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CONVERGENCE OF IMPLICIT NOOR ITERATIVE SEQUENCE TO THE FIXED POINT OF A GENERALIZED NON-EXPANSIVE MAPPING IN UNIFORMLY CONVEX SPACE

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ABSTRACT. We establish the convergence and stability of an implicit iterative sequence in approximating fixed points of a class of nonexpansive mappings in uniformly convex Banach space and contractive mappings in Banach space. The result obtained is an extension of several others in the literature.

1. INTRODUCTION

Let (X, d) be a metric space. A mapping $W : X \times X \times [0,1] \longrightarrow X$ is called a convex structure on X if for each $(x, y, \lambda) \in X \times X \times [0,1]$ and $u \in X$, we have

$$d(u, W(x, y, \lambda)) \leq \lambda d(u, x) + (1 - \lambda)d(u, y).$$

The metric space X, together with convex structure W, is called a convex metric space. A nonempty subset K of X is said to be convex if $W(x, y, \lambda) \in K$ for all $(x, y, \lambda) \in K \times K \times [0,1]$. All normed spaces and their convex subsets are convex metric spaces, but not all convex metric spaces are embedded in a normed space [20].

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2. PRELIMINARIES

Consider the selfmapping $T : K \longrightarrow K$ and sequences $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ of real numbers in $[0,1]$. The following scheme in convex metric spaces is called Noor iteration:

$x_n = W(x_{n-1}, Ty_{n-1}, a_n)$
$y_{n-1} = W(x_{n-1}, Tz_{n-1}, b_n)$
$z_{n-1} = W(x_{n-1}, Tx_{n-1}, c_n), \quad n = 0, 1, 2, ...$

It is well known that the Ishikawa and Mann iterations are easily obtainable from the Noor iteration. The convergence of the three-step iterations to fixed points and common fixed points have been studied extensively. For example, see Owojori and Imoru [17,18], Noor [15], Khan et al. [14], Chugh et al. [5] and so on.

Given any initial approximation $x_0 \in K$, and sequences $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ of real numbers in $[0,1]$, the implicit Noor iteration is defined in a convex metric space as:

$x_n = W(y_{n-1}, Tx_n, a_n)$
$y_{n-1} = W(z_{n-1}, Ty_{n-1}, b_n)$
$z_{n-1} = W(x_{n-1}, Tz_{n-1}, c_n), \quad n = 0, 1, 2, ...$

The Implicit Noor iterative scheme is expressed in a linear space as follows.

$$\begin{aligned} x_n &= a_n y_{n-1} + (1-a_n)Tx_n, \\ y_{n-1} &= b_n z_{n-1} + (1-b_n)Ty_{n-1} \\ z_{n-1} &= c_n x_{n-1} + (1-c_n)Tz_{n-1}, \end{aligned} \tag{1}$$

$n = 0, 1, 2, ...$, where $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ are sequences of real numbers in $[0,1]$.

The Implicit Ishikawa and Implicit Mann iterations are obtained by putting $c_n = 1$ and $c_n = b_n = 1$ respectively.

T is called a contraction if there exists $k \in [0,1)$ such that $d(Tx, Ty) \leq kd(x,y)$ for all $x, y \in X$. In case $k = 1$, so that $d(Tx, Ty) \leq d(x,y)$, the mapping T is said to be nonexpansive.

In a complete metric space, the Banach's contraction mapping principle guarantees the existence of a fixed point of a contraction mapping T, it also establishes convergence of the Picard iterative sequence $x_{n+1} = Tx_n, n = 0, 1, 2, ...$ to the fixed point. In addition, the theorem gives the following error estimates involved in the approximation of the fixed point.

$d(x_n, p) \leq k^n(1-a)^{-1}d(x_0, x_1), \ n \geq 1$ (a priori error estimate),
$d(x_n, p) \leq (1-a)^{-1}d(x_n, x_{n+1}), \ n \geq 1$ (a posteriori error estimate).

Considerable effort has been made by several authors targeted at extending the Banach's contraction principle. In this direction, Kannan [12], Zamfirescu [21], Berinde [4], Akram [2], among several others, introduced interesting classes of contractive mappings such as weak contractions, φ-contractions, A-contractions, et c.
In 2015, Chugh et al.[5] used the following contractive definition to obtain convergence and stability results for the implicit Noor iteration in convex metric spaces:

Let $\varphi : \mathbf{R}^+ \longrightarrow \mathbf{R}^+$ be a monotonic increasing function, with $\varphi(0) = 0$, such that $d(Tx, Ty) \leq \varphi(d(x, Tx)) + ad(x, y), \quad a \in [0, 1), \quad \forall x, y \in X$.

Chugh et al proved that the sequence $\{x_n\}$ defined by the implicit Noor scheme converges faster than the implicit Mann and implicit Ishikawa iterations to the fixed point of T. Moreover, implicit iterations generally converge faster than the corresponding explicit iterations. (See Theorem 13 of [5])
An iterative process $f(T, x_n)$ is said to be T-stable if and only if a sequence $\{\nu_n\}$ in X (arbitrarily close to $\{x_n\}$) converges to the fixed point of T. In 1988, Harder and Hicks [7,8] gave the formal definition of the stability of general iterative procedures. Some of the authors who contributed remarkably to the study of stability of iterative processes are Berinde[3], Imoru and Olatinwo[9], Jachymski[11], Osilike[16] and others.

Chugh et al [5] proved that the implicit Noor iteration is T-stable for the class T of quasi-contractive operators satisfying $d(Tx, Ty) \leq \varphi(d(x, Tx)) + ad(x, y), \quad a \in [0, 1), \quad \forall x, y \in X$.
Definition 1: A Banach space X is said to be uniformly convex if given any positive number ϵ there exists $\delta > 0$ such that for all $x, y \in X$ with $||x|| \leq 1, ||y|| \leq 1$ and $||x - y|| \geq \epsilon$, the inequality $||x + y|| \leq 2(1 - \delta)$ holds.

The modulus of convexity of X is the function $\delta_X : (0, 2] \longrightarrow [0, 1]$ defined by

$$\delta_X(\epsilon) = \inf\{1 - \frac{||x + y||}{2} : ||x|| = ||y|| = 1, ||x - y|| = \epsilon\}. \qquad (2)$$

X is uniformly convex if and only if $\delta_X(\epsilon) > 0 \ \forall \ \epsilon \in (0, 2]$

Several authors have presented quite a number of interesting results on the convergence of multiple-step iterations to fixed points and common fixed points in convex metric spaces. A few of them are highlighted below.

In 2006, Lin Wang [23] introduced the following iteration scheme:

$$\begin{gathered} x_1 \in K; \\ x_{n+1} = P((1-\alpha_n)x_n + \alpha_n T_1(PT_1)^{n-1}y_n); \\ y_n = P((1-\beta_n)x_n + \beta_n T_2(PT_2)^{n-1}x_n, n \geq 1, \end{gathered} \qquad (W)$$

where $\{\alpha_n\}, \{\beta_n\}$ are real sequences in $[\epsilon, 1-\epsilon]$ for some $\epsilon > 0$. He took K as a nonempty closed convex nonexpansive retract of a real uniformly convex Banach space E and $T_1, T_2 : K \longrightarrow E$ as nonself asymptotically nonexpansive mappings with sequences $\{k_n\}, \{l_n\} \subset [1, \infty), \lim_{n\to\infty} k_n = 1, \lim_{n\to\infty} l_n = 1$. In proving the convergence of the sequence generated by the scheme (W) to a common fixed point of T_1 and T_2, the following conditions were imposed:
(i) One of T_1 and T_2 must be completely continuous or demicompact;
(ii) $\sum_{n=1}^{\infty} (k_n - 1) < \infty,\ \sum_{n=1}^{\infty} (k_n - 1) < \infty$.

Shahzad and Udomene [22] obtained similar convergence results in a nonempty closed convex subset of a real uniformly convex Banach space. In their work, they studied the sequence $\{x_n\}$ generated by

$$x_1 \in K,\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n S^n[(1-\beta_n)x_n + \beta_n T^n x_n], n \geq 1, \quad (S)$$

where $\{\alpha_n\}, \{\beta_n\}$ are sequences in $[0, 1]$. Here, S and T are asymptotically quasi-nonexpansive selfmappings of K

Yildirim [24], 2013, considered the following iterative procedure:

$$x_0 \in K, \quad x_n = \alpha_n x_{n-1} + \beta_n I^n x_n + \gamma_n T_1^n +_n T_2^n x_n, \quad n \in \mathbf{N}, \quad (Y)$$

where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ and $\{\theta_n\}$ are real sequences in $(0, 1)$ satisfying $\alpha_n + \beta_n + \gamma_n + \theta_n = 1$. I is taken to be an asymptotically quasi-nonexpansive mapping, and T_1, T_2 are asymptotically quasi I-nonexpansive mappings. It was proved that if K is a nonempty closed convex subset of real uniformly convex Banach space X, and if T_1, T_2 and I are uniformly L-Lipschitzian, then the iterative sequence (Y) converges to the common fixed point of T_1, T_2 and I, provided the following conditions are satisfied:
(i) $\alpha_n + \beta_n + \gamma_n + \theta_n = 1$;
(ii) $\delta M < 1$;
(iii) $\sum_{n=1}^{\infty} (1-\alpha_n)(h_n^2 - 1) < \infty$,
where $\delta = \sup_n (1-\alpha_n), M = \sup_n h_n^2 \geq 1$.

In this work, we shall prove the convergence of the implicit Noor iteration (1) to the fixed point of the class of nonepansive mappings T satisfying

$$||Tx-Ty|| \leq \max\{||x-y||, \frac{1}{2}(||x-Tx||+||y-Ty||), \frac{1}{2}(||x-Ty||+||y-Tx||)\} \tag{3}$$

for all $x, y \in X$.

Our results are intended to be obtained with minimal conditions being imposed on the control sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$. The following Lemmas shall be useful in the proof of our main theorems.

Lemma 1 (Groetsch[6]): Let X be a uniformly convex Banach space, and let $u, v \in X$. If $||u|| \leq 1, ||v|| \leq 1$ and $||u-v|| \geq \epsilon > 0$, then $||\lambda u + (1-\lambda)v|| \leq 1 - 2\lambda(1-\lambda)\delta(\epsilon)$ for $0 \leq \lambda \leq 1$.

Lemma 2:(See [4, 5, 14]) Let δ be a real number such that $0 \leq \delta < 1$, and let $\{\epsilon_n\}_{n=0}^{\infty}$ be a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$. If a sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfies

$$u_{n+1} \leq \delta u_n + \epsilon_n, \quad n = 0, 1, 2, ...,$$

then, $\lim_{n\to\infty} u_n = 0$.

3. MAIN RESULTS

The statements and proofs of our theorems are presented in this section.

Theorem 1: Let K be a nonempty closed convex subset of a uniformly convex space X, and let a self-mapping $T : K \longrightarrow K$ satisfy inequality (3). If T has a fixed point in K, then the implicit Noor iteration (1) converges to a fixed point of T, provided $0 < \alpha \leq a_n \leq \beta < 1$.

Proof: For any $x \in K$ and $p \in F_T = \{x \in K : x = Tx\}$, using (3), with the triangle inequality $||x-Tx|| \leq ||x-p|| + ||Tx-p||$, we have

$$||Tx-p|| = ||Tx-Tp|| \leq \max\{||x-p||, \frac{||x-p|| + ||Tx-p||}{2}\},$$

that is,

$$||Tx-p|| \leq ||x-p||. \tag{4}$$

Now, from (1) and (4),

$$\begin{aligned}||z_{n-1}-p|| &= ||c_n x_{n-1}+(1-c_n)Tz_{n-1}-p|| \\ &= ||c_n(x_{n-1}-p)+(1-c_n)(Tz_{n-1}-p)|| \\ &\le c_n||x_{n-1}-p||+(1-c_n)||Tz_{n-1}-p|| \\ &\le c_n||x_{n-1}-p||+(1-c_n)||z_{n-1}-p||\end{aligned}$$

Therefore,

$$||z_{n-1}-p|| \le ||x_{n-1}-p|| \tag{5}$$

Also,

$$\begin{aligned}||y_{n-1}-p|| &= ||b_n z_{n-1}+(1-b_n)Ty_{n-1}-p|| \\ &= ||b_n(z_{n-1}-p)+(1-b_n)(Ty_{n-1}-p)|| \\ &\le b_n||z_{n-1}-p||+(1-b_n)||Ty_{n-1}-p|| \\ &\le b_n||z_{n-1}-p||+(1-b_n)||y_{n-1}-p||\end{aligned}$$

Therefore, $||y_{n-1}-p|| \le ||z_{n-1}-p|| \le ||x_{n-1}-p||$.
Moreover,

$$\begin{aligned}||x_n-p|| &= ||a_n y_{n-1}+(1-a_n)Tx_n-p|| \\ &= ||a_n(y_{n-1}-p)+(1-a_n)(Tx_n-p)|| \\ &\le a_n||y_{n-1}-p||+(1-a_n)||Tx_n-p|| \\ &\le a_n||y_{n-1}-p||+(1-a_n)||x_n-p||\end{aligned}$$

Therefore,

$$||x_n-p|| \le ||y_{n-1}-p|| \le ||z_{n-1}-p|| \le ||x_{n-1}-p||. \tag{6}$$

$\{||x_n-p||\}$ is a monotone decreasing sequence in $\mathbf{R}^+$. It therefore converges to a real number $l \ge 0$.
Let $u=\frac{y_{n-1}-p}{||x_{n-1}-p||}$, $v=\frac{Tx_n-p}{||x_{n-1}-p||}$, then $||u|| \le 1, ||v|| \le 1, ||u-v|| = \frac{||y_{n-1}-Tx_n||}{||x_{n-1}-p||} \ge \frac{||y_{n-1}-Tx_n||}{D}$, where $D=||x_0-p|| \ne 0$ Then, from Lemma 1,

$$||a_n u+(1-a_n)v|| \le 1-2a_n(1-a_n)\delta\left(\frac{||y_{n-1}-Tx_n||}{D}\right).$$

That is,

$$||x_n-p|| \le \left[1-2a_n(1-a_n)\delta\left(\frac{||y_{n-1}-Tx_n||}{D}\right)\right]||x_{n-1}-p||.$$

It follows by induction that,

$$||x_n-p|| \le D\prod_{k=1}^{n}\left[1-2a_k(1-a_k)\delta\left(\frac{||y_{k-1}-Tx_k||}{D}\right)\right].$$

Since $||y_{k-1}-Tx_k|| \le ||y_{k-1}-p||+||Tx_k-p|| \le 2||x_{k-1}-p||$, then $\{||y_{k-1}-Tx_k||\}$ is a bounded sequence, and $\limsup ||y_{k-1}-Tx_k|| =$

$h \in [0, \infty)$. There exists some natural number N such that $||y_{n_i-1} - Tx_{n_i}|| \geq \frac{h}{2}$ for all $n_i > N$. Thus, since $\delta(\epsilon)$ is nondecreasing,

$$||x_n - p|| \leq D \prod_{j=N+1}^{n} \left[1 - 2a_j(1-a_j)\delta\left(\frac{h}{2D}\right)\right]$$

and using the condition: $0 < \alpha \leq a_n \leq \beta < 1$, we obtain

$$||x_n - p|| \leq D \left[1 - 2\alpha(1-\beta)\delta\left(\frac{h}{2D}\right)\right]^{n-N}$$

Observe that $1 - 2\alpha(1-\beta)\delta\left(\frac{h}{2D}\right) \in (0,1)$. Then, $||(x_n - p)|| \to 0$ as $n \to \infty$. The sequence $\{x_n\}$ converges to p.

Theorem 2: Let K be a nonempty closed convex subset of a Banach space X, and let T be a self-mapping of K satisfying

$$||Tx - Ty|| \leq \lambda max\{||x-y||, \frac{1}{2}(||x-Tx|| + ||y-Ty||), \frac{1}{2}(||x-Ty|| + ||y-Tx||)\} \tag{7}$$

for all $x, y \in X$ with $\lambda \in [0,1)$. Then, for $x_0 \in K$, the Implicit Noor iterative sequence $\{x_n\}$ generated by (1), with $0 \leq a_n b_n c_n < \beta < 1$, converges to a fixed point of T.

Proof. Let $x \in K$ and $p \in F_T$. Then (7) yields

$$||Tx - p|| \leq \lambda ||x - p||. \tag{8}$$

Therefore, given $x_0 \in K$,

$$\begin{aligned} ||x_n - p|| &= ||a_n(y_{n-1} - p) + (1-a_n)(Tx_n - p)|| \\ &\leq a_n||y_{n-1} - p|| + (1-a_n)||Tx_n - Tp|| \\ &\leq a_n||b_n(z_{n-1} - p) + (1-b_n)(Ty_{n-1} - p)|| + \lambda(1-a_n)||x_n - p|| \end{aligned}$$

That is,

$$\begin{aligned} ||x_n - p|| &\leq \tfrac{a_n}{1-\lambda(1-a_n)}||b_n(z_{n-1} - p) + (1-b_n)(Ty_{n-1} - p)|| \\ &\leq \tfrac{a_n b_n}{1-\lambda(1-a_n)}||c_n(x_{n-1} - p) + (1-c_n)(Tz_{n-1} - p)|| + \tfrac{\lambda a_n(1-b_n)}{1-\lambda(1-a_n)}||y_{n-1} - p|| \\ &\leq \tfrac{a_n b_n c_n}{1-\lambda(1-a_n)}||x_{n-1} - p|| + \tfrac{\lambda a_n b_n(1-c_n)}{1-\lambda(1-a_n)}||z_{n-1} - p|| + \tfrac{\lambda a_n(1-b_n)}{1-\lambda(1-a_n)}||y_{n-1} - p|| \end{aligned}$$

Since $||y_{n-1} - p|| \leq ||z_{n-1} - p|| \leq ||x_{n-1} - p||$, we have

$$||x_n - p|| \leq \frac{a_n b_n c_n(1-\lambda) + \lambda a_n}{1 - \lambda(1-a_n)}||x_{n-1} - p||.$$

Applying the condition $0 \leq a_n b_n c_n < \beta < 1$, we have

$$||x_n - p|| \leq \frac{\beta(1-\lambda) + \lambda a_n}{1 - \lambda(1-a_n)}||x_{n-1} - p||$$

Now, put $k = \frac{\beta(1-\lambda)+\lambda a_n}{1-\lambda(1-a_n)}$, then,

$$\begin{aligned} 1-k &= \tfrac{(1-\beta)(1-\lambda)}{1-\lambda(1-a_n)} \\ &\geq (1-\beta)(1-\lambda), \end{aligned}$$

so that $0 \leq k \leq 1-(1-\beta)(1-\lambda) < 1$
Thence,

$$\begin{aligned} ||x_n - p|| &\leq k||x_{n-1} - p|| \\ &\leq k^n||x_0 - p||. \end{aligned}$$

As $n \to \infty$, we have $||x_n - p|| \to 0$, so that the sequence $\{x_n\}$ converges to p.

Next we prove that the implicit Noor iteration is T-stable. Lemma 2 shall be useful in our proof.

Theorem 3: Let K be a nonempty closed convex subset of a Banach space X, and let T be a self-mapping of K satisfying (7) with $F_T \neq \phi$. Then, for $x_0 \in K$, the Implicit Noor iterative sequence $\{x_n\}$ generated by (1), with $0 \leq a_n b_n c_n < \beta < 1$, is T-stable.

Proof. Suppose $\{\nu_n\}_{n=0}^{\infty} \subset K$ and $\epsilon_n = ||\nu_n - [a_n\mu_{n-1} + (1-a_n)T\nu_n]||$, $n = 1, 2, 3, ...$, where $\mu_{n-1} = b_n\omega_{n-1} + (1-b_n)T\mu_{n-1}$, $\omega_{n-1} = c_n\nu_{n-1} + (1-c_n)\omega_{n-1}$. Let $\epsilon_n \to 0$ as $n \to \infty$.
Then, using (8),

$$\begin{aligned} ||\nu_n - p|| &\leq ||\nu_n - [a_n\mu_{n-1} + (1-a_n)T\nu_n]|| + ||a_n\mu_{n-1} + (1-a_n)T\nu_n - p|| \\ &= \epsilon_n + ||a_n(\mu_{n-1} - p) + (1-a_n)(T\nu_n - p)|| \\ &\leq \epsilon_n + a_n||\mu_{n-1} - p|| + (1-a_n)||T\nu_n - p|| \\ &\leq \epsilon_n + a_n||\mu_{n-1} - p|| + (1-a_n)\lambda||\nu_n - p|| \\ &\leq \tfrac{\epsilon_n}{1-\lambda(1-a_n)} + \tfrac{a_n}{1-\lambda(1-a_n)}[b_n||\omega_{n-1} - p|| + (1-b_n)||T\mu_{n-1} - p||] \\ &\leq \tfrac{\epsilon_n}{1-\lambda(1-a_n)} + \tfrac{a_n b_n}{1-\lambda(1-a_n)}[c_n||\nu_{n-1} - p|| + (1-c_n)||T\omega_{n-1} - p||] \\ &\quad + \tfrac{\lambda a_n(1-b_n)}{1-\lambda(1-a_n)}||\mu_{n-1} - p|| \\ &\leq \tfrac{\epsilon_n}{1-\lambda(1-a_n)} + \tfrac{a_n b_n}{1-\lambda(1-a_n)}[c_n||\nu_{n-1} - p|| + \lambda(1-c_n)||\omega_{n-1} - p||] \\ &\quad + \tfrac{\lambda a_n(1-b_n)}{1-\lambda(1-a_n)}||\mu_{n-1} - p|| \\ &\leq \tfrac{\epsilon_n}{1-\lambda(1-a_n)} + \tfrac{a_n b_n c_n(1-\lambda)+\lambda a_n}{1-\lambda(1-a_n)}||\nu_{n-1} - p||, \end{aligned}$$

since it is also true that $||\mu_{n-1} - p|| \leq ||\omega_{n-1} - p|| \leq ||\nu_{n-1} - p||$.

As in the proof of Theorem 2, we have $k \in [0, 1)$ such that

$$||\nu_n - p|| \leq k||\nu_{n-1} - p|| + \frac{\epsilon_n}{1-\lambda(1-a_n)}.$$

Since the sequence $\{a_n\}$ is bounded between 0 and 1, and $\lim_{n\to\infty} \epsilon_n = 0$, by Lemma 2 we have $\lim_{n\to\infty} ||\nu_n - p|| = 0$.

Conversely, if $\lim_{n\to\infty} \nu_n = p$ then,

$$\begin{aligned}\epsilon_n &= ||\nu_n - (a_n\mu_{n-1} + (1-a_n)T\nu_n)|| \\ &\leq [1+\lambda(1-a_n)]||\nu_n - p|| + [\beta(1-\lambda) + \lambda a_n]||\nu_{n-1} - p|| \\ &\leq (1+\lambda+\beta-\lambda\beta)||\nu_{n-1} - p|| \to 0 \ as \ n \to \infty.\end{aligned}$$

This completes the proof.

4. CONCLUDING REMARKS

The classes of nonexpansive mappings (3) and contractions (7) contain several other prominent classes of mappings in fixed point theory and applications, including the celebrated Banach contractions. Chugh et al emphasized that the implicit Noor iteration has better convergence rate compared with Mann, Ishikawa, Noor, implicit Mann and implicit Ishikawa iterative procedures.

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