y^2}+L_2\frac{\partial^2 u}{\partial x\partial y}+h_1\frac{\partial^2 \mathrm{v}}{\partial x^2}-(1+\nu_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial y}, \qquad (20)$$

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=\left(n_1\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\theta+\varepsilon\left(n_1\frac{\partial}{\partial t}+n_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}\right) \qquad (21)$$

$$\mu_T\sigma_{xx}=A_{11}u_{,x}+A_{12}v_{,y}-A_{22}(1+v_0\frac{\partial}{\partial t})\theta, \qquad (22)$$

$$\mu_T\sigma_{yy}=A_{22}v_{,y}+A_{12}u_{,x}-A_{22}(1+v_0\frac{\partial}{\partial t})\theta, \qquad (23)$$

$$\mu_T\sigma_{zz}=A_{12}u_{,x}+\lambda v_{,y}-A_{22}(1+v_0\frac{\partial}{\partial t})\theta, \qquad (24)$$

$\mu_T\sigma_{xy}=\mu_L(u_{,y}+v_{,x}),$ $\qquad\qquad$ $\sigma_{zx}=\sigma_{zy}=0.$ and

similar relations in $\coprod_2$ with $\rho,\lambda,\mu,\alpha,\gamma,\mu_T,\mu_L,\gamma,u,v,\theta,\sigma_{ij}$ replaced by $\rho',\lambda',\mu',\alpha',\gamma',\mu_T',\mu_L',\gamma',u',\mathrm{v}',\theta',\sigma_{ij}'$

To get the exact solutions without any assumed restrictions on temperature, displacement and stress distributions. The solution of the considered physical variable can be decomposed in terms as the following form

$$\begin{aligned}
u(x,y,t)&=u^*(x)\exp i\,\omega(y-ct),\\
\mathrm{v}(x,y,t)&=\mathrm{v}^*(x)\exp i\,\omega(y-ct),\\
\theta(x,y,t)&=\theta^*(x)\exp i\,\omega(y-ct),\\
\sigma_{ij}(x,y,t)&=\sigma_{ij}^*(x)\exp i\,\omega(y-ct)
\end{aligned} \qquad (25)$$

where, $u^*(x)$, $\mathrm{v}^*(x)$, $\theta^*(x)$, and $\sigma_{ij}^*(x)$ are undetermined amplitude vectors of the field quantities. Since ω is the circular frequency and the wave velocity c obtained from Eq. (25) depends on the particular value of ω which indicates to the dispersion of the general wave form and on the magnetic field, imposing a certain changes in the waves form.

Substituting from Eq. (5) into Eqs. (19)-(24) we get set of differential equations for medium $\coprod_1$ as follows

$$[L_{11}D^2 - A_1]u^* + [i\omega L_2 D]v^* = QD\theta^*, \tag{26}$$

$$[h_1 D^2 - A_2]v^* + [i\omega L_2 D]u^* = i\omega Q\theta^*, \tag{27}$$

$$A_3 Du^* + i\omega A_3 v^* = [D^2 - A_4]\theta^*, \tag{28}$$

$$\mu_T \sigma^*_{xx} = A_{11}Du^* + i\omega A_{12}v^* - A_{22}Q\theta^*, \tag{29}$$

$$\mu_T \sigma^*_{yy} = A_{12}Du^* + i\omega A_{22}v^* - A_{22}Q\theta^*, \tag{30}$$

$$\mu_T \sigma^*_{zz} = A_{12}Du^* + i\omega\lambda v^* - A_{22}Q\theta^*, \tag{31}$$

$$\mu_T \sigma^*_{xy} = \mu_L (i\omega u^* + Dv^*), \qquad \sigma^*_{zx} = \sigma^*_{zy} = 0 \tag{32}$$

where

$$A_1 = h_1\omega^2 - \alpha_1\omega^2 c^2, \qquad A_2 = iL_{22}\omega^2 - \alpha_1\omega^2 c^2, \qquad Q = 1 - i\omega c v_0,$$

$$A_3 = i\omega c\varepsilon(n_1 - i\omega c n_0 \tau_0), \qquad A_4 = \omega^2 + i\omega c(n_1 + i\omega c\tau_0), \qquad D = \frac{d}{dx}.$$

Eliminating $\theta^*(x)$ and $v^*(x)$ between Eqs. (26)-(28), we obtain the partial differential equation satisfied by $u^*(x)$

$$[D^6 - AD^4 + BD^2 - C]u^*(x) = 0 \tag{33}$$

where

$$A = \frac{1}{h_1 h_{11}}\left\{L_{11}A_2 + h_1 A_1 + h_1 A_3 Q + h_1 L_{11} A_3 - L_2^2\omega^2\right\}, \tag{34}$$

$$B = \frac{1}{h_1 h_{11}}\left\{A_1 A_2 + L_{11}A_2 A_3 + h_1 A_1 A_3 + Q(L_{11}\omega^2 A_4 + A_2 A_4 - 2L_2\omega^2 A_4) - L_2^2\omega^2 A_3\right\}, \tag{35}$$

$$C = \frac{1}{h_1 L_{11}}\left\{A_1 A_2 A_3 + A_1 A_4 \omega^2 Q\right\}. \tag{36}$$

In a similar manner, we can show that $v^*(x)$ and $\theta^*(x)$ satisfy the equation

$$[\,D^6 - AD^4 + BD^2 - C\,]\left\{ v^*(x),\ \theta^*(x)\right\} = 0. \tag{37}$$

It can be factorized as

$$\left(D^2 - k_1^2\right)\left(D^2 - k_2^2\right)\left(D^2 - k_3^2\right) u^*(x) = 0 \tag{38}$$

where, k_n^2, $(n = 1, 2, 3)$ are the roots of the following characteristic equation

$$K^6 - AK^4 + BK^2 - C = 0. \tag{39}$$

where, $u^*(x)$, $v^*(x)$ and $\theta^*(x)$ will describe surface waves, they must become vanishingly small as $x \to \infty$.

Since the attenuation coefficient is defined as

$$Q_i = \operatorname{Im} g(k_n) \qquad n = 1,2,3$$

The solution of Eq. (33) which is bounded as $x \to \infty$, for medium $\coprod_1$ is given by

$$u^*(x) = \sum_{n=1}^{3} M_n(c,\omega)\exp(-k_n x). \tag{40}$$

Similarly

$$v^*(x) = \sum_{n=1}^{3} M_n'(c,\omega)\exp(-k_n x), \tag{41}$$

$$\theta^*(x) = \sum_{n=1}^{3} M_n''(c,\omega)\exp(-k_n x) \tag{42}$$

where, M_n, M_n' and M_n'' are some parameters depending on c and ω.

Substituting from Eqs. (41) and (42) into Eqs. (26)-(28), we get

$$M'_n(c,\omega)=H_{1n}M_n(c,\omega), \qquad n=1,2,3. \tag{43}$$

$$M''_n(c,\omega)=H_{2n}M_n(c,\omega), \qquad n=1,2,3. \tag{44}$$

Thus, we have

$$\mathrm{v}^*(x)=\sum_{n=1}^{3}H_{1n}\,M_n(c,\omega)\exp(-k_n x), \tag{45}$$

$$\theta^*(x)=\sum_{n=1}^{3}H_{2n}M_n(c,\omega)\exp(-k_n x), \tag{46}$$

$$\sigma_{xx}^*=\sum_{n=1}^{3}H_{3n}\,M_n(c,\omega)\exp(-k_n x), \tag{47}$$

$$\sigma_{yy}^*=\sum_{n=1}^{3}H_{4n}\,M_n(c,\omega)\exp(-k_n x), \tag{48}$$

$$\sigma_{zz}^*=\sum_{n=1}^{3}H_{5n}\,M_n(c,\omega)\exp(-k_n x), \tag{49}$$

$$\sigma_{xy}^*=\sum_{n=1}^{3}H_{6n}\,M_n(a,\omega)\exp(-k_n x) \tag{50}$$

where

$$H_{1n}=\frac{-i\,\omega L_{11}k_n^2+i\,\omega A_1+i\,\omega L_2k_n^2}{L_2k_n\omega^2+h_1k_n^3-A_2k_n}, \qquad n=1,2,3, \tag{51}$$

$$H_{2n}=\frac{-L_{11}k_n^2+A_1+i\,\omega L_2k_nH_{1n}}{k_n}, \qquad n=1,2,3, \tag{52}$$

$$H_{3n}=\frac{-A_{11}k_n+i\,\omega A_{12}H_{1n}-(\lambda+2\mu_T)H_{2n}}{\mu_T}, \qquad n=1,2,3, \tag{53}$$

$$H_{4n}=\frac{-A_{12}k_n+i\,\omega A_{22}H_{1n}-(\lambda+2\mu_T)H_{2n}}{\mu_T}, \qquad n=1,2,3, \tag{54}$$

$$H_{5n}=\frac{-A_{12}k_n+i\,\omega\lambda H_{1n}-(\lambda+2\mu_T)H_{2n}}{\mu_T}, \qquad n=1,2,3, \tag{55}$$

$$H_{6n}=\frac{\mu_l(i\omega-k_nH_{1n})}{\mu_T}, \qquad n=1,2,3. \tag{56}$$

Also for medium $\amalg_2$, set differential equations for medium $\amalg_2$ as follow

$$[L'_{11}D^2 - A'_1]u'^* + [i\omega L'_2 D]\mathrm{v}'^* = QD\theta'^*, \tag{57}$$

$$[h'_1 D^2 - A'_2]\mathrm{v}'^* + [i\omega L'_2 D]u'^* = i\omega Q\theta'^*, \tag{58}$$

$$A_3 Du'^* + i\omega A_3 \mathrm{v}'^* = [D^2 - A_4]\theta'^*, \tag{59}$$

$$\mu'_T \sigma'^*_{xx} = A'_{11}D' u^* + i\omega A'_{12}\mathrm{v}'^* - A'_{22}Q\theta'^*, \tag{60}$$

$$\mu'_T \sigma'^*_{yy} = A'_{12}Du'^* + i\omega A'_{22}\mathrm{v}'^* - A'_{22}Q\theta'^*, \tag{61}$$

$$\mu'_T \sigma'^*_{zz} = A'_{12}Du'^* + i\omega\lambda'\,\mathrm{v}'^* - A'_{22}Q\theta'^*, \tag{62}$$

$$\mu'_T \sigma'^*_{xy} = \mu'_L(i\,\omega u'^* + D\,\mathrm{v}'^*), \qquad \sigma^*_{zx} = \sigma^*_{zy} = 0 \tag{63}$$

where

$$A'_1 = h'_1\omega^2 - \alpha_1\omega^2 c^2, \qquad A'_2 = iL'_{22}\omega^2 - \alpha'_1\omega^2 c^2.$$

Eliminating $\theta'^*(x)$ and $\mathrm{v}'^*(x)$ between Eqs. (57)-(59), we obtain the partial differential equation satisfied by $u'^*(x)$ for medium $\amalg_2$ we get

$$[D^6 - A'D^4 + B'D^2 - C']u'^*(x) = 0 \tag{64}$$

where

$$A' = \frac{1}{h'_1 h'_{11}}\left\{L'_{11}A'_2 + h'_1 A'_1 + h'_1 A_3 Q + h'_1 L'_{11} A_3 - L'^2_2\omega^2\right\}, \tag{65}$$

$$B' = \frac{1}{h_1' h_{11}'} \left\{ A_1' A_2' + L_{11}' A_2' A_3 + h_1' A_1' A_3 + Q(L_{11}' \omega^2 A_4 + A_2' A_4 - 2L_2' \omega^2 A_4) - L_2'^2 \omega^2 A_3 \right\}, \quad (66)$$

$$C' = \frac{1}{h_1' L_{11}'} \left\{ A_1' A_2' A_3 + A_1' A_4 \omega^2 Q \right\}. \quad (67)$$

In a similar manner, we can show that $\theta'^*(x)$ and $\mathrm{v}'^*(x)$, satisfy the equation

$$[D^6 - A' D^4 + B' D^2 - C'] \left\{ \mathrm{v}'^*(x), \theta'^*(x) \right\} = 0. \quad (68)$$

The above equation can be factorized as

$$\left(D^2 - k_1'^2\right)\left(D^2 - k_2'^2\right)\left(D^2 - k_3'^2\right) u'^*(x) = 0 \quad (69)$$

where, $k_n'^2\ (n = 1,2,3)$ are the roots of the following characteristic equation

$$k'^6 - A' k'^4 + B' k'^2 - C' = 0 \quad (70)$$

where $\mathrm{u}'^*(x)$, $\mathrm{v}'^*(x)$, and $\theta'^*(x)$ will describe surface waves, they must become vanishingly small as $\mathrm{x} \to \infty$. The solution of Eq. (64) which is bounded as $\mathrm{x} \to \infty$, for medium $\coprod_2$ is given by

$$u'^*(x) = \sum_{n=1}^{3} \tilde{M}_n(c, \omega) \exp(-k_n' x) \quad (71)$$

Similarly

$$\mathrm{v}'^*(x) = \sum_{n=1}^{3} \tilde{M}_n'(c, \omega) \exp(-k_n' x) \quad (72)$$

$$\theta'^*(x) = \sum_{n=1}^{3} \tilde{M}_n''(c, \omega) \exp(-k_n' x) \quad (73)$$

Where $\tilde{M}_n$, $\tilde{M}_n'$ and $\tilde{M}_n''$ are some parameters depending on c and ω for medium $\coprod_2$

Substituting from Eqs. (72)-(73) into Eqs. (57)-(59), we get

$$\tilde{M}'_n(c,\omega) = H'_{1n}\tilde{M}_n(c,\omega), \qquad n = 1,2,3, \tag{74}$$

$$\tilde{M}''_n(c,\omega) = H'_{2n}\tilde{M}_n(c,\omega), \qquad n = 1,2,3. \tag{75}$$

Thus, we have

$$\mathrm{v}'^{*}(x) = \sum_{n=1}^{3} H'_{1n}\,\tilde{M}_n(c,\omega)\exp(-k'_n x), \tag{76}$$

$$\theta'^{*}(x) = \sum_{n=1}^{3} H'_{2n}\tilde{M}_n(c,\omega)\exp(-k'_n x), \tag{77}$$

$$\sigma'^{*}_{xx} = \sum_{n=1}^{3} H'_{3n}\,\tilde{M}_n(c,\omega)\exp(-k'_n x), \tag{78}$$

$$\sigma'^{*}_{yy} = \sum_{n=1}^{3} H'_{4n}\,\tilde{M}_n(c,\omega)\exp(-k'_n x), \tag{79}$$

$$\sigma'^{*}_{zz} = \sum_{n=1}^{3} H'_{5n}\,\tilde{M}_n(c,\omega)\exp(-k'_n x), \tag{80}$$

$$\sigma'^{*}_{xy} = \sum_{n=1}^{3} H'_{6n}\,\tilde{M}_n(a,\omega)\exp(-k'_n x) \tag{81}$$

where

$$H'_{1n} = \frac{-i\,\omega L'_{11}k'^2_n + i\,\omega A'_1 + i\,\omega L'_2 k'^2_n}{L'_2 k'_n\omega^2 + h'_1 k'^3_n - A'_2 k'_n}, \qquad n = 1,2,3, \tag{82}$$

$$H'_{2n} = \frac{-L'_{11}k'^2_n + A'_1 + i\,\omega L'_2 k'_n\, H'_{1n}}{k'_n}, \qquad n = 1,2,3, \tag{83}$$

$$H_{3n} = \frac{-A'_{11}k'_n + i\,\omega A'_{12}H'_{1n} - (\lambda' + 2\mu'_T)H'_{2n}}{\mu'_T}, \qquad n = 1,2,3, \tag{84}$$

$$H'_{4n} = \frac{-A'_{12}k'_n + i\,\omega A'_{22}H'_{1n} - (\lambda' + 2\mu'_T)H'_{2n}}{\mu'_T}, \qquad n = 1,2,3, \tag{85}$$

$$H'_{5n} = \frac{-A'_{12}k'_n + i\,\omega\lambda' H'_{1n} - (\lambda' + 2\mu'_T)H'_{2n}}{\mu'_T}, \qquad n = 1,2,3, \tag{86}$$

$$H'_{6n} = \frac{\mu'_l(i\,\omega - k'_n H'_{1n})}{\mu'_T}, \qquad n = 1,2,3. \tag{87}$$

The solution for the medium $\coprod_1$ and $\coprod_2$ for the physical components takes the form

$$\{u, \mathrm{v}, \theta\}(x, y, t) = \{\sum_{n=1}^{3} M_n(c, \omega)\exp(-k_n x), \sum_{n=1}^{3} H_{1n} M_n(c, \omega)\exp(-k_n x), \sum_{n=1}^{3} H_{2n} M_n(c, \omega)\exp(-k_n x)\}\exp i\,\omega(y - ct), \tag{88}$$

$$\{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}\}(x, y, t) = \{\sum_{n=1}^{3}[H_{3n}, H_{4n}, H_{5n}, H_{6n}]M_n(a, \omega)\exp(-k_n x)\} \quad \exp i\,\omega(y - ct), \tag{89}$$

$$\{u', \mathrm{v}', \theta'\}(x, y, t) = \{\sum_{n=1}^{3} \tilde{M}_n(c, \omega)\exp(-k'_n x), \sum_{n=1}^{3} H'_{1n} \tilde{M}_n(c, \omega)\exp(-k'_n x), \sum_{n=1}^{3} H'_{2n} \tilde{M}_n(c, \omega)\exp(-k'_n x)\}\exp i\,\omega(y - ct), \tag{90}$$

$$\{\sigma'_{xx}, \sigma'_{yy}, \sigma'_{zz}, \sigma'_{xy}\}(x, y, t) = \{\sum_{n=1}^{3}[H'_{3n}, H'_{4n}, H'_{5n}, H'_{6n}]\tilde{M}_n(a, \omega)\exp(-k'_n x)\} \quad \exp i\,\omega(y - ct). \tag{91}$$

4. Applications

4.1. Thermal shock problem

To investigate the possibility of thermal shock in anisotropic fibre-reinforced elastic media, we replace medium $\coprod_2$ by a vacuum, in the preceding problem. Since the boundary z = 0 is adjacent to vacuum, it is free from surface traction. So thermal boundary and the stress boundary condition in this case may be expressed as:

1) Thermal boundary conditions that the surface of the half-space subjected to thermal shock

$$\theta(x, y, t) = f(y, t), \qquad at \quad x = 0. \tag{92}$$

2) Mechanical boundary condition that surface of the half-space is traction free

$$\sigma_{xx}(0,y,t)=\sigma_{xy}(0,y,t)=0. \tag{93}$$

Substituting the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{n=1}^{3} H_{2n}M_n = f^*, \tag{94}$$

$$\sum_{n=1}^{3} H_{3n}M_n = 0, \tag{95}$$

$$\sum_{n=1}^{3} H_{6n}M_n = 0. \tag{96}$$

Solving Eqs.(94)-(96), we get the parameters $M_n (n=1,2,3)$ are defined as the follow

$$M_1=\frac{\Delta_1}{\Delta},\qquad M_2=\frac{\Delta_2}{\Delta},\qquad M_3=\frac{\Delta_3}{\Delta} \tag{97}$$

where

$$\Delta=H_{21}(H_{32}H_{63}-H_{33}H_{62})-H_{22}(H_{31}H_{63}-H_{61}H_{33})+H_{23}(H_{31}H_{62}-H_{32}H_{61}), \tag{98}$$

$$\Delta_1=f^*(H_{32}H_{63}-H_{33}H_{62}), \tag{99}$$

$$\Delta_2=-f^*(H_{31}H_{63}-H_{61}H_{33}), \tag{100}$$

$$\Delta_3=f^*(H_{31}H_{62}-H_{32}H_{61}). \tag{101}$$

4.2. Rayleigh waves

To investigate the possibility of Rayleigh waves in anisotropic fibre-reinforced elastic media, also we replace medium $\amalg_2$ by a vacuum. Since the boundary z = 0 is adjacent to vacuum, it is free from surface traction. So the stress boundary condition in this case may be expressed as

$$\sigma_{xx}(x,y,t)=0 \qquad at \quad x=0, \tag{102}$$

$$\sigma_{yy}(x,y,t)=0 \qquad at \quad x=0, \tag{103}$$

$$\sigma_{xy}(x,y,t)=0 \qquad at \quad x=0. \tag{104}$$

The wave velocity for Rayleigh waves in isotropic elastic medium it gives

$$\sum_{n=1}^{3} H_{3n} M_n = 0, \tag{105}$$

$$\sum_{n=1}^{3} H_{4n} M_n = 0, \tag{106}$$

$$\sum_{n=1}^{3} H_{6n} M_n = 0. \tag{107}$$

Also, by solving Eqs. (105)-(107), we get the parameters $M_n (n=1,2,3)$, to determine the wave velocity for Rayleigh waves.

4.3. The boundary conditions between interfaces

(i) The displacement components at the boundary surface between the media $\amalg_1$ and $\amalg_2$ must be continuous at all times and positions. This means that

$$[u, \text{v}]\amalg_1 = [u', \text{v}']\amalg_2 \tag{108}$$

then we obtain

$$\begin{aligned}\{\sum_{n=1}^{3} M_n(c,\omega)\exp(-k_n x), \quad \sum_{n=1}^{3} H_{1n} M_n(c,\omega)\exp(-k_n x),\} = \\ \{\sum_{n=1}^{3} \tilde{M}_n(c,\omega)\exp(-k'_n x), \sum_{n=1}^{3} H'_{1n} \tilde{M}_n(c,\omega)\exp(-k'_n x)\}\end{aligned} \tag{109}$$

(ii) The stress components σ_{xx}, σ_{yy} and σ_{zz} must be continuous at the boundary

$$[\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}]\amalg_1 = [\sigma'_{xx}, \sigma'_{yy}, \sigma'_{zz}, \sigma'_{xy}]\amalg_2 \tag{110}$$

$$\begin{aligned}
\{&\sum_{n=1}^{3} H_{3n}\, M_n(c,\omega)\exp(-k_n x),\sum_{n=1}^{3} H_{4n}\, M_n(c,\omega)\exp(-k_n x),\\
&\sum_{n=1}^{3} H_{5n}\, M_n(c,\omega)\exp(-k_n x),\sum_{n=1}^{3} H_{6n}\, M_n(a,\omega)\exp(-k_n x) =\\
&\sum_{n=1}^{3} H'_{3n}\, \tilde{M}_n(c,\omega)\exp(-k'_n x),\sum_{n=1}^{3} H'_{4n}\, \tilde{M}_n(c,\omega)\exp(-k'_n x),\\
&\sum_{n=1}^{3} H'_{5n}\, \tilde{M}_n(c,\omega)\exp(-k'_n x),\sum_{n=1}^{3} H'_{6n}\, \tilde{M}_n(a,\omega)\exp(-k'_n x).
\end{aligned} \qquad (111)$$

Invoking the boundary conditions (108) and (110) at the surface $x = 0$ of the plate, we obtain a system of six equations. After applying the inverse of matrix method, we have the values of the six constants by eliminating $\mathrm{M}_j, \tilde{\mathrm{M}}_j$ from equation (109)-(111) we get

$$Det \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \mathrm{H}_{11} & \mathrm{H}_{12} & \mathrm{H}_{13} & \mathrm{H}'_{11} & \mathrm{H}'_{12} & \mathrm{H}'_{13} \\ \mathrm{H}_{31} & \mathrm{H}_{32} & \mathrm{H}_{32} & \mathrm{H}'_{31} & \mathrm{H}'_{32} & \mathrm{H}'_{33} \\ \mathrm{H}_{41} & \mathrm{H}_{42} & \mathrm{H}_{43} & \mathrm{H}'_{41} & \mathrm{H}'_{42} & \mathrm{H}'_{43} \\ \mathrm{H}_{51} & \mathrm{H}_{52} & \mathrm{H}_{53} & \mathrm{H}'_{51} & \mathrm{H}'_{52} & \mathrm{H}'_{53} \\ \mathrm{H}_{61} & \mathrm{H}_{62} & \mathrm{H}_{63} & \mathrm{H}'_{61} & \mathrm{H}'_{62} & \mathrm{H}'_{63} \end{pmatrix} = 0. \qquad (112)$$

Hence, we obtain the expressions of displacements, temperature distribution and another physical quantities of the plate.

4.4. Stoneley waves

It is the generalized form of Rayleigh waves in which we assume that the waves are propagated along the common boundary of two semi-infinite media $\coprod_1$ and $\coprod_2$. Therefore equation (112) determines the wave velocity equation for Stoneley waves in anisotropic fiber- reinforced solid elastic media under the influence of gravity.

Clearly from equation (112), it is follows that wave velocity of the Stoneley waves depends upon the parameters for fiber-reinforced of the material medium, gravity and the densities of both media. Since the wave velocity equation (112) for Stoneley waves under the present circumstances depends on the particular value of ω and creates a dispersion of a general wave form.

Further equation (112), of course, is in complete agreement with the corresponding classical result, when the effect of gravity and parameters of the fibre-reinforcement are ignored.

5. Numerical results

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variation of temperature, displacement and stress fields in the context of three theories. To study the effect of rotation and reinforcement on wave propagation, we now present some numerical results for the physical constants [7]

$\lambda = 7.59\,\mathrm{x}\,10^9$ N/m^2, $\mu_T = 1.89\,\mathrm{x}\,10^9$ N/m^2, $\mu_L = 2.45\,\mathrm{x}\,10^9$ N/m^2,

$k = 386$, $\alpha = -1.28\,\mathrm{x}\,10^9$ N/m^2, $\beta = 0.32\,\mathrm{x}\,10^9$ N/m^2,

$\rho = 7800$ kg/m^2, $C_E = 383.1$ J / $\alpha_t = 1.78\,\mathrm{x}\,10^{-5}$ N/m^2,

$\nu_0 = 0.03$, $\tau_0 = 0.02$, $a = 1$ $T_0 = 293$ K $f^* = 1$,

$\mu = 3.86 \times 10^{10}$ kg/ms^2, $\omega = \omega_0 + i\xi$, $\omega_0 = 2$, $\xi = 1$.

The computations were carried out for a value of time $t = 0.1$. The numerical technique, outlined above, was used for the distribution of the real part of the thermal temperature θ, the displacement u and v, the stresses σ_{xx}, σ_{yy} and σ_{xy} distribution for the problem. The field quantities, temperature, displacement components u, v and stress components σ_{xx}, σ_{yy} and σ_{xy} depend not only on space x and time t but also on the thermal relaxation times τ_0 and ν_0. Here, all the variables are taken in non dimensional form. When there two thermal relaxation time in the absence and the presence of magnetic field, the results are shown in Figs. 1–11. The graph shows the three curves predicted by different theories of thermo-elasticity. In these figures, the solid lines represent the solution in the Coupled theory, the dotted lines represent the solution in the generalized LS theory and dashed lines represent the solution derived using the GL theory. We notice that the results for the temperature, the displacement and stresses distribution when the relaxation time is including in the heat equation are distinctly different from those when the relaxation time is not mentioned in heat equation, because the thermal waves in the Fourier's theory of heat equation travel with an infinite speed of propagation as opposed to finite speed in the non-Fourier case. This demonstrates clearly the difference between the coupled and the theory of generalized thermo-elasticity. For the value of y, namely y = 1, were substituted in performing the computation. It should be noted (Fig.1). It is clear from the graph that θ has decreases in the presence of magnetic field to arrive the minimum value at the beginning and hence increases with tacking the wave behaviors.

But in the case of absence of magnetic field the temperature increases in the beginning to arrive the maximum amplitudes and smooth decreases, also move in the wave propagation. The value of temperature converges to zero with increasing the distance x. The effect of magnetic field on temperature decreases the value of amplitude of θ and its nature for the medium with magnetic field.

Fig. 2 depicts the horizontal displacement u, begins from the negative values in the presence of magnetic field and then increases to arrive the maximum amplitudes, also moves in the wave propagation. However in the absence of magnetic field, the displacement start from the positive value at the beginning, the behavior is smooth in the first and then decreases again to reach its minimum, beyond it u falls again to try to retain zero at infinity, but the effect in the presence of magnetic field illustrated in the figure that the value of u for magnetic field is smaller when compared to those in the absence of magnetic field. The behavior of displacement u in (CD), (LS) and (GL) theories for two different values of magnetic field is similar in two cases.

Fig. 3, shows that the vertical displacement v always starts from the positives value and terminates at the zero value but v always starts from the positive values without magnetic field sharp increases in the beginning to arrive the maximum amplitudes and smooth decreases. However in the presence of magnetic field, v starts from the positive value and terminates at the zero value but v always starts from the positive value and smooth decreases in the beginning to arrive the minimum amplitudes and smooth increases. Also v in this case reaches minimum value, beyond reaching zero at the infinity (state of particles equilibrium). The displacements u and v show different behaviors, because of the elasticity of the solid tends to resist vertical displacements in the problem under investigation. Fig.4 explains that the stress component σ_{xx} reaches coincidence with zero value and satisfies the boundary condition at $x = 0$, starts sharp increases and reaches the maximum value in two cases and converges to zero with increasing the distance x, behavior of three theories are similar. In case of absence of magnetic field, σ_{xx} increases in the start and start decreases in the context of the three theories until reaching the zero value with increases the distance. These trends obey elastic and thermoelastic properties of the solid under investigation. The magnetic field caused relaxes the wave propagation and mores the amplitudes of the stress value. Fig. 5, shows that the stress component σ_{yy} has a different behavior in the presence and absence of magnetic field. It decreases in the start and start increases (maximum) in the context of the three theories and propagation until reaching the zero value at infinity in the absence of magnetic field, but in the presence of magnetic field, it increases in the start and start decreases in the context of the three theories until reaching zero value. These trends obey elastic and thermoelastic properties of the solid under investigation. Fig. 6, shows that the stress component σ_{xy} satisfies the boundary condition, it sharply decreases in the start and start increases in the

presence of magnetic field but in the absence of magnetic field σ_{xy} takes the same behavior nearly. The lines without magnetic field has the highest gradient when compared with magnetic field. These trends obey elastic and thermoelastic properties of the solid.
Fig.7 depicts that the stress component σ_{zz} reaches coincidence with positives value start smooth increases and reaches the maximum value in two cases and converges to zero with increasing the distance x, behavior of three theories are similar. In case of absence of magnetic field, σ_{zz} increases in the start and start decreases in the context of the three theories until reaching the zero value with increasing the distance. However, in the presence of magnetic field the curves of σ_{zz} are large compared with the curves in the absence of magnetic field. These trends obey elastic and thermoelastic properties of the solid under investigation.
Figs. 8-11, show the comparison between the temperature θ, the normal displacement component v and the force stress components σ_{xx} and σ_{xy} in case of material with reinforced constants and case of material without reinforced constants in the context of the three theories under the same value of magnetic field.
Fig. 8 shows that the temperature θ with reinforced constants is greater than that without reinforced constants since start from the positive values and satisfy the boundary conditions. The material with and without reinforced constants start smooth decreases and reaches the minimum value in two cases and then increases to

reach the maximum value in two cases and converges to zero with increasing the distance x. Fig. 9 shows that the normal displacement v with reinforced constants is greater than that without reinforced constants and tacks the wave propagation in all ranges but we observed that the displacement without reinforced constants tacks the exponential behavior. Fig. 10 shows that the stress component σ_{xx} with reinforced constants is greater than that without reinforced constants. Fig. 11 shows that the stress component σ_{xy} with reinforced constants are greater than that without reinforced constants.

By comparing the figures which obtained under the three thermoelastic theories, important phenomena are observed:- The curves in the context of the (CD), (LS) and (GL) theories decrease exponentially with increasing x, this indicates that the thermoelastic waves are untenanted and non-dispersive, where purely thermoelastic waves undergo both attenuation and dispersion.

Fig. 12 displays the temperature distribution with varies values of c with respect to y, it is shown that θ starts from zero, increasing to its maximum value and decreases with the increased values of y to attend again to zero that indicated to the vanishing of the temperature distribution at the boundary and the high values of the coordinate y. It is appearing that θ increases, decreases and increases with an increasing of c tends to zero as y tends to infinity. From Fig. 13, it is seen that the distribution of σ_{yy} starts from zero, decreasing to its minimum value and increases with the increased values of y to attend again to zero that indicated to the vanishing of the stress distribution at the boundary and the high values of the coordinate y. It is showing that σ_{yy} decreases, increases and decreases with an increasing of c tends to zero as y tends to infinity.

Fig. 14 shows that the distribution of σ_{xx} starts from zero, increasing to its maximum value and decreases with an increasing of y to attend again to zero that indicated to the vanishing of the stress distribution at the boundary and the high values of the coordinate y. It is showing that σ_{xx} decreases and increases with an increasing of c arrives to zero as y tends to infinity. From Fig. 15, it is clear that the distribution of σ_{xy} starts from zero, increasing to its maximum value and decreases with an increasing of y to attend again to its minimum value. It is seen that σ_{xy} increases with an increasing of c. Figs. 16 and 17 schematic the variation of phase velocity with respect to depth and magnetic field, respectively, it is shown that c decreases to interrupt to zero with an increasing of the depth and magnetic field. It is clear from Figs. 18 and 19 that Stoneley waves velocity and attenuation coefficient affect strongly with the magnetic field and phase velocity c that tend to zero if the magnetic field vanishes and for high magnetic field value.

Figs. 20 and 21 plot the variation of the temperature distribution θ and vertical displacement distribution v with respect to the depth y and coordinate x, it is seen that θ and v increase and decrease periodically with x tend to zero and increases with the depth y.

Fig. 22 displays the variation of σ_{xx} with respect the depth and coordinate x, it is obvious that σ_{xx} decreases, increases periodically tending to zero as x tends to infinity and decreases and decreasing with the increased values of y. From Fig. 23, it is appear that σ_{xx} affects strongly with variation of x and y which tends to zero as x tends to zero.

Figs. 24 and 25 display variation the attenuation coefficient of Rayleigh waves with respect to magnetic field, phase velocity and depth, phase velocity, respectively. It is clear that it affects with all parameters and takes the harmonic form with variation of the depth y. Fig. 26 displays the variation of the attenuation coefficient for Rayleigh waves with variation of magnetic field and depth, it is obvious that it decreases periodically with magnetic field and decreases with an increasing of y tends nearly to zero. Fig. 27 displays the variation of the attenuation coefficient for Stoneley waves with variation of magnetic field and depth, it is obvious that it increases tends to zero with the increasing of magnetic field and increases and decreases with an increasing of y tends nearly to zero.

Finally, Figs. 28 and 29 plot the variation the attenuation coefficient of Stoneley waves with respect to phase velocity, depth and magnetic field, phase velocity, respectively. It is clear that it affects with all parameters and takes the harmonic form with variation of the depth y.

6. Conclusions

Due to the complicated nature of the governing equations of the elasticity fiber-reinforced theory, the work done in this field is

unfortunately limited in number. The method used in this study provides a quite successful in dealing with such problems. This method gives exact solutions in the elastic medium without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered. Important phenomena are observed in all these computations:

1. The curves of the physical quantities with (CD) theory in most of figures are lower in comparison with those under (LS) theory and (GL) theory, due to the relaxation times.
2. Analytical solutions based upon normal mode analysis for themoelastic problem in solids have been developed and utilized.
3. The value of all the physical quantities converges to zero with an increase in distance x and all functions are continuous.
4. The fibre-reinforced has an important role on the distribution of the field quantities.
5. The method which used in the present article is applicable to a wide range of problems in thermodynamics and thermoelasticity.
6. Deformation of a body depends on the nature of forced applied as well as the type of boundary conditions.
7. It was found that for large values of time they give close results.

The solutions obtained in the context of elasticity theory, however, exhibit the behavior of speeds of wave propagation.

8. By comparing Figs. (13-29), it was found that the wave velocity has the same behavior in both media. But with the passage of time and magnetic field, numerical values of wave velocity in the elastic medium are large in comparison due to the influences of magnetic field .
9. The results presented in this paper should prove useful for researchers in material science, designers of new materials.
10. Study of the phenomenon of relaxation time and magnetic field is also used to improve the conditions of oil extra

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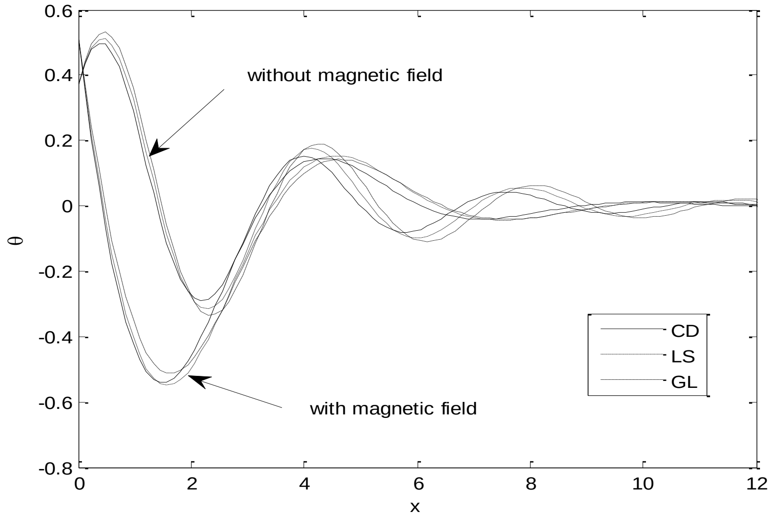

Fig. 1: Temperature distribution in the presence and absence of magnetic field.

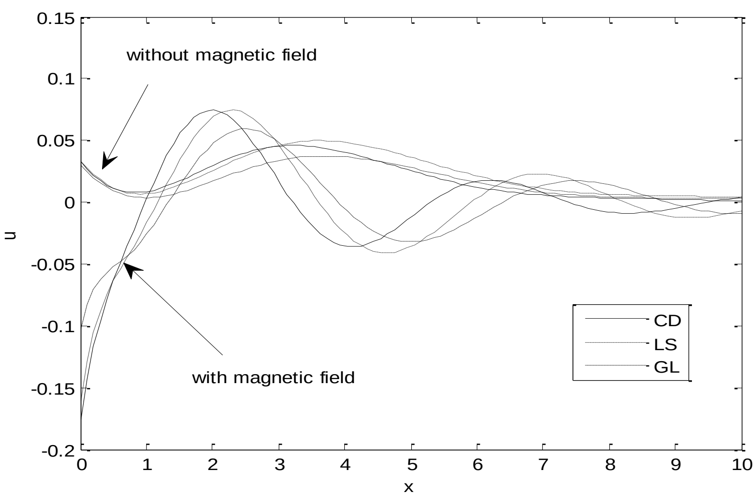

Fig. 2: Horizontal displacement distribution u in the presence and absence of magnetic field.

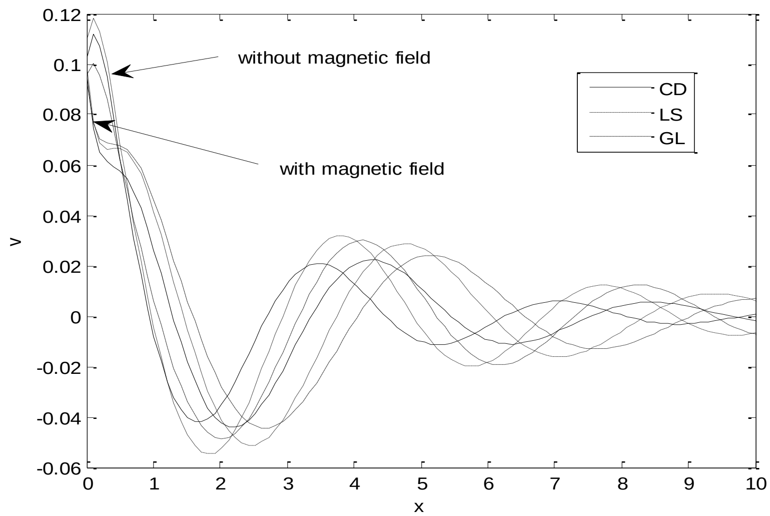

Fig. 3: Vertical displacement distribution v in the presence and absence of magnetic field.

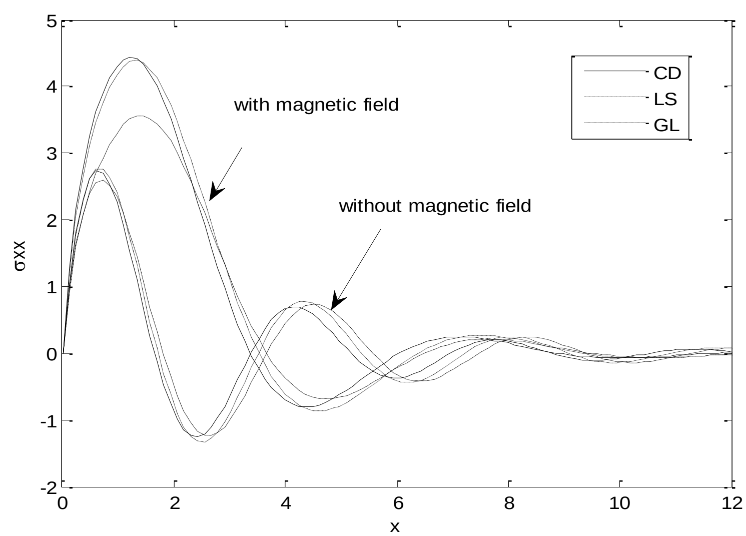

Fig. 4: The distribution of stress component σ_{xx} in the presence and absence of magnetic field.

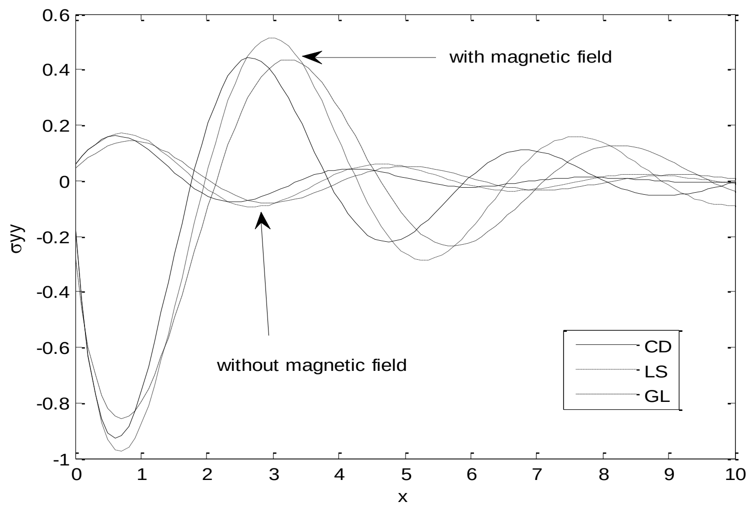

Fig. 5: The distribution of stress component σ_{yy} in the presence and absence of magnetic field.

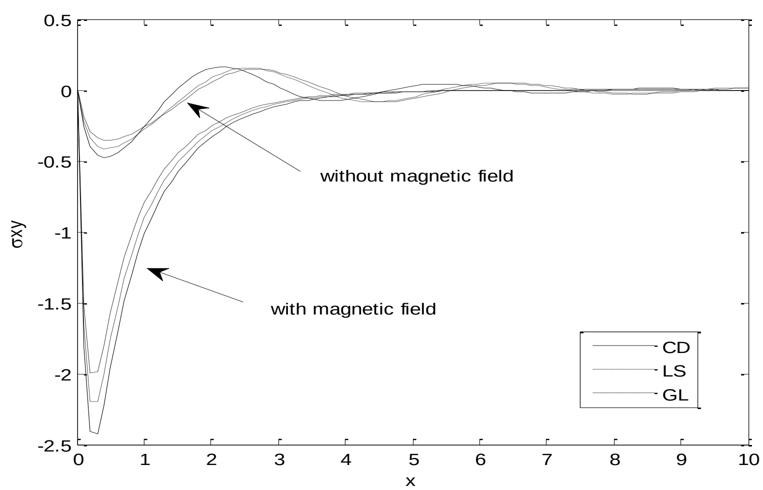

Fig. 6: The distribution of stress component σ_{xy} in the presence and absence of magnetic field.

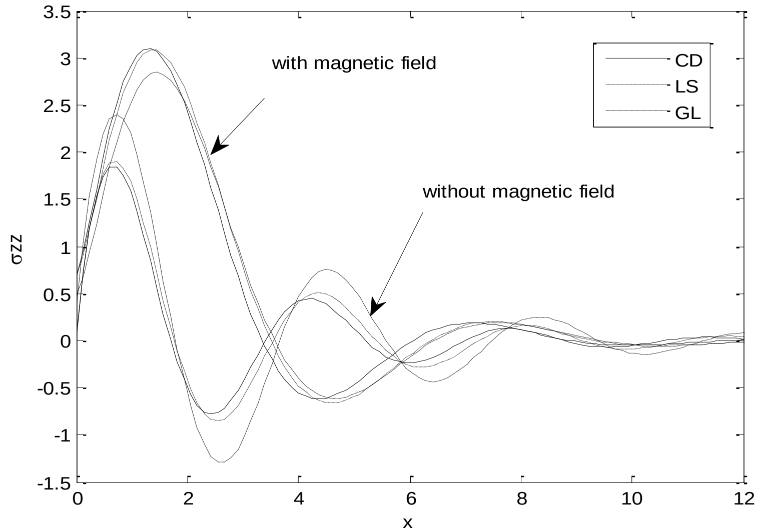

Fig .7: The distribution of stress component σ_{zz} in the presence and absence of magnetic field.

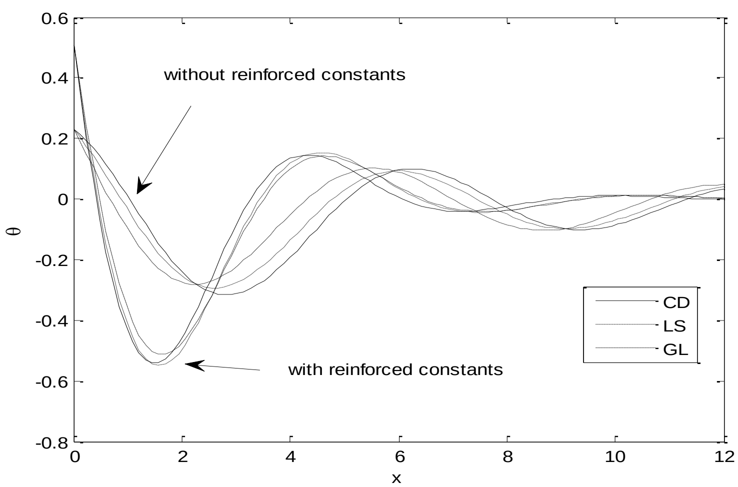

Fig. 8: Temperature distribution with and without reinforced constants at constant magnetic field.

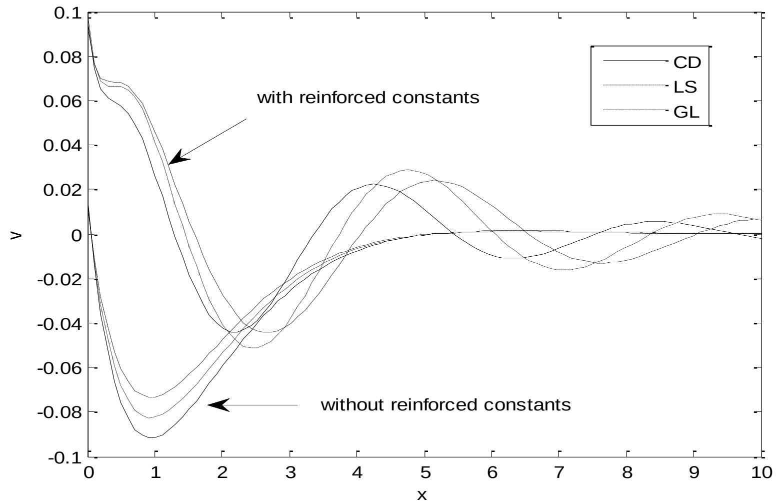

Fig. 9: Vertical displacement distribution v with and without reinforced constants at constant magnetic field.

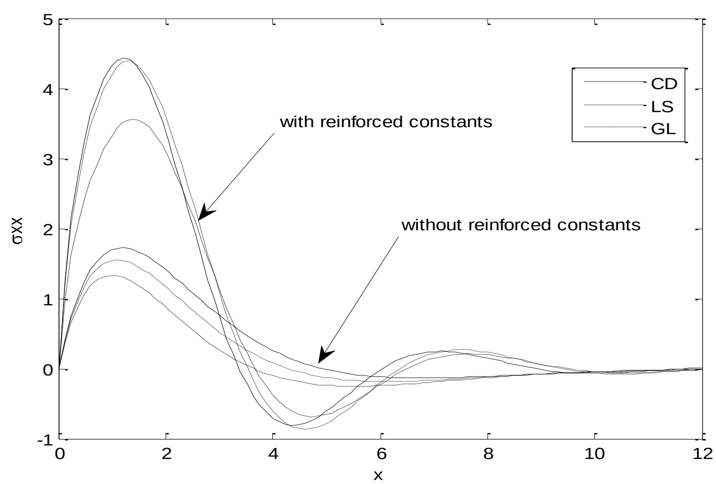

Fig. 10: The distribution of stress component σ_{xx} with and without reinforced constants at constant magnetic field.

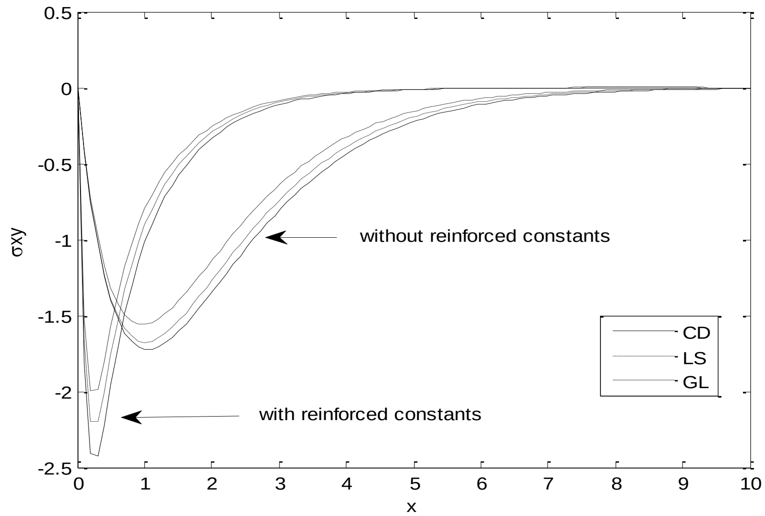

Fig. 11: The distribution of stress component σ_{xy} with and without reinforced constants at constant magnetic field.

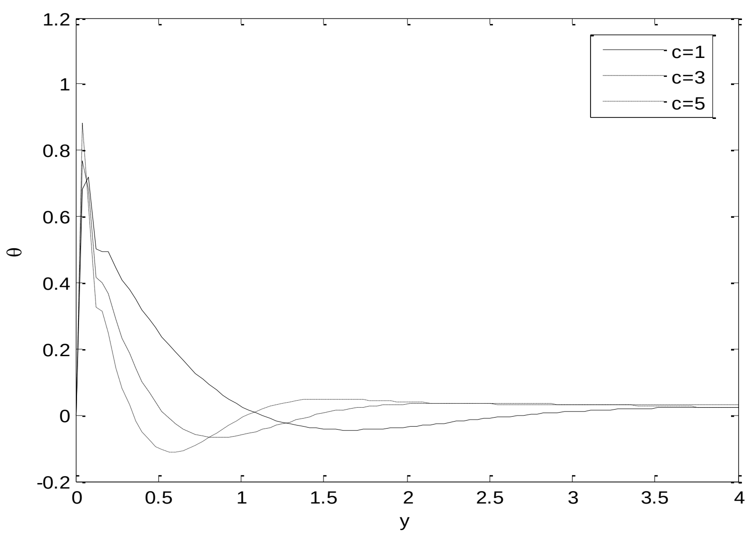

Fig. 12: Temperature distribution with varies values of c with respect to y

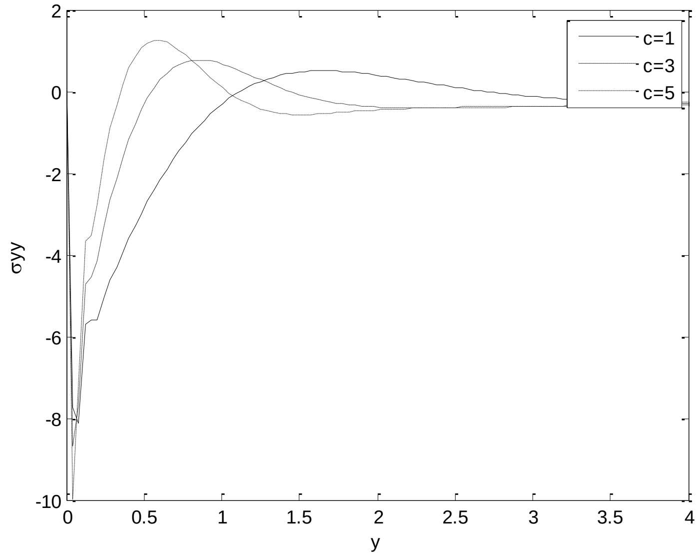

Fig. 13: The distribution of stress component σ_{yy} with varies values of c with respect to y

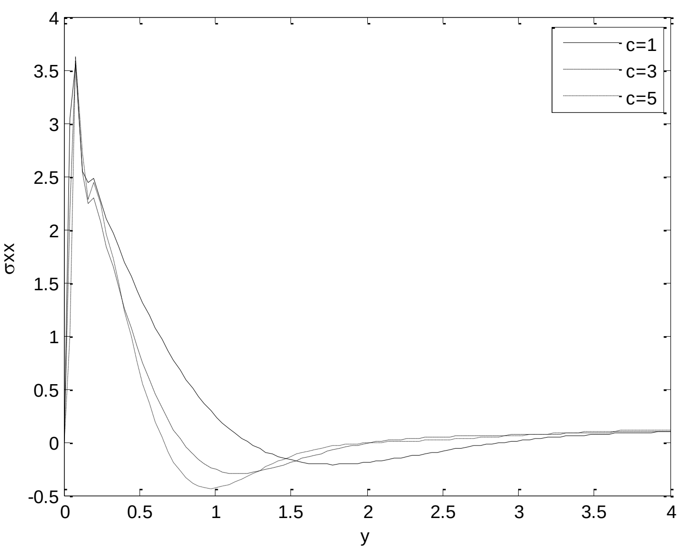

Fig. 14: The distribution of stress component σ_{xx} with varies values of c with respect to y.

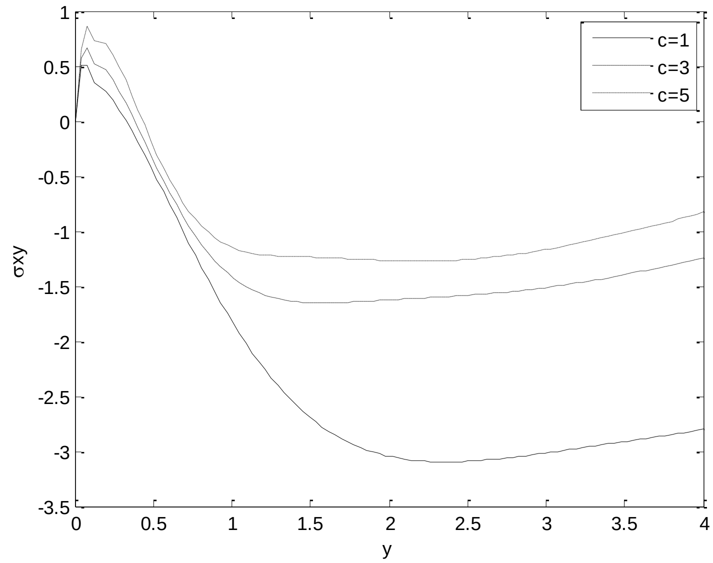

Fig. 15: The distribution of stress component σ_{xy} with varies values of c with respect to y

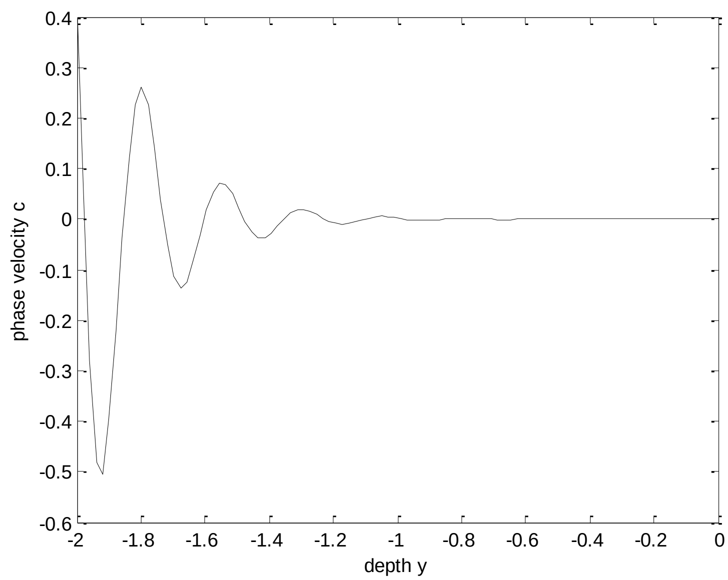

Fig. 16: Phase velocity c for Rayleigh waves with respect to depth

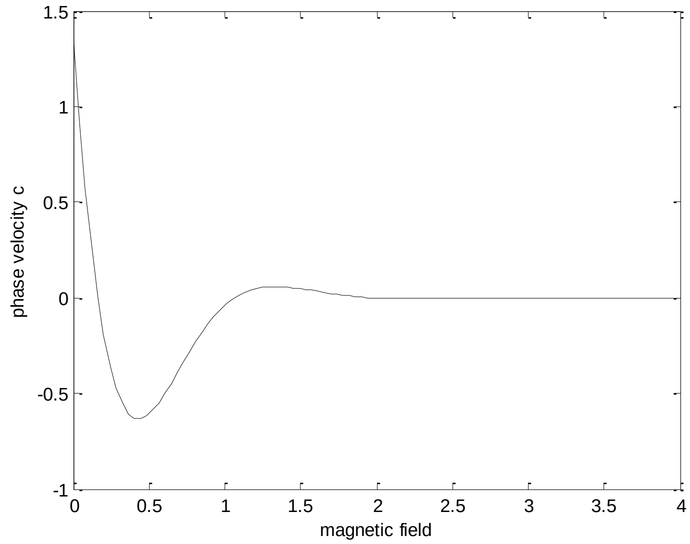

Fig. 17: Phase velocity c for Rayleigh waves with respect to magnetic field

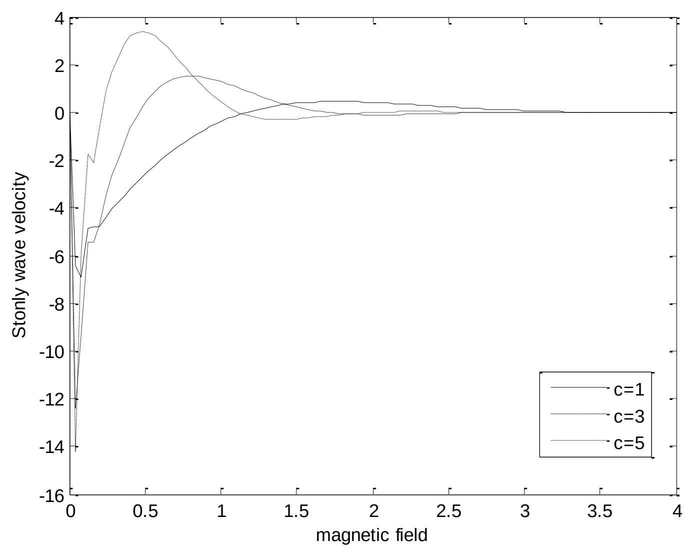

Fig. 18: Variation of Stoneley wave velocity with varies values of c with respect to magnetic field

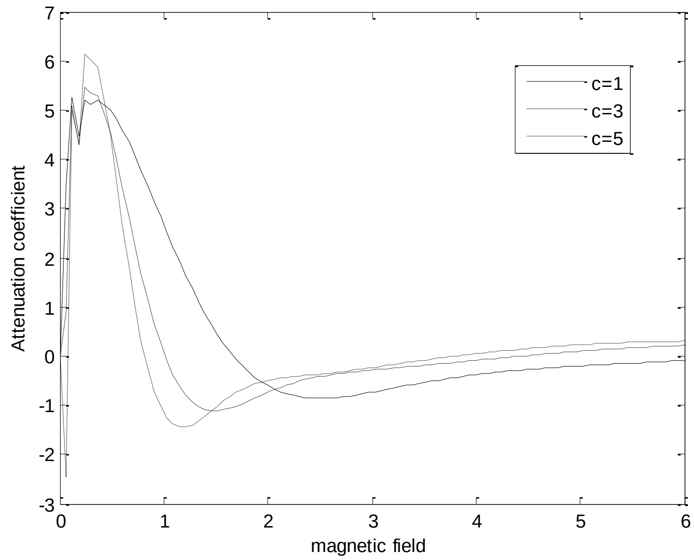

Fig. 19: Variation of attenuation coefficient with varies values of c with respect to magnetic field

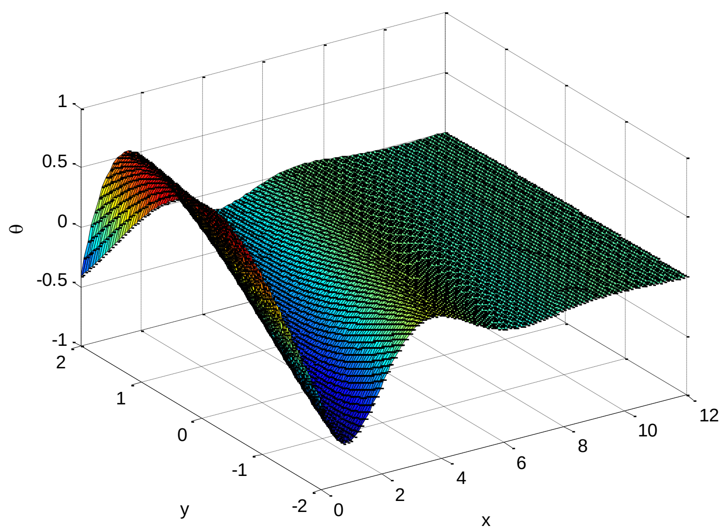

Fig. 20: 3D Temperature distribution between two medium with a constant of magnetic field

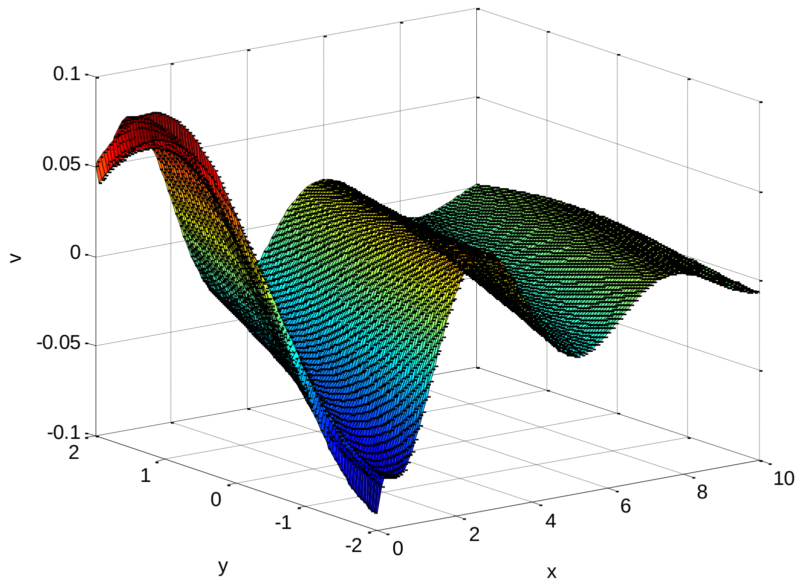

Fig. 21: Vertical displacement distribution v in the presence of magnetic field in medium 1 with a constant c

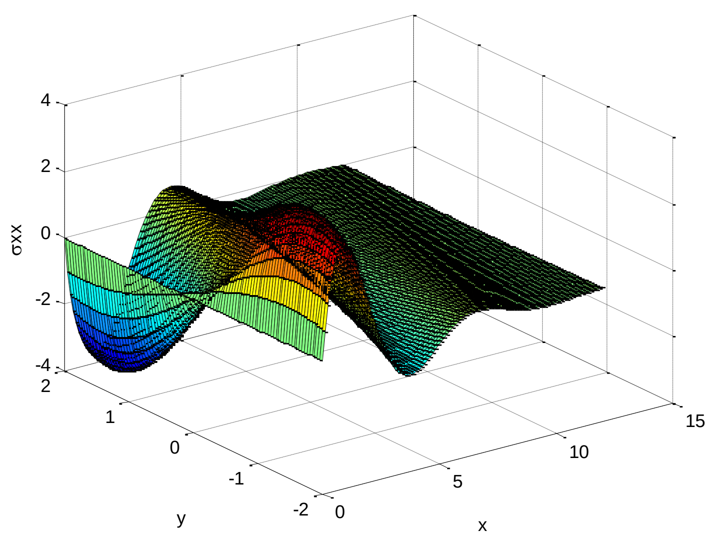

Fig. 22: 3D distribution of stress component σ_{xx} in the presence of magnetic field between two medium with a constant c

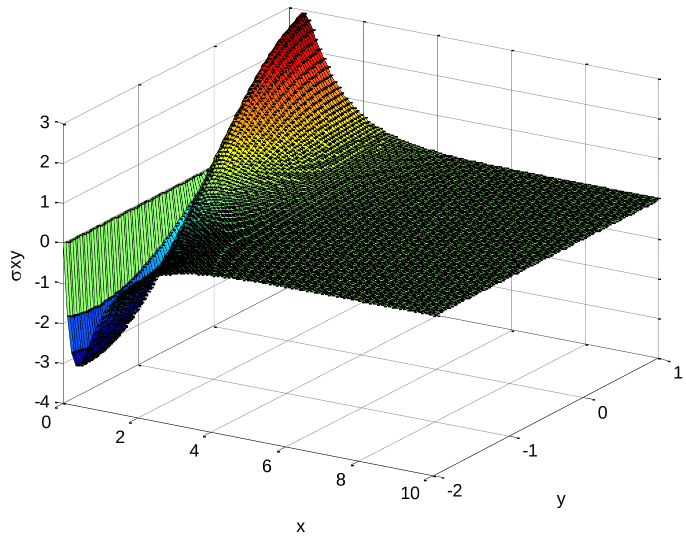

Fig. 23: 3D distribution of stress component σ_{xy} in the presence of magnetifield between two medium with a constant c

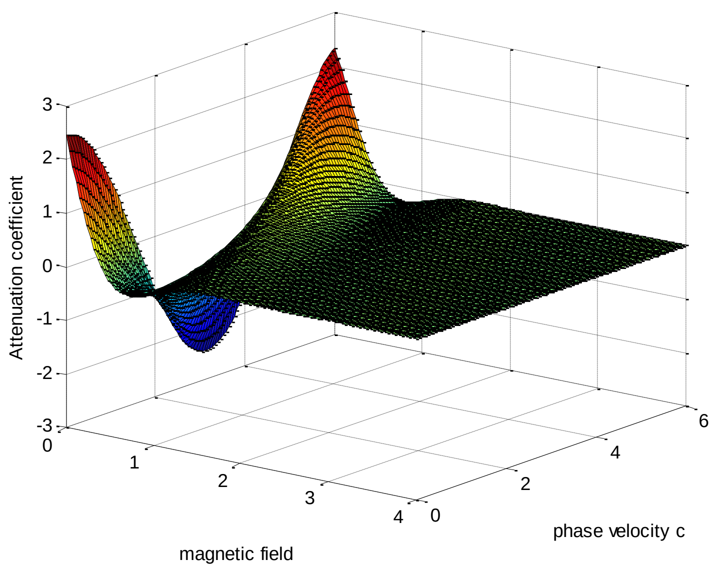

Fig. 24: Variation of the attenuation coefficient with magnetic field and phase velocity for Rayleigh waves

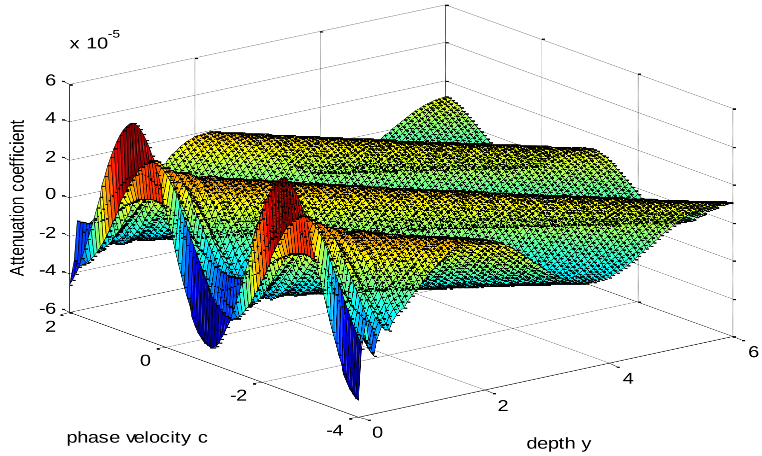

Fig. 25: Variation of the attenuation coefficient with phase velocity and depth for Rayleigh waves

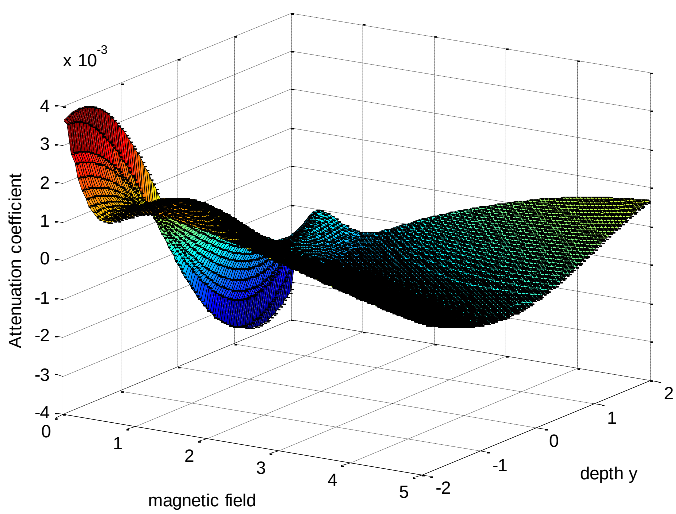

Fig. 26: Variation of the attenuation coefficient with magnetic field and depth for Rayleigh waves

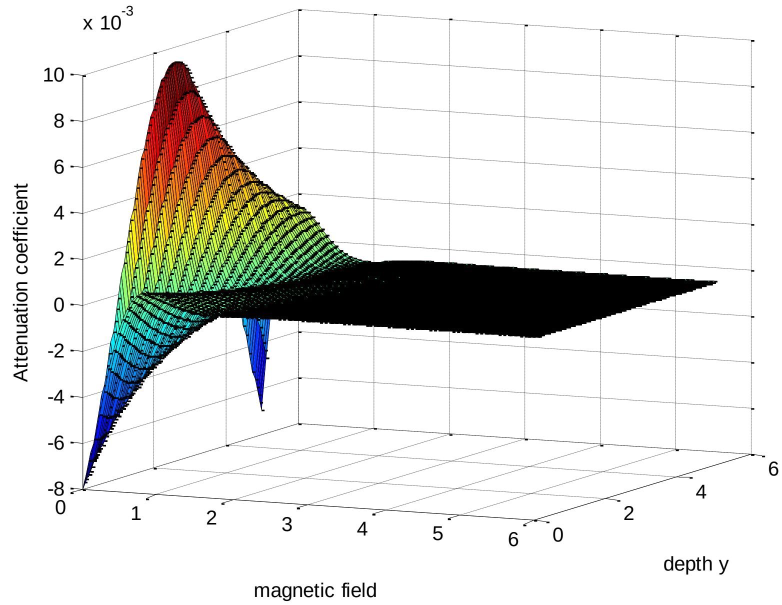

Fig. 27: Variation of the attenuation coefficient with magnetic field and depth for Stoneley waves

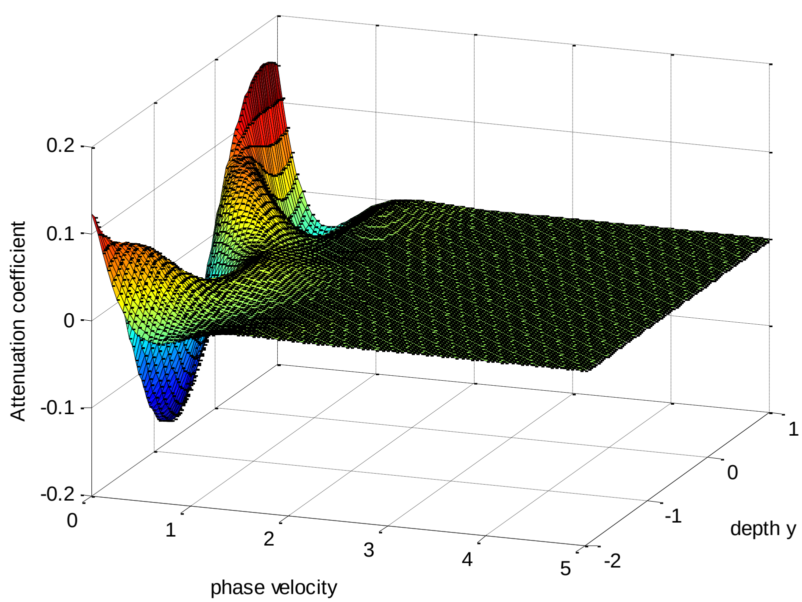

Fig. 28: Variation of the attenuation coefficient with phase velocity and depth for Stoneley waves

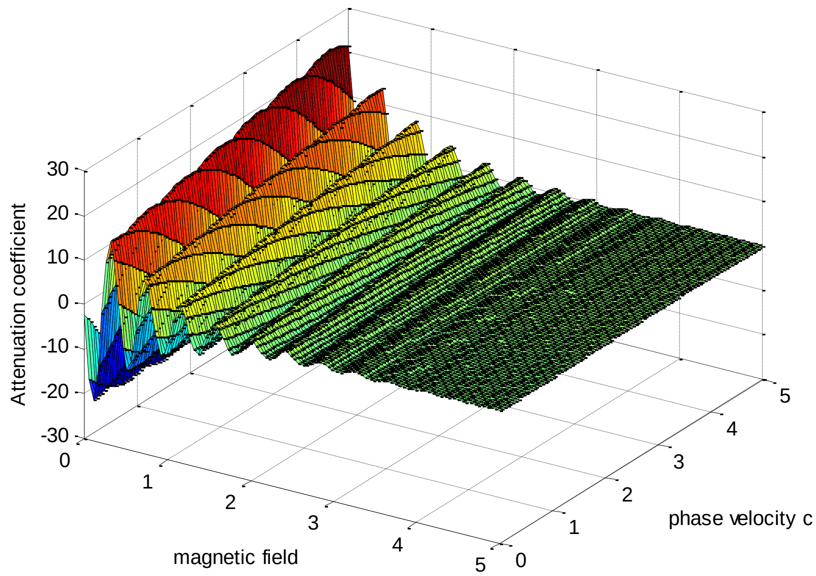

Fig. 29: Variation of the attenuation coefficient with magnetic field and phase velocity for Stoneley waves