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ON ANALYSIS OF A MATHEMATICAL MODEL OF CHOLERA USING CAPUTO FRACTIONAL ORDER

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ABSTRACT. Cholera, a dangerous waterborne disease, disproportionately affect developing nations. This study employs a Caputo fractional order derivative cholera model to assess control strategies (immunization, therapy, water sanitation). Qualitative analysis validates the model, and the basic reproductive ratio is calculated to gauge control effectiveness.The results of the homotopy perturbation method, simulated using MAPLE, showed that a combined strategy of vaccines, treatment, and water sanitation can effectively eradicate the cholera virus. It is recommended that the insights from this study be utilized to guide decision-making and enhance public health outcomes.

1. INTRODUCTION

Cholera is a serious waterborne illness caused by the Vibrio cholera bacterium, which spreads through contaminated water and food. It can lead to severe diarrhea, vomiting, dehydration, and even death if left untreated. Cholera affects between 1.3 and 4 million people worldwide each year, leading to 21,000 to 143,000 deaths annually. Africa carries a heavy burden, accounting for over 40% of global cases, with countries like Nigeria, the Democratic Republic of Congo, and Mozambique frequently experiencing outbreaks. In Africa, mortality rates during these outbreaks can exceed 5%[1], largely due to poor water, sanitation, and healthcare systems. Tackling cholera in Africa requires strengthening these critical services and ensuring a rapid response to outbreaks. In Nigeria, cholera is a serious public health issue, especially in rural areas with inadequate water and sanitation infrastructure [2]. Studies on

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recent cholera outbreaks in Nigeria indicate that the disease has affected 25 of the country's 36 states, including Cross River State. By September 2021, over 70,000 suspected cases and more than 2,000 deaths had been reported nationwide [3]. In order to achieve the 2030 cholera outbreak control strategy, which aims to gain a 0.9 rate advantage over cholera deaths, it is necessary to improve infrastructure such as health care centers and combat cholera. Several researchers have investigated different measures for controlling the spread of cholera. A study by Abdul et al. [4] investigated the transmission dynamics of cholera at the early stages of an outbreak and found that human behavior change could lower infection levels and delay disease peaks. Spatial heterogeneity was also found to have a significant impact on disease spread. Meanwhile, another study showed that reducing the contact rate, successful recovery rates, and appropriate hygiene are essential for eliminating cholera in the community. In [5], vaccination was found to be less effective than controlling long and short-cycle transmission routes in combating cholera in Uganda.

To effectively study the spread of cholera disease, researchers commonly utilize mathematical modeling techniques. This approach involves employing mathematical equations and methods to comprehend the dynamics of disease transmission as well as devising strategies for its control and prevention. These models are employed to analyze the transmission of the disease from person to person, enabling the estimation of the potential number of new cases within a specific population [6]. Additionally, mathematical models can help identify risk factors for cholera infection, such as age, gender, socioeconomic status, and exposure to contaminated water or food [7]. They can evaluate the effectiveness of various vaccination strategies, including routine immunization, targeted vaccination of high-risk populations, or mass vaccination campaigns [8]. Furthermore, these models can assess different treatment approaches, such as antibiotic usage or supportive care, in terms of their impact on disease progression and the risk of complications [9].

Mathematical models often play crucial roles in evaluating the effectiveness of control and prevention measures [44], such as improved sanitation practices [45], hygiene, and food safety protocols, and aid in identifying the most cost-effective strategies for reducing the burden of diseases [10], such as cholera. For instance, a study [11] employed both stochastic and deterministic models to determine that reducing contact rates, improving treatment rates, and enhancing environmental sanitation were essential activities for eradicating cholera. Optimal control of

these problems was also examined using Pontryagin's maximum principle in [12], and their study utilized interventions like educational campaigns and water treatment to optimize the objective function.
Vaccination is an important strategy in the control of cholera. The use of cholera vaccines can help prevent outbreaks and reduce the burden of the disease in endemic areas. Several studies have been conducted on the modeling and control methods of cholera disease using vaccines. In [13], it was presented that increasing the immunization coverage rate can result in the eradication of the disease, while authors in [14] revealed that vaccination can easily reduce the virus. Vaccination alone is unlikely to result in cholera eradication. Short- and long-term carriers also have a role in transmission; therefore, treatment of infected individuals can further limit the sting of the disease so as to effectively lower the mortality rate. This treatment typically involves antibiotics, hydration, and good hygiene practices [15]. Treatment typically lasts between 10 and 14 days, with extended durations in severe cases. A lot of research has been put forward by different scientists on mathematical modeling of cholera disease. These include Mukandavire et al. [16], who conducted a modeling study on the impact of vaccination and antibiotic treatment on the transmission dynamics of cholera disease in endemic settings. A mathematical model was developed in their study to evaluate the impact of vaccination and antibiotic treatment on the transmission of cholera disease in endemic settings. They found that vaccination and antibiotic treatment can significantly reduce the incidence of cholera disease in these settings. In another study conducted by Hendriksen et al. [17], the impact of optimizing antibiotic treatment for cholera disease was carried out. Their study used a mathematical model to evaluate the optimal duration of antibiotic treatment for cholera disease, and it was discovered in their results that shorter courses of antibiotic treatment can be effective in treating cholera disease and reducing the risk of antibiotic resistance. Also, the control of cholera disease via hygienic practices cannot be underestimated. As a matter of fact, a study conducted by Maliki et al. [18] on the mathematical modeling of the impact of improved water, sanitation, and hygiene services on cholera claimed that with proper hygiene, the disease can be completely eradicated. Some interventions are also considered by other researchers. Furthermore, the impact of public health educational campaigns, vaccination, and treatment was analyzed as control strategies for curtailing cholera disease [19]. Their study showed that these parameters are potent in reducing the disease's spread, especially when they are interdependent.

Researchers have utilized fractional derivatives to extend classical derivative concepts to non-integer orders, enabling the analysis of functions with non-integer independent variables [28] & [43]. Fractional calculus operators, such as Riemann-Liouville [33], Caputo [34], Caputo-Fabrizo [35], Antagana-Baleanu [36], and Yang-Abdel-Cattani [37], are valuable for investigating chaotic and fractal phenomena in various fields. Recent studies that have demonstrated effective application of these derivatives include Baba et al. [38] that presented a well-posed fractional-order cholera model with a saturated incidence rate. Their study provides valuable insights into the dynamics of cholera transmission. Their work contributes to the understanding of fractional order models in disease modeling and offers potential applications in cholera control strategies. Also, Helikumi et al. [39] published a work on a fractional-order model for cholera disease transmission with control strategies. The study highlights the importance of considering fractional-order dynamics in modeling cholera transmission and explores the effectiveness of control strategies. Their findings contribute to the field of mathematical biology and provide valuable information for designing effective interventions against cholera. Rosa and Torres [40] conducted research on fractional-order modeling and optimal control of cholera transmission. Their work focuses on developing mathematical models that capture the complexities of cholera dynamics using fractional calculus. The study explores optimal control strategies to mitigate the spread of cholera, and it could be said that their research contributed to the field by demonstrating the utility of fractional-order modeling and optimization techniques in understanding and controlling cholera transmission.

In Nigeria, limited access to clean water and hygiene has consistently resulted in high cholera mortality. Neglecting new eradication insights could potentially trigger a pandemic, underscoring the urgency of a robust control plan. While previous studies have explored cholera control strategies [20-21], [41-42], our research introduces a novel approach by simultaneously integrating vaccines, therapies, water sanitation, and monitoring of non-local disease dynamics. This is accomplished by modifying Mukandavire's cholera model, as proposed in [16], which was originally analyzed for assessing cholera transmission dynamics and control strategies in Zimbabwe and Haiti but has not been solved for simulations.

2. PRELIMINARY

Here, we present some basic definitions and properties of fractional calculus applicable in this study.

Riemann-Liouville Interal

The Riemann-Liouville fractional integration of order $\theta \geq 0$ of a positive real function $\beta(t) \in C_\mu,\ \mu \geq -1\ t > 0$ is defined as :

$$I^\eta \beta(t) = \frac{1}{\Gamma(\theta)} \int_0^t (t-x)^{\theta-1} \beta(x) dx.$$

The following properties hold for fractional integral operators I^η for$\beta(t) \in C_\mu,\ \mu \geq -1\ \theta, \alpha \geq 0$ and $\beta \geq -1$:

1. $I^\theta I^\alpha \beta(t) = I^{\theta+\alpha} \varphi(t)$,
2. $I^\theta I^\sigma \beta(t) = I^\theta I^\sigma \varphi(t)$,
3. $I^\theta t^\vartheta = \frac{\Gamma(\vartheta+1)}{\Gamma(\sigma+\vartheta+1)} t^{\theta+\vartheta}$.

Caputo-Fractional derivative The Caputo fractional derivative of a positive real function $\varphi(t)$ given as $D^\eta \varphi(t)$ is :

$$D^\eta \varphi(t) = \frac{1}{\Gamma(n-\beta)} \int_0^t (t-x)^{n-\beta-1} \varphi^{(n)}(x) dx, n-1 < \theta \leq n,\ n \in N,\ t > 0,\ \varphi \in c_{-1}^n.$$

The following property holds for fractional integration of the Caputo fractional derivative

$$I^\theta D^\theta \varphi(t) = \varphi(t) - \sum_{k=0}^{n-1} \varphi^{(k)}(0) \frac{t^k}{k!} \qquad n-1 < \theta \leq n,\ n \in N,\ \varphi \in c_{-1}^n, \mu \geq -1.$$

Homotopy Perturbation Method

The homotopy perturbation method introduced by He [23] relies on creating a homotopy function that connects an initial approximation to the actual solution, simplifying the problem for iterative resolution. The perturbation technique is then applied to obtain a series solution that converges to the accurate solution. We illustrate this method for solving Caputo-fractional order differential equations [24-26] with the following equation:

$$\frac{{}^c d^{\alpha_1} \gamma_k(x)}{dx^{\alpha_1}} = g(x, \gamma_1, \gamma_2, \gamma_3, \cdots \gamma_n) \qquad k, n \in N. \tag{1}$$

We can construct a homotopy for (1):

$$(1-p) \frac{{}^c d^{\alpha_1} \gamma_k(x)}{dx^{\alpha_1}} - p \left(\frac{{}^c d^{\alpha_1} \gamma_k(x)}{dx^{\alpha_1}} - g(x, \gamma_1, \gamma_2, \gamma_3, \cdots \gamma_n \right) = 0, \qquad k, n \in \mathrm{N}. \tag{2}$$

Simplifying (2) yields

$$\frac{{}^{c}d^{\alpha_1}\gamma_k(x)}{dx^{\alpha_1}} = p\left(g(x,\gamma_1,\gamma_2,\gamma_3,\cdots\gamma_n)\right), \qquad k,n \in \mathrm{N} \qquad (3)$$

According to He [23], we can naturally assume a series solution of the form:

$$\gamma_k(x) = \lambda_{i0} + p\lambda_{i1} + p^2\lambda_{i2} + p^3\lambda_{i3}, \cdots p^n\lambda_{in}, \qquad (4)$$

Using (4) to evaluate (1) and comparing equal powers of p, the following system are obtained:

$$\begin{array}{ll} p^{0:} & \frac{{}^{c}d\gamma_{i0}{}^{\alpha_1}(x)}{dx^{\alpha_1}} = 0, \\ p^{1}: & \frac{{}^{c}d\gamma_{i1}{}^{\alpha_1}(x)}{dx^{\alpha_1}} = g_{i1}(x,\lambda_{10},\lambda_{20},\lambda_{30},\cdots\lambda_{n0}), \\ p^{2}: & \frac{{}^{c}d\gamma_{i2}{}^{\alpha_1}(x)}{dx^{\alpha_1}} = g_{i2}(x,\lambda_{10},\lambda_{20},\lambda_{30},\cdots\lambda_{n0},\lambda_{11},\lambda_{21},\lambda_{31},\cdots\lambda_{n1}), \end{array} \qquad (5)$$

e.t.c. In turn, the Riemann-Liouville fractional integral operator I^{α_1} is applied on the system to obtain $\gamma_{i1}(x), \gamma_{i2}(x), \gamma_{i3}(x)\ldots$and the solution of (1) is obtained as:

$$\varpi_{ik}(t) = \sum_{n=0}^{K-1} w_{in}(t).$$

3. METHODS

3.1. **The Mathematical Method.** The proposed model for the conceptual dynamics of Cholera disease transmission in a physical system is given by:

$$\left.\begin{array}{l} {}^{c}D^{\theta_1}S(t) = \rho - \varphi S(t)I(t) - (\psi + u + \omega\lambda)S(t), \\ {}^{c}D^{\theta_2}I(t) = \omega\lambda S(t) + \varphi S(t)I(t) - (\gamma + \delta + \psi)I(t) - TI(t), \\ {}^{c}D^{\theta_3}B(t) = \varepsilon I(t) \ \ - (\tau + z)B(t), \\ {}^{c}D^{\theta_4}R(t) = \gamma I(t) + uS(t) + TI(t) - \psi R(t). \end{array}\right\} \qquad (6)$$

Subject to the following starting conditions:

$$S(0) = s_0, I(0) = i_0, B(0) = b_0, R = \ \ R_0$$

Figure 1 presents the schematic diagram of the model, and Table 1 presents the description of variables, parameters, their corresponding values, and sources.

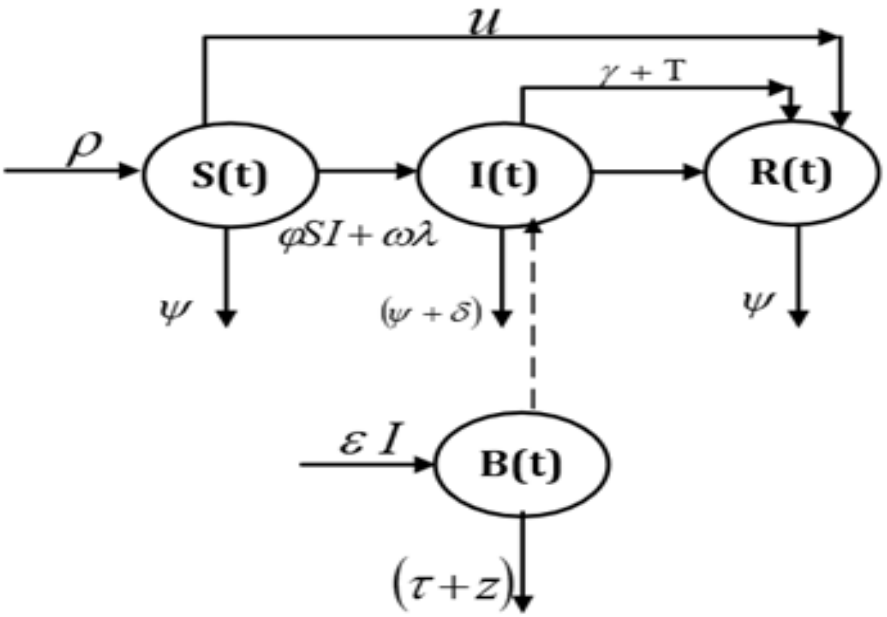


Figure 1: Schematic diagram of the model

Description

The model utilizes the following parameters: $S(t)$, $I(t)$, $R(t)$ as state variables. This respectively represents the population of vulnerable, infected, and recovered individuals. The concentration of vibrio cholera in water at time t is $B(t)$. ρ represents the influx rate of individuals into the system. It covers the birth of newborn babies and immigrants. ω is the ingestion rate of vibrio cholera from contaminated sources; $\lambda = \frac{B}{k+B}$ represents the force of infection, where k is the average saturation of vibrio cholera. φ represents the contracting rate of vibrio cholera from human contact, ψ is the natural mortality rate, and δ is the cholera-induced death rate. The recovery rate of individuals treated from cholera infection is denoted by γ, parameters $\varepsilon$and τ respectively represents the clearance rate of Vibrio cholera by humans through untreated waste and sanitization practices, respectively, while the incorporated control parameters u, represents the immunization of vulnerable individuals, and z denotes the rate at which individuals of the population adopt water sanitization practices. Lastly, the therapeutic response

action T for the sick individuals $I(t)$ is a piecewise linear function defined by Wang in [29]; it ranges on the interval $0 < v \leq 1$ and is related to the prevalence of infected individuals such that $TI(t) = rI(t)$ and system (1) yields:

$$^{c}D^{\theta_1}S(t) = \rho - \varphi S(t)I(t) - (\psi + u + \omega\lambda)\,S(t), \tag{7}$$

$$^{c}D^{\theta_2}I(t) = \omega\lambda S(t) + \varphi S(t)I(t) - (\gamma + \delta + \psi + r)I(t), \tag{8}$$

$$^{c}D^{\theta_3}B(t) = \varepsilon I(t) \ - (\tau + z)B(t), \tag{9}$$

$$^{c}D^{\theta_4}R(t) = \gamma I(t) + uS(t) + TI(t) - \psi R(t). \tag{10}$$

Table 1. Description of variables, parameters, their corresponding values and references

Variables	**Description**	**Values**	**References**
s_0	Starting population of susceptible individuals	150000	Assumed
i_0	Starting population of infected individuals	12300	Assumed
b_0	Starting concentration of vibrio cholera bacteria	26240	Assumed
R_0	Initial population of recovered individuals	6000	Assumed
Parameters	**Description**	**Values**	**References**
ρ	Influx rate	0.000913 /day	[27]
ψ	Natural death rate	0.000034/day	[46]
γ	recovery rate	0.2 /day	[27]
τ	Death rate of vibrio cholera bacteria	0.33 /day	[30]
k	Average saturation rate of vibrio cholera bacteria	10∧6 day	[31]
ε	Clearance rate of vibrio cholera bacteria	10 cells/day	[32]
ω	Human ingestion rate of vibrio cholera bacteria	0.214 cells/ml-day	[20]
φ	Transmission rate of vibrio cholera via human contact	0.02 cells/day	[27]
δ	cholera induced death rate	0.0013/day	[32]
Control Parameters			
z	Adoption rate of water sanitization practices	0, $0 < z \leq 1$	-
u	Rate of vaccine uptake	0,0 $< u \leq 1$	-
v	Rate of therapy administration	0, $0 < v \leq 1$	-

3.2. **Qualitative Analysis.**

3.2.1. *Positivity of Solution.* **Lemma.[43]:**
Let $\mathbb{R}^4_+ = \left\{ \kappa \in \mathbb{R}^4_+,\ \kappa \geq 0 \right\} \geq 0$, then $\kappa(t) = \left(S^{\theta_1}(t),\ I^{\theta_2}(t),\ B^{\theta_3}(t),\ R^{\theta_{41}}(t) \right)^T$ must be positive in $\mathbb{R}^4$.
Proof: Let $f(\nu) \in \delta[m,n]$ and $D^\theta f(\nu) \in \delta[m,n]$ for $0 < \theta \leq 1$. Then we have

$$f(\nu) = f(a) + \frac{1}{(\theta+1)} D^\theta f(\upsilon)(\nu - m)^\theta,$$

$\forall\, \nu \in (m,n]$ which exists and is positive in $\mathbb{R}^4_+$.

3.2.2. *Existence and Uniqueness of Solution.* **Theorem 1.[43]:** The solution of Equations (7)–(10) exists and remains in $\mathbb{R}^4$ for $\omega \geq 0$.
Proof: We prove the existence and uniqueness of the solution in (7)–(10) for $(0, \theta_i)$, $i = 1..4$. We need to show that the domain $\mathbb{R}^4,\ \omega \geq 0$ is positively invariant. Thus,

$$D^\theta S|_{s=0} = \rho \geq 0, \quad D^\theta I|_{i=0} = \omega\lambda S \geq 0,$$

$$D^\theta B|_{b=0} = \varepsilon I \geq 0, \quad D^\theta R|_{r=0} = \gamma + uS + TI \geq 0.$$

Since the positive solution satisfies the vector field pointing into $\mathbb{R}^4_+$, thus the solution exists and is unique.

3.2.3. *Cholera-Free Equilibrium.* When cholera doesn't exist in the system, $I = 0$. Thus, equating the left hand side of (7) to (10) to zero and concurrently solving the system at $I = 0$ yields the cholera-free equilibrium of the system:

$$(\ S_0, I_0, B_0,\ \ R_0) \ = \ \left(\frac{\rho}{u+\psi},\ 0,\ 0,\ \frac{u}{u+\psi} \right) \tag{11}$$

3.2.4. *Basic reproduction number.* The number of secondary infection that can arise from one single case is the basic reproductive number R_0. The R_0 of the mathematical model is computed to be:

$$R_0 \ = \ \frac{\varphi\rho}{(\gamma+\psi+\delta+v)(u+\psi)}. \tag{12}$$

Evaluating this reproductive number without the implementation of the control scheme i.e. $(u = 0, v = 0)$ using the baseline values of each parameters defined on Table 1, the reproductive number is calculated as $R_0 = 1.90519743$**.** According to this estimation, it suggests that

each infected individual can potentially transmit the disease to approximately two other individuals, indicating a significant rate of transmission. To reduce the prevalence of the disease spread, the impact of the two control parameters incorporated into the model present in the R_0 is varied on the interval. $0 \leq u \leq 1$, $0 \leq v \leq 1$,and the following results are obtained for R_0:

Table 2: Estimation of incorporated parameters on R_0

S/N	u, v	R_0
1	$u = 0, v = 0$	1.90519743
2	$u = 0.5, v = 0.5$	5.122×10^{-6}
3	$u = 1, v = 1$	1.5×10^{-6}

The results presented in Table 2 provide justification for implementing the proposed control scheme to reduce the prevalence of the disease. It is evident that with the full combined implementation of these strategies, specifically at values $u = 1, v = 1$, there are positive outcomes in terms of disease control and mitigation; the R_0 is reduced to minimum and $R_0 < 1$. This signifies that the disease will eventually be wiped out of the system as time progresses, and the system will be asymptotically stable.

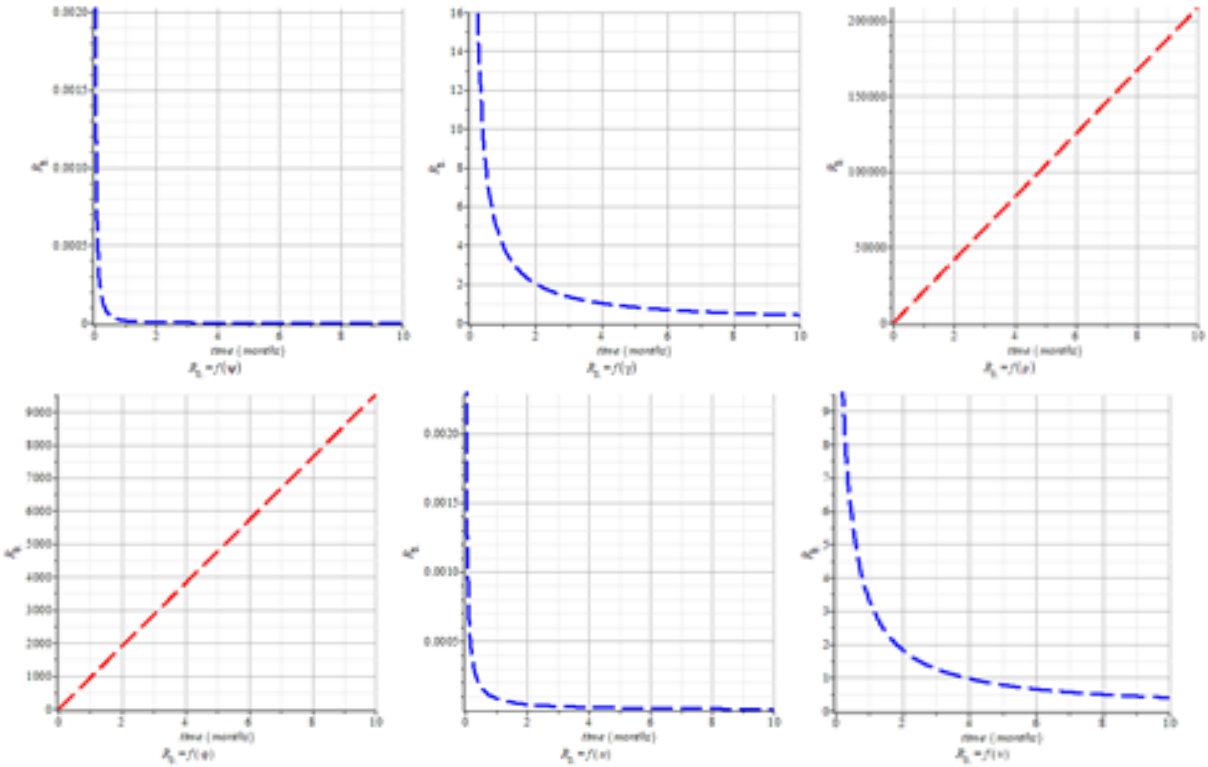
3.2.5. *Sensitivity Analysis.* To enhance the research, we evaluate the robustness of each parameter on the basic reproductive ratio using a normalized sensitivity analysis of $\Re_0$, calculated with the following formula:

$$S_F^{R_0} = \frac{\partial R_0}{\partial F} \cdot \frac{F}{R_0} \tag{13}$$

Here, $F = (\varphi, \gamma, \rho, \psi, u, \delta, v)$. Thus using (13) and evaluating the outcomes using the baseline values of each parameter presented on Table 1, we obtained the following results presented on Table 2:
Table 2: Sensitivity indices of each parameter on R_0

Parameter	Value
φ	1
γ	-0.9389472972
ρ	1
ψ	-1.0000021127
δ	-0.016103137445
v	0
u	0

TABLE 1. Parameter values used in the model



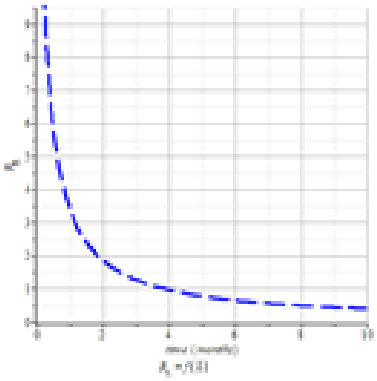


Figure 1: Sensitivity of model parameters on R_0

3.2.6. *Semi-analytic solution via homotopy perturbation method.* To acquire the solution of the fractional order model, we apply the describe methodology of the HPM in section 2.1. Thus constructing a homotopy for (7) - (10):

$$(1-p)^c D^{\theta_1} S(t) + p\left(D^{\theta} S(t) - (\rho - \omega\lambda S(t) - \varphi S(t)I(t) - \psi S(t) - uS(t))\right) = 0 \tag{14}$$

$$(1-p)^c D^{\theta_2} I(t) + p\left(D^{\theta} I(t) - (\omega\lambda S(t) + \varphi S(t)I(t) - (\gamma+\delta+\psi)I(t) - vI(t))\right) = 0 \tag{15}$$

$$(1-p)^c D^{\theta_3} B(t) + p\left(D^{\theta} B(t) - (\varepsilon I(t) - \tau B(t) - zB(t))\right) = 0 \tag{16}$$

$$(1-p)^c D^{\theta_4} R(t) + p\left(D^{\theta} R(t) - (\gamma I(t) - \psi R(t) + vI(t) + uS(t))\right) \tag{17}$$

We suggest the following series solution embedding parameter p for the system such that:

$$S(t) = s_0(t) + ps_1(t) + p^2 s_2(t) + \cdots p^n s_n(t) \tag{18}$$

$$I(t) = i_0(t) + pi_1(t) + p^2 i_2(t) + \cdots p^n i_n(t) \tag{19}$$

$$B(t) = b_0(t) + pb_1(t) + p^2 b_2(t) + \cdots p^n b_n(t) \tag{20}$$

$$R(t) = R_0(t) + pr_1(t) + p^2 r_2(t) + \cdots p^n r_n(t) \tag{21}$$

Evaluating (14)-(17) using (18)-(21) and comparing coefficients of p^n, the following results are obtained:

For p^0 :

$$D^\theta i_0(t)=0,\ D^\theta i_0(t)=0,\ D^\theta b_0(t)=0,\ D^\theta\ \ R_0(t)=0 \tag{22}$$

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Applying Riemann-Liouville integral operator $I^\theta$to (22) at the given initial condition, yields the following initial approximations

$$s_0(t)=s_0,\ \ i_0(t)=i_0, b_0(t)=b_0,\quad R_0(t)=\ \ R_0 \tag{23}$$

Also, the coefficient of p^1 yields:

$$\begin{aligned}
&D^\theta s_1(t)=\rho-\omega\lambda s_0(t)-\varphi s_0(t)i_0(t)-\psi s_0(t)-us_0(t)\\
&D^\theta i_1(t)=\omega\lambda s_0(t)+\varphi s_0(t)i_0(t)-(\gamma+\delta+\psi)i_0(t)-vi_0(t)\\
&D^\theta b_1(t)=\varepsilon i_0(t)\ -\tau b_0(t)-\ \ zb_0(t)\\
&D^\theta r_1(t)=\gamma i_0(t)-\ \ \psi\ \ R_0(t)\ +\ \ vi_0(t)+us_0(t)
\end{aligned} \tag{24}$$

Similarly applying the Riemann-Liouville integral operator I^θon (24) as in the previous iteration,

$$\begin{aligned}
&s_1(t)=(\rho-\omega\lambda s_0-\varphi s_0i_0-\psi s_0-us_0)\tfrac{t^\theta}{\Gamma(\theta+1)}\\
&i_1(t)=(\omega\lambda s_0+\varphi s_0i_0-(\gamma+\delta+\psi)i_0-vi_0)\tfrac{t^\theta}{\Gamma(\theta+1)}\\
&b_1(t)=(\varepsilon i_0\ -\tau b_0-\ \ zb_0\ )\tfrac{t^\theta}{\Gamma(\theta+1)}\\
&r_1(t)=(\gamma i_0-\ \ \psi\ \ R_0\ +\ \ vi_0+us_0)\tfrac{t^\theta}{\Gamma(\theta+1)}
\end{aligned}$$

Also, the coefficient of p^2 yields:

$$\begin{aligned}
&D^\theta s_2(t)=-\omega\lambda s_1(t)-\varphi s_1(t)i_1(t)-\psi s_1(t)-us_1(t)\\
&D^\theta i_2(t)=\omega\lambda s_1(t)+\varphi s_1(t)i_1(t)-(\gamma+\delta+\psi)i_1(t)-vi_1(t)\\
&D^\theta b_2(t)=\varepsilon i_1(t)\ -\tau b_1(t)-\ \ zb_1(t)\\
&D^\theta r_2(t)=\gamma i_1(t)-\ \ \psi r_1(t)\ +\ \ vi_1(t)+us_1(t)
\end{aligned} \tag{25}$$

Applying the integral operator $I^\theta$to (25) equally yields

$$s_2(t)=\begin{pmatrix}
2\psi us_0-\omega\lambda\rho+\omega^2\lambda^2s_0-\varphi^2s_0^2i_0^2+\varphi^2i_0^2s_0-\varphi i_0\rho+3\psi\varphi s_0i_0\\
+2\psi\omega\lambda s_0+u^2s_0+2\omega\lambda us_0-\varphi s_0^2\omega\lambda+\varphi s_0\delta i_0+\varphi s_0\gamma i_0\\
+2\varphi i_0us_0-\psi\rho+\psi^2s_0+2\omega\lambda\varphi s_0i_0-u\rho
\end{pmatrix}\frac{t^{2\theta}}{\Gamma(2\theta+1)}$$

$$I_2(t)=\begin{pmatrix}
\omega\lambda\rho-\omega^2\lambda^2s_0+\varphi^2s_0^2i_0^2-\varphi^2i_0^2s_0+\varphi i_0\rho-\gamma\omega\lambda s_0-v\omega\lambda s_0\\
-2\omega\lambda\varphi s_0i_0+2\gamma\delta i_0+2\psi i_0+2v\gamma i_0+2v\psi i_0+2\delta\psi i_0\\
+\gamma i_0^2+v^2i_0+\gamma^2i_0+\psi^2i_0-3\psi\varphi s_0i_0-2\psi\omega\lambda s_0-\omega\lambda us_0\\
-2\varphi\delta s_0i_0-2\varphi vs_0i_0-2\varphi\gamma s_0i_0-\varphi us_0i_0+2\gamma vi_0+\varphi s_0^2\omega\lambda
\end{pmatrix}\frac{t^{2\theta}}{\Gamma(2\theta+1)}$$

$$b_2(t)=\left(\varepsilon\omega\lambda s_0+\varepsilon\varphi s_0i_0-\varepsilon\delta i_0-\varepsilon\psi i_0-\varepsilon vi_0-\varepsilon\delta i_0-\tau\varepsilon i_0+\tau^2b_0+2\tau zb_0+z^2b_0\right)\frac{t^{2\theta}}{\Gamma(2\theta+1)}$$

$$r_2(t) = \begin{pmatrix} \omega\gamma\lambda s_0 + \delta\varphi s_0 i_0 - \gamma\delta i_0 - 2\gamma\psi i_0 - 2v\gamma i_0 - \gamma i_0 - 2\psi u s_0 + \psi^2 R_0 + u\rho \\ -2\psi v i_0 + v\omega\lambda s_0 + v\varphi s_0 i_0 - v\delta i_0 - v^2 i_0 - u\varphi s_0 i_0 - u^2 s_0 - u\omega\lambda s_0 \end{pmatrix} \frac{t^{2\theta}}{\Gamma(2\theta+1)}$$

The approximate solution for each class is obtained by summing the partial iterative solutions. In our case, these are given as follows:

$$S(t) = \sum_{n=0}^{2} s_n(t), \quad I(t) = \sum_{n=0}^{2} i_n(t), \quad B(t) = \sum_{n=0}^{2} b_n(t), \quad R(t) = \sum_{n=0}^{2} r_n(t).$$

4. RESULTS AND EXPERIMENTS

4.1. **Results.** In this section, we assess the raw results obtained from the iterative scheme of the homotopy perturbation method for each state variable. The evaluation process utilizes the standard data cholera related data acquired from literatures presented in Table 1. Subsequently, we generate the following results, which include three control parameters at an arbitrary time order.

$$S(t) = 15000 + \frac{(-125.84511 - 0.060401u)\,t^{\theta}}{\Gamma(\theta+1)} \tag{26}$$

$$+ \frac{\left(0.2397549836u + 0.0002212299650 + 0.015091420v + 0.00617544u^2\right) t^{2\theta}}{\Gamma(2\theta+1)} \tag{27}$$

$$I(t) = 12300 + \frac{(-104.0304670868 - 0.000124u)\,t^{\theta}}{\Gamma(\theta+1)} \tag{28}$$

$$+ \frac{\left(-0.1185294118u - 0.0003258536941 + 0.04259523382v + 0.022000000u^2\right) t^{2\theta}}{\Gamma(2\theta+1)} \tag{29}$$

$$B(t) = 26240 + \frac{(243.40 - 2.15z)\,t^{\theta}}{\Gamma(\theta+1)} \tag{30}$$

$$+ \frac{\left(-250u - 80.01732913 - 236.80z + 131.554z^2\right) t^{2\theta}}{\Gamma(2\theta+1)} \tag{31}$$

$$R(t) = 6000 + \frac{(740 + 0.1273u + 0.0013244v)\,t^{\theta}}{\Gamma(\theta+1)} \tag{32}$$

$$+ \frac{\left(-0.0200952382v + 0.0000598095235 - 0.1212255718u - 0.0001383u^2\right) t^{2\theta}}{\Gamma(2\theta+1)} \tag{33}$$

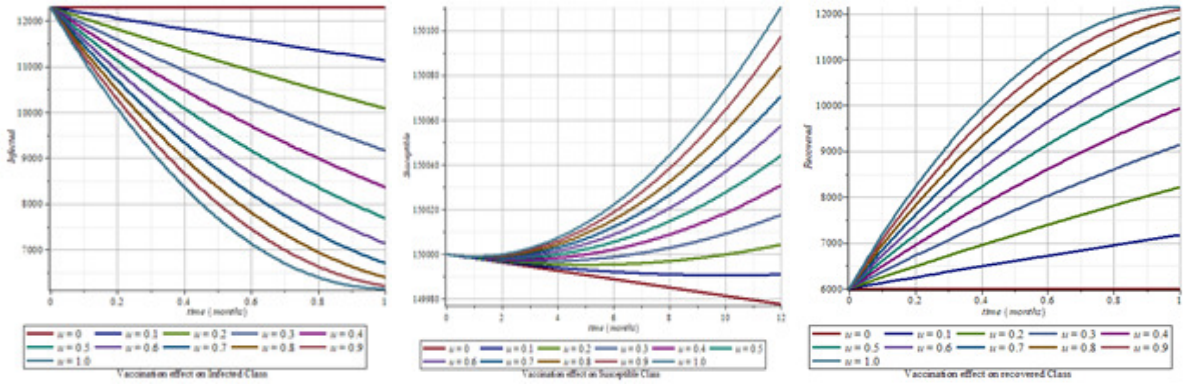
The results obtained will form the foundation for further experiments in the next section.

4.2. **Numerical Experiments.** In this section, our focus will be on using the model results for performing different scenarios of experiments to determine the most effective approach for controlling a cholera pandemic.

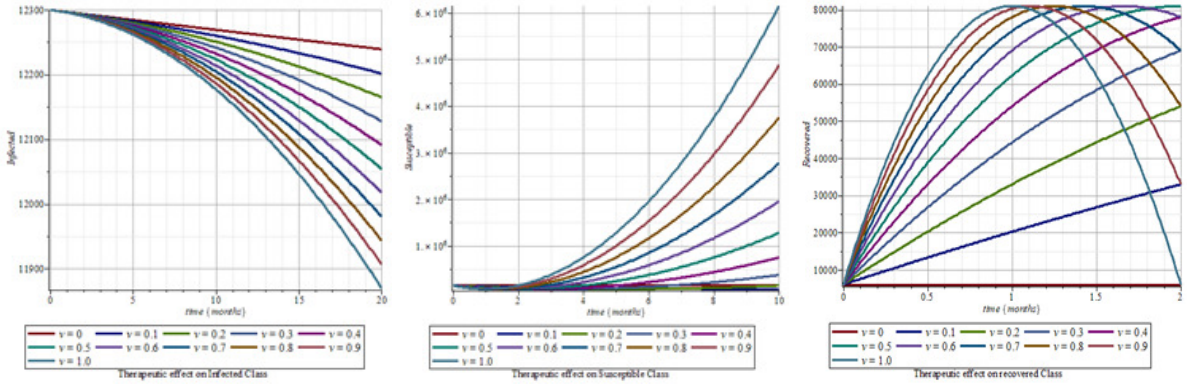
Experiment A:

Independent applications of each control function.

This experiment involves applying each control function separately, evaluating their individual effectiveness in containing the spread of cholera.



Figures 2: Dynamical impact of vaccination on SIR class



Figures 3: Dynamical impact of therapeutic actions on SIR class

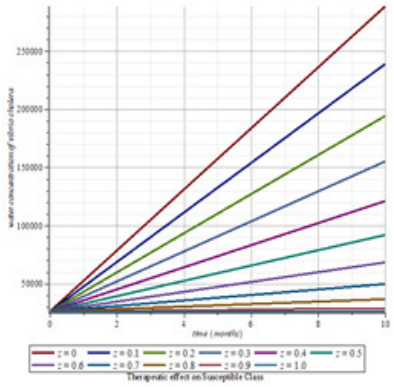


Figure 4: Impact of water sanitation practices on Vibrio cholera concentration

Experiment B:

Combined applications of the control functions

We explore the combined application of multiple control functions simultaneously. We assess the synergistic effects and potential improvements in pandemic control achieved through their combined implementation.

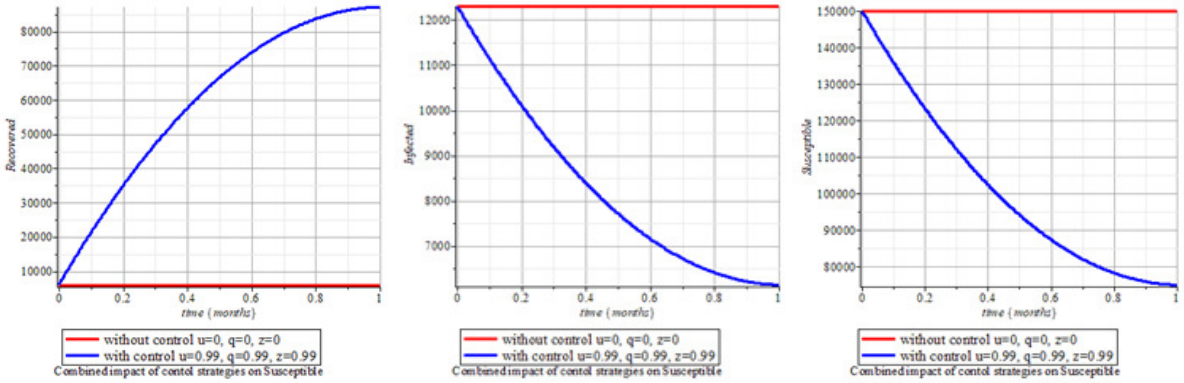


Figure 5: Impact of water sanitation practices on Vibrio cholera concentration

Experiment C:

Time-fractional analysis of the effectiveness of combine implementation of the control strategy.

In this section, our focus is to investigate the time-fractional aspect of the effectiveness of implementing the control functions for controlling the cholera outbreak. By conducting a thorough analysis of these scenarios, we aim to gain a comprehensive understanding of the behavioral dynamics of the disease. This knowledge will enable us to determine the most suitable approach for effectively managing and mitigating the

impact of a cholera pandemic. Our ultimate goal is to develop strategies that provide optimal control over the outbreak and minimize its consequences.

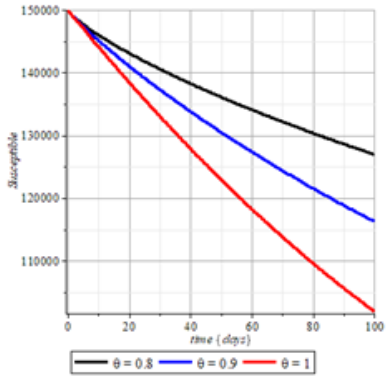


Figure 6: Time-fractional impact of individual enlightenment to partake in control scheme on of infected population

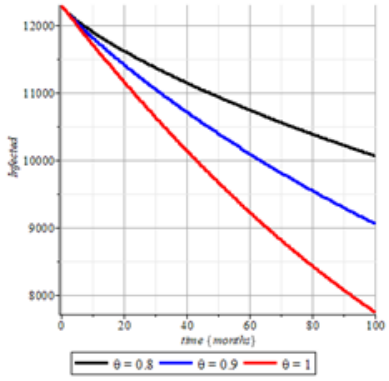


Figure 7: Time-fractional impact of individual enlightenment to partake in control scheme of infected population

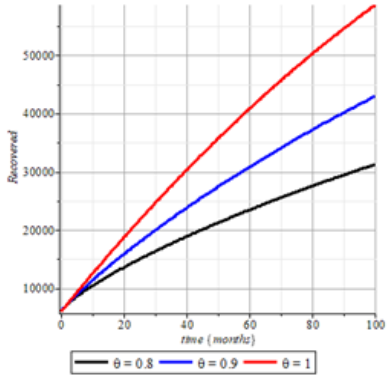


Figure 8: Time- fractional impact of individual enlightenment to partake in control scheme on Recovered population;

5. DISCUSSION

The sensitivity analysis presented in Table 2 provides valuable insights into the factors influencing the dynamics of cholera transmission. Among the parameters considered, the transmission rate parameter (φ) stands out with a sensitivity index value of 1, indicating its utmost significance in determining the spread of cholera. Variations in this parameter have a substantial impact on transmission dynamics, as demonstrated in Figure 1C.

Similarly, the recovery rate parameter (γ) exhibits a sensitivity index value of -0.9389472972, highlighting its influential negative effect on transmission dynamics. Changes in this parameter strongly affect the overall dynamics of cholera transmission, albeit in an inverse manner. Furthermore, the influx rate parameter (ρ) also plays a significant role in shaping transmission dynamics, as indicated by its sensitivity index value of 1. This parameter's impact is closely linked to the transmission rate parameter, emphasizing its importance in understanding and managing the spread of cholera. On the other hand, the death rate of Vibrio cholera bacteria (δ) has a sensitivity index value of -0.016103137445, suggesting that it has a relatively minor effect on transmission dynamics compared to the other parameters examined. However, it is still considered in the analysis, though with a smaller influence on the overall transmission dynamics of cholera. Figure 1 presents the experimental dynamics of R_0 with respect to the variation of each parameter included in the analysis.

In addition to the sensitivity analysis, the impact of vaccination, treatment, and water sanitation practices on controlling cholera transmission was investigated through a series of numerical simulations presented in Figures 2 to 5. Figure 2 focuses on the effect of vaccination on the SIR class, revealing a significant reduction in the number of susceptible individuals when vaccination is implemented. This reduction in susceptibility plays a crucial role in limiting the transmission of cholera within the population and consequently decreasing the number of infected cases. Figure 3 highlights the importance of effective treatment in controlling cholera by reducing the duration, severity, and infectiousness of the illness. By enhancing therapeutic actions such as providing appropriate medical care and rehydration therapy, the recovery rate of infected individuals improves, leading to a decrease in the infected population within the model.

Furthermore, Figure 4 underscores the vital role of water sanitation practices in preventing cholera outbreaks. Implementing measures such

as ensuring clean drinking water and proper waste disposal minimizes the contamination of water sources by Vibrio cholera bacteria. This, in turn, reduces the likelihood of individuals becoming infected. Figure 5 displays the combined impact of implementing all three control strategies concurrently on the SIR population. The figure clearly illustrates a sharp decline in the population of susceptible and infected individuals, accompanied by a steep increase in the recovered population. This highlights the importance of implementing all three control measures together to effectively eradicate the spread of cholera. Also, the long-term behavioral dynamics of the population's response to the implemented control strategies were investigated, as depicted in Figures 6–8. These figures provide insights into the effectiveness of individuals adopting the three control schemes over time. The analysis reveals the dynamic impact of individual freedom in adopting the control strategies within each population subgroup. Notably, it was discovered that a higher level of acceptability and implementation of these control factors corresponds to a more rapid eradication of the disease's spread. This underscores the crucial role played by individual awareness and acceptance in effectively controlling and preventing cholera outbreaks. The findings emphasize the significance of promoting and fostering a collective understanding and adoption of control measures to combat the spread of cholera effectively.

6. CONCLUSION

In conclusion, this paper addresses the critical healthcare issue of cholera infection in developing countries, highlighting the urgent need for effective control strategies. The proposed fractional-order mathematical model incorporates vital control parameters such as vaccination, treatment, and water sanitation practices. Through comprehensive analysis, the study establishes the well-posedness properties of the model and underscores the importance of parameter estimation through sensitivity analysis. The key findings of this study emphasize the necessity of combining multiple control strategies to effectively combat cholera outbreaks. The numerical simulations clearly demonstrate the potential impact of the proposed control measures, indicating their efficacy in reducing the spread of cholera. Additionally, the study highlights the significance of considering well-posedness properties when interpreting conclusions derived from mathematical models. Furthermore, the findings underscore the urgent need to employ a fractional-order

approach in constructing mathematical models, as it captures the disease's hereditary properties and enhances accuracy through memory description ability. However, the study acknowledges the importance of incorporating awareness programs into each population state to ensure effective participation in cholera prevention, control, and public health interventions. In summary, this research emphasizes the critical importance of integrating various control strategies and considering implementing awareness programs to effectively control cholera. Additionally, the study recommends further exploration of agent-based approaches to enhance decision-making capabilities and improve public health interventions in the fight against cholera.

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