Journal of the Nigerian Mathematical Society
Vol.44, Issue 2, pp.229 - 243, 2025

CONTROL OF SOME NONLINEAR FUZZY SYSTEMS WITH FUZZY CONTROL INPUTS

B. O. ONASANYA[1], O. I. OLUSOLA[2], J. FELIX[3], N. M. ADO[4] AND O. C. AKEREMALE[5]

ABSTRACT. The study of control theory has become very popular with mathematicians, engineers and computer scientists because of its wide applications in communications and the design of artificially intelligent machines. More often than not, researchers focus on building models whose control parameters are crisp when the real life situations they are modelling are fuzzy. In the recent times, some have built models which have fuzziness into the control input. The limitation of this is that, while control may be fuzzy, uncertainty and the fuzziness in the system is not captured. This research focuses on modelling a system whose information matrix and the control input(s) are both fuzzy. The model developed was applied to a real life Chua electrical system and the numerical simulation confirms that the model is more efficient.

1. NOTATIONS, SYMBOLS AND PRELIMINARIES

Henceforth, $\mathbb{P}$ is a positive definite symmetric constant matrix whose transpose is $\mathbb{P}^T = \mathbb{P} > 0$ ($\mathbb{P} > 0 \Rightarrow \lambda_i > 0$, where $\lambda_i' s$ are the eigenvalues of $\mathbb{P}$), and $\lambda_N(\mathbb{P}), \lambda_X(\mathbb{P})$ represent the eigenvalue of $\mathbb{P}$ with minimum value and eigenvalue of $\mathbb{P}$ with maximum value, respectively. Consider $\mathbb{R}$ as the set of real numbers and $||y|| = (\sum_{i=1}^{n} y_i^2)^{\frac{1}{2}}$ as the Euclidean norm on $y \in \mathbb{R}^n$. Fuzzy less than, approximately and approximately less or equal to are respectively $\lesssim$, $\approxeq$ and $\lessapprox$.

Received by the editors October 15, 2024; Revised: March 03, 2025; Accepted: April 04, 2025

www.nigerianmathematicalsociety.org; Journal available online at https://ojs.ictp. it/jnms/

2010 *Mathematics Subject Classification.* 15B15, 37G35, 93C42, 37N35.

Key words and phrases. Fuzzy systems, Fuzzy system parameter, Fuzzy control inputs.

2. BACKGROUND TO THE STUDY

Many methods has been deployed to stabilize non-linear system [3]. Some of these methods include switching control system, impulsive control system, intermittent control system e.t.c [1]. Pulse control system has been applied in different fields such as artificial satellites, information communication technology and chaotic synchronization [18]. Most researchers considered an instantaneous stochastic impulse occurring at some specific points in time. But [3] introduced two time windows which are subdivisions of the period T within which the control input can occur alternately. In 2019, Feng *et al*[4] introduced a kind of bounded control which occurs at any point within the time window of the period. This method, however still introduces a crisp control which does not take care of the uncertainties or fuzziness that normally would occur in real life.

Zadeh [18], in his seminal paper, was able to establish that physical and electrical systems require fuzziness because the classical method cannot adequately give a general solution to the uncertainties that may occur in real life. He was able to construct a membership function such that each element of a set could have partial membership.

Moreover, [20] was able to come up with a realistic problems, in which case the control input was fuzzy and the impulsive control system of such better models the alternate control system was. In other words, his research was able to take care of the uncertainties that may arise with $\mathbb{P}$. One limitation of the ensuing model is that the Linear Matrix Inequality (LMI) obtained was very lengthy and might require a more complicated computer skill or complexity. [22] was able to investigate similar situation but introduced a delay to the system, which makes a significant improvement to the previous work that was fuzzy but the challenges encountered at [20] previously still remain.

The goal of this paper is to develop a model that leads to a simpler LMI and that responds better to uncertainties in a more realistic way. Indeed, we developed a system where the control inputs and the system information matrix are all fuzzy. This system will better represent realities.

3. MOTIVATION AND MODEL FORMULATION

A nonlinear system of the form

$$\begin{cases} \dot{x}(t) = \mathbb{A}x(t) + f(x(t)) + u(t), \\ x(t_0) = x_0, \end{cases} \tag{1}$$

is such that $x(t) \in I\!R^n$ is the state variable, $\mathbb{A} \in I\!R^{n\times n}$ is a constant matrix which contains the system information, and the value of mapping $f : I\!R^n \longrightarrow I\!R^n$ at 0 is 0. Besides, $\mathbb{D} = diag(d_1, d_2, \cdots, d_n) \geq 0$ is a diagonal matrix such that $||f(x)||^2 \leq x^T \mathbb{D}x$.

Consider that

$$u(t) = \begin{cases} Kx(t), nT \leq t < nT + \mu, \\ 0, nT + \mu \leq t < (n+1)T. \end{cases} \tag{2}$$

Then, $u(t)$ in system (1) is as illustrated in (2), where K is the control that occurs intermittently. But Feng *et al.* [3] proposed an alternate control

$$u(t) = \begin{cases} \mathbb{C}_1 x(t), nT \leq t < nT + \mu, \\ \mathbb{C}_2 x(t), nT + \mu \leq t < (n+1)T, \end{cases} \tag{3}$$

where constant matrices $\mathbb{C}_1, \mathbb{C}_2 \in \mathbb{R}^{n\times n}$ are control matrices, the nonnegative T is the period and $\mu \in (0, T)$.

Thus, (1) becomes

$$\begin{cases} \dot{x}(t) = \mathbb{A}x(t) + f(x(t)) + \mathbb{C}_1 x(t), nT \leq t < nT + \mu, \\ \dot{x}(t) = \mathbb{A}x(t) + f(x(t)) + \mathbb{C}_2 x(t), nT + \mu \leq t < (n+1)T. \end{cases} \tag{4}$$

In case where $\mathbb{C}_2 = 0$, (2) is obtained from (3).

However, no such precise values of $\mathbb{C}_1$ and $\mathbb{C}_2$ exist in real life [4]. Hence, Onasanya *et al.* [20] introduced fuzzy intensity (readers can also check [21]) of the controls set as

$$\mathbb{C}_1 \leq \mathbb{C}_{\alpha_1},\ \mathbb{C}_2 \leq \mathbb{C}_{\alpha_2}$$

where $\alpha_1 \mathbb{C}_1 = \mathbb{C}_{\alpha_1}, \alpha_2 \mathbb{C}_2 = \mathbb{C}_{\alpha_2}$ are fuzzy control matrices, with $\alpha_1, \alpha_2 \in [0, 1]$.

In this case, the control is either fuzzy intermittent (when either α_1 or α_2 is zero) or fuzzy alternate (when $\alpha_1, \alpha_2 \in (0, 1]$).

Modifying (4), we obtain

$$\begin{cases} \dot{x}(t) \approxeq \mathbb{A}x(t) + f(x(t)) + \mathbb{C}_{\alpha_1} x(t), nT \leq t < nT + \mu_n, \\ \dot{x}(t) \approxeq \mathbb{A}x(t) + f(x(t)) + \mathbb{C}_{\alpha_2} x(t), nT + \mu_n \leq t < (n+1)T. \end{cases} \tag{5}$$

Thus, the fuzzified form of the control (3) is

$$u(t)_\alpha = \begin{cases} \mathbb{C}_{\alpha_1} x(t), nT \leq t < nT + \mu, \\ \mathbb{C}_{\alpha_2} x(t), nT + \mu \leq t < (n+1)T. \end{cases} \tag{6}$$

(6) is equivalent to

$$u(t)_\alpha = \begin{cases} \alpha_1 \mathbb{C}_1 x(t), nT \leq t < nT + \mu, \\ \alpha_2 \mathbb{C}_2 x(t), nT + \mu \leq t < (n+1)T. \end{cases} \tag{7}$$

From the foregoing, it can be seen that both the intermittent (2) and alternate control (3) have been generalized by (6).

However, the system parameter matrix $\mathbb{A}$ has always been left crisp, which is also not realistic. This paper was motivated by the works of [3] and [20]. It is of interest to build a model with fewer LMI terms in which, not only the control inputs are fuzzy but also, the system parameter matrix $\mathbb{A}$ is fuzzy. This system will be much more flexible to adapt to changing uncertainties in the real life situations it models.

4. PRELIMINARY

Definition 1:[18] A fuzzy subset $\mathbb{A}$ of X, with grade of membership of $x \in \mathbb{A}$ as $\mu_{\mathbb{A}} : X \longrightarrow [0,1]$, is the ordered pair $\mathbb{A} = \{(x, \mu_{\mathbb{A}}(x)) : x \in X\}$.
Lemma 1:[11] Let Δ_1, Δ_2 be $n \times s$ real matrices and M an $n \times n$ real matrix such that matrix $M = M^T > 0$, then

$$\Delta_1^T \Delta_2 + \Delta_2^T \Delta_1 \leq \Delta_1^T M \Delta_1 + \Delta_2^T M^{-1} \Delta_2. \tag{8}$$

Remark 1:[3, 4, 20] Note that for $M = M^T$, $(\alpha M)^T = \alpha M^T$ for a real α. Let $M = \varepsilon \Delta_3$, where $\varepsilon > 0$ is a scalar. Then, $M^T = (\varepsilon \Delta_3)^T = \varepsilon \Delta_3^T = \varepsilon \Delta_3$ and $M^{-1} = (\varepsilon \Delta_3)^{-1} = \varepsilon^{-1} \Delta_3^{-1}$. Hence, the inequality in (8) becomes

$$\Delta_1^T \Delta_2 + \Delta_2^T \Delta_1 \leq \varepsilon \Delta_1^T \Delta_3 \Delta_1 + \varepsilon^{-1} \Delta_2^T \Delta_3^{-1} \Delta_2. \tag{9}$$

This result implies that

$$2x^T y \leq \varepsilon x^T x + \frac{1}{\varepsilon} y^T y. \tag{10}$$

Lemma 2:[1] The LMI

$$\begin{bmatrix} M(y) & W(y) \\ W(y)^T & N(y) \end{bmatrix} > 0,$$

such that $M(y) = M(y)^T, N(y) = N(y)^T$ and $W(y)^T$ affinely depend on y, is equivalent to

$$N(y) > 0, \;\; M(y) - W(y) N(y)^{-1} W(y)^T > 0.$$

5. MAIN RESULTS

5.1 Classical (Non-fuzzy) Control

Theorem 1: Subject to the fact that

$$||x(t)|| < \sqrt{\frac{\lambda_X(\mathbb{P})}{\lambda_N(\mathbb{P})}}||x_0||e^{[-\beta(t-T)]},$$

where $\beta = \frac{|a|\mu - b(T-\mu)}{2T} > 0$, then the solution of the system (4) assumes exponential stability given that there exist constants $\delta_1, \delta_2, b > 0$, $a < 0$ and a square matrix $\mathbb{P}^T = \mathbb{P} > 0$ such that

(1) $\delta_1\mathbb{P}^2 + \frac{3}{\delta_1}(\mathbb{A}^T\mathbb{A} + \mathbb{D} + \mathbb{C}_1^2) + a\mathbb{P} \leq 0$,

(2) $\delta_2\mathbb{P}^2 + \frac{3}{\delta_2}(\mathbb{A}^T\mathbb{A} + \mathbb{D} + \mathbb{C}_2^2) - b\mathbb{P} \leq 0$

(3) $|a|\mu - b(T-\mu) > 0$, for any $t > 0$.

Proof: Consider the function

$$V = x^T\mathbb{P}x, \tag{11}$$

whence

$$\lambda_N(\mathbb{P})||x||^2 \leq V \leq \lambda_X(\mathbb{P})||x||^2. \tag{12}$$

For $nT \leq t < nT + \mu_m$, using (4), (9), (10) and (11)

$$\begin{aligned}
\dot{V}(x(t)) &= 2x^T\mathbb{P}\dot{x} \\
&\leq \delta_1(\mathbb{P}x)^T(\mathbb{P}x) + \frac{1}{\delta_1}\dot{x}^T\dot{x} \\
&\leq \delta_1 x^T\mathbb{P}^2 x + \frac{1}{\delta_1}[\mathbb{A}x + f(x) + \mathbb{C}_1]^T[\mathbb{A}x + f(x) + \mathbb{C}_1] \\
&\leq x^T[\delta_1\mathbb{P}^2 + \frac{3}{\delta_1}(\mathbb{A}^T\mathbb{A} + \mathbb{D} + \mathbb{C}_1^2) + a\mathbb{P}]x - aV \\
&\leq -aV, \text{for some real number } a.
\end{aligned} \tag{13}$$

In particular,

$$V(x(t)) \leq V(x(nT)^-)e^{-a(t-nT)}. \tag{14}$$

Similarly, for $nT + \mu \leq t < (n+1)T$,

$$\begin{aligned}
\dot{V}(x(t)) &= 2x^T\mathbb{P}\dot{x} \\
&= x^T[\delta_2\mathbb{P}^2 + \frac{3}{\delta_2}(\mathbb{A}^T\mathbb{A} + \mathbb{D} + \mathbb{C}_2^2) - b\mathbb{P}]x + bV \\
&\leq bV, \text{for some real number } b.
\end{aligned} \tag{15}$$

This implies that

$$V(x(t)) \leq V(x(nT+\mu)^-)e^{b(t-nT-\mu)}. \tag{16}$$

From the proof in [3], for any positive t, we can obtain that

1) if $nT \leq t < nT+\mu$, i.e., $\frac{t-\mu}{T} < n \leq \frac{t}{T}$, then

$$V(x(t)) \leq V(x_0)e^{-na\mu-a(t-nT)+nb(T-\mu)}. \tag{17}$$

So

$$V(x((nT+\mu)^-)) \leq V(x_0)e^{-(n+1)a\mu+nb(T-\mu)}.$$

2) Also, if $nT+\mu \leq t < (n+1)T$, i.e., $\frac{t}{T} < n+1 \leq \frac{t+T-\mu}{T}$, then we have that

$$\begin{aligned} V(x(t)) &\leq V(x((nT+\mu)^-))e^{b(t-nT-\mu)} \\ &\leq V(x_0)e^{-(n+1)a\mu+nb(T-\mu)+b(t-nT-\mu)}. \end{aligned} \tag{18}$$

Case 1 If $b \geq 0$: From (17) we can have

$$V(x(t)) \leq V(x_0)e^{-na\mu+nb(T-\mu)}, \tag{19}$$

where $nT \leq t < nT+\mu$. Then, $\exists\ a < 0$ with $|a| > b$ and $-n|a|\mu + nb(T-\mu) < 0$ such that

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-n|a|\mu+nb(T-\mu)} \\ &\leq V(x_0)e^{-na\mu+nb(T-\mu)} \end{aligned} \tag{20}$$

Hence, from (20),

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-n|a|\mu+nb(T-\mu)} \\ &= V(x_0)e^{-(|a|\mu-b(T-\mu))n} \\ &\leq V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-\mu}{T}} \\ &\leq V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-T}{T}}. \end{aligned} \tag{21}$$

Also, from (18) we have

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-(n+1)a\mu+nb(T-\mu)} \\ &\leq V(x_0)e^{-(n+1)a\mu+(n+1)b(T-\mu)}, \end{aligned} \tag{22}$$

where $nT+\mu \leq t < (n+1)T$ and $\frac{t}{T} < (n+1)$.
Then, there exists such $a < 0$ with $|a| > b$ such that

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-(n+1)|a|\mu+(n+1)b(T-\mu)} \\ &\leq V(x_0)e^{-(n+1)a\mu+(n+1)b(T-\mu)}. \end{aligned} \tag{23}$$

Hence, from (23), we have

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-(n+1)|a|\mu+(n+1)b(T-\mu)} \\ &= V(x_0)e^{-(|a|\mu-b(T-\mu))(n+1)} \\ &\leq V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t}{T}} \\ &\leq V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-\mu}{T}} \\ &\leq V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-T}{T}}. \end{aligned} \tag{24}$$

Case 2 If $b < 0$:
From (17) we can have

$$V(x(t)) \leq V(x_0)e^{-na\mu+nb(T-\mu)}, \tag{25}$$

where $nT+\mu \leq t < (n+1)T$ and $\frac{t}{T} < (n+1)$. Then, $\exists\ a < 0$ with $|a| > b$ and $-n|a|\mu+nb(T-\mu) < 0$ such that

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-n|a|\mu+nb(T-\mu)} \\ &\leq V(x_0)e^{-na\mu+nb(T-\mu)} \end{aligned} \tag{26}$$

Hence, from (26),

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-n|a|\mu+nb(T-\mu)} \\ &= V(x_0)e^{-(|a|\mu-b(T-\mu))n} \\ &\not\leq V(x_0)e^{-(|a|\mu-b(T-\mu))(n+1)} \\ &< V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t}{T}} \\ &< V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-\mu}{T}} \\ &< V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-T}{T}}. \end{aligned} \tag{27}$$

Hence, the inequalities (21) and (24) are not obtainable.

Also, from (18) we have

$$V(x(t)) \leq V(x_0)e^{-(n+1)a\mu+nb(T-\mu)}, \tag{28}$$

where $nT+\mu \leq t < (n+1)T$ and $\frac{t}{T} < (n+1)$.

Then, such $a < 0$, with $|a| > b$, exists such that

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-(n+1)|a|\mu+nb(T-\mu)} \\ &\leq V(x_0)e^{-(n+1)a\mu+nb(T-\mu)}. \end{aligned} \tag{29}$$

Hence, from (29), we have

$$\begin{aligned} V(x(t)) &\leq V(x_0)e^{-(n+1)|a|\mu+nb(T-\mu)} \\ &\not\leq V(x_0)e^{-(n+1)|a|\mu-(n+1)b(T-\mu)} \\ &= V(x_0)e^{-(|a|\mu+b(T-\mu))(n+1)} \\ &\leq V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t}{T}} \\ &< V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-\mu}{T}} \\ &< V(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-T}{T}}. \end{aligned} \tag{30}$$

$$V(x(t)) \leq V(x_0)e^{-(|a|\mu - b(T-\mu))\frac{t-T}{T}}, \tag{31}$$

and $V(x(t)) \to 0$ as $t \to \infty$ for $|a|\mu - b(T-\mu) > 0$, and by (11), (12) and (31),

$$||x(t)|| < \sqrt{\frac{\lambda_x(\mathbb{P})}{\lambda_n(\mathbb{P})}}||x_0||e^{[-\beta(t-T)]},$$

where $\beta = \frac{|a|\mu - b(T-\mu)}{2T}$.

Furthermore, conditions (1) and (2) in Theorem 1 are respectively equivalent to

$$\begin{bmatrix} \frac{3}{\delta_1}(\mathbb{A}^T\mathbb{A} + \mathbb{D} + \mathbb{C}_1^2) + a\mathbb{P} & -\mathbb{P} \\ -\mathbb{P} & -\delta_1^{-1}I \end{bmatrix} \leq 0, \tag{32}$$

and

$$\begin{bmatrix} \frac{3}{\delta_2}(\mathbb{A}^T\mathbb{A} + \mathbb{D} + \mathbb{C}_2^2) - b\mathbb{P} & -\mathbb{P} \\ -\mathbb{P} & -\delta_2^{-1}I \end{bmatrix} \leq 0,. \tag{33}$$

5.2 Fuzzy Control

Also, we can modify system (5) to become

$$\begin{cases} \dot{x}(t) \approxeq \mathbb{A}_{\alpha_3}x(t) + f(x(t)) + \mathbb{C}_{\alpha_1}x(t), nT \leq t < nT + \mu_n, \\ \dot{x}(t) \approxeq \mathbb{A}_{\alpha_3}x(t) + f(x(t)) + \mathbb{C}_{\alpha_2}x(t), nT + \mu_n \leq t < (n+1)T. \end{cases} \tag{34}$$

in which case we can obtain a fuzzy symmetric positive definite matrix $\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)}$ which depends on the uncertainties $\alpha_i \in (0,1](i = 1,2,3)$ in the system. Hence the following theorem is a generalised form of Theorem 1.

Theorem 2: Subject to the fact that

$$||x(t)|| < \sqrt{\frac{\lambda_X(\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)})}{\lambda_N(\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)})}}||x_0||e^{[-\beta(t-T)]},$$

where $\beta = \frac{|a|\mu - b(T-\mu)}{2T} > 0$, then the solution of the system (34) assumes exponential stability given that there exist constants $\delta_1, \delta_2, b > 0$, $a < 0$ and a square matrix $\mathbb{P}^T_{(\alpha_1\alpha_2\alpha_3)} = \mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} > 0$ such that

(1) $\delta_1\mathbb{P}^2_{(\alpha_1\alpha_2\alpha_3)} + \frac{3}{\delta_1}(\mathbb{A}^T_{\alpha_3}\mathbb{A}_{\alpha_3} + \mathbb{D} + \mathbb{C}^2_{\alpha_1}) - a\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} \leq 0$,
(2) $\delta_2\mathbb{P}^2_{(\alpha_1\alpha_2\alpha_3)} + \frac{3}{\delta_2}(\mathbb{A}^T_{\alpha_3}\mathbb{A}_{\alpha_3} + \mathbb{D} + \mathbb{C}^2_{\alpha_2}) - a\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} \leq 0$,
(3) $|a|\mu - b(T-\mu) > 0$, for any $t > 0$.

Proof: For $nT \leq tT + \mu$, it can be seen from the proof of Theorem 1 that

$$\dot{V}_{\alpha}(x(t)) \lessapprox -aV_{\alpha}. \tag{35}$$

Particularly,

$$V_{\alpha}(x(t)) \lessapprox V_{\alpha}(x(nT)^{-})e^{(-a(t-nT))}.$$

Similarly, for $nT+\mu \leq t < (n+1)T$,

$$\dot{V}_{\alpha}(x(t)) \quad \lessapprox bV_{\alpha}. \tag{36}$$

Furthermore, for any positive t,

$$V_{\alpha}(x(t)) \lesssim V_{\alpha}(x_0)e^{-(|a|\mu-b(T-\mu))\frac{t-T}{T}}. \tag{37}$$

If $\beta = \frac{|a|\mu-b(T-\mu)}{2T} > 0$, $V_{\alpha}(x(t))$ approaches zero as t tends to become large, and by (11), (12) and (37),

$$||x(t)|| \lesssim \sqrt{\frac{\lambda_X(\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)})}{\lambda_N(\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)})}}||x_0||e^{[-\beta(t-T)]}.$$

In addition, condition (1) and (2) in Theorem 2 are respectively equivalent to

$$\begin{bmatrix} \frac{3}{\delta_1}(\mathbb{A}_{\alpha_3}^T\mathbb{A}_{\alpha_3}+\mathbb{D}+\mathbb{C}_{\alpha_1}^2)+a\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} & -\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} \\ -\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} & -\delta_1^{-1}I \end{bmatrix} \leq 0, \tag{38}$$

and

$$\begin{bmatrix} \frac{3}{\delta_2}(\mathbb{A}_{\alpha_3}^T\mathbb{A}_{\alpha_3}+\mathbb{D}+\mathbb{C}_{\alpha_2}^2)+bP_{(\alpha_1\alpha_2\alpha_3)} & -\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} \\ -\mathbb{P}_{(\alpha_1\alpha_2\alpha_3)} & -\delta_2^{-1}I \end{bmatrix} \leq 0. \tag{39}$$

6. NUMERICAL SIMULATIONS

Remark 2: It is important to note that the control matrices $\mathbb{C}_1$ and $\mathbb{C}_2$ in [3] will not simultaneously produce the $\mathbb{P}$ and the control graph it reported.

Example 1: Consider the Chua's system

$$\begin{cases} \dot{x}_1 &= \varpi(x_2-x_1-i(x_1)) \\ \dot{x}_2 &= x_1-x_2-x_3 \\ \dot{x}_3 &= -\eta x_2 \end{cases} \tag{40}$$

for which

$i(x_1) = dx_1+0.5(c-d)(|x_1+1|-|x_1-1|), c<d<0$ are constants.

Then, (40) can be written as

$$\dot{\mathrm{x}} = \mathbb{A}\mathrm{x} + h(\mathrm{x}),$$

where $\varpi = -9.2156$, $\eta = 15.9946$, $c = -1.24905$, $d = -0.75735$ and

$$\mathbb{A} = \begin{bmatrix} -2.2362 & 9.2156 & 0 \\ 1 & -1 & 1 \\ 0 & -15.9946 & 0 \end{bmatrix}.$$

For

$$h(x) = \begin{bmatrix} -0.5(c-d)(|x+1|-|x-1|) \\ 0 \\ 0 \end{bmatrix},$$

and

$$||f(x)||^2 \leq \omega^2(c-d)^2,$$

it can be taken that $\mathbb{D} = diag(\omega^2(c-d)^2, 0, 0)$.

In what follows in Figure 1 is the chaotic graph of the Chua's system 40.

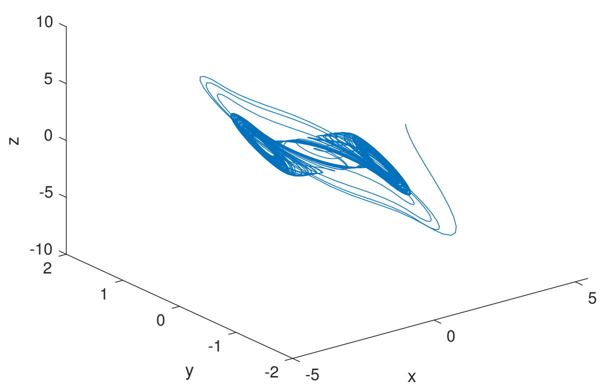

FIGURE 1. Chaotic Classical Chua's System

When $T = 10$, $\mu = 3$, $-a\mu - b(T-\mu) > 0$ and $a = -4.6667 \times 10^5$, then $b = 2.0 \times 10^5$. Choosing

$$\mathbb{C}_1 = diag(-3.5, -2.1, -1.2),$$

$$\mathbb{C}_2 = diag(-3.0, -1.0, -1.0),$$

and setting these as the constraints for the LMI (32), a feasible solution $\mathbb{P}$ was obtained. The chaotic system was controlled as shown in Figure 2.

$$\mathbb{P} = \begin{bmatrix} 0.1273 & 0.0013 & 0.0011 \\ 0.0013 & 0.1088 & 0.0000 \\ 0.0011 & 0.0000 & 0.0920 \end{bmatrix} \times 10^{-03}$$

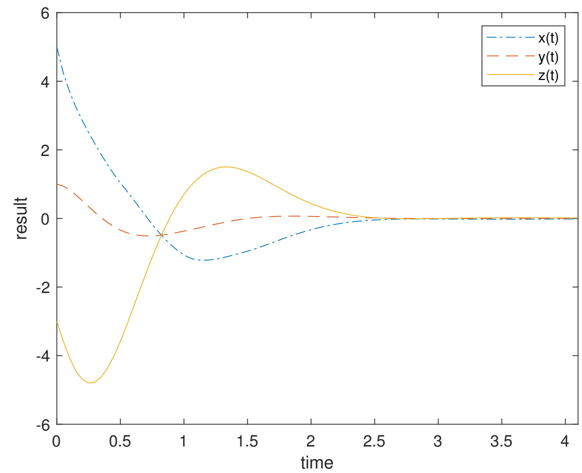

FIGURE 2. Controlled Classical Chua's System

Even for fuzzy matrix $\mathbb{A}_{0.9}$, the system (40) is still chaotic as shown in Figure 3. Hence, the system parameters can be around what they usually are.

Also, choosing

$$\mathbb{C}_{\alpha_1} = \mathbb{C}_{0.8} = diag(-2.8, -1.68, -0.96),$$

$$\mathbb{C}_{\alpha_2} = \mathbb{C}_{0.9} = diag(-2.7, -0.9, -0.9),$$

and

$$\mathbb{A}_{0.9} = \begin{bmatrix} -1.8113 & 7.4646 & 0 \\ 0.8100 & -0.8100 & 0.8100 \\ 0 & -12.9556 & 0 \end{bmatrix}$$

as the constraints for the LMIs (38), we obtain a feasible solution $\mathbb{P}_{(\alpha_1,\alpha_2,\alpha_3)}$ which depends on the uncertainty levels of $\mathbb{A}_{\alpha_3}$, $\mathbb{C}_{\alpha_1}$ and $\mathbb{C}_{\alpha_2}$ in the system. When

$$\mathbb{P}_{(0.8,0.9,0.9)} = \begin{bmatrix} 0.8041 & -0.0029 & -0.0001 \\ -0.0029 & 0.8452 & -0.0001 \\ -0.0001 & -0.0001 & 0.8094 \end{bmatrix} \times 10^{-03}$$

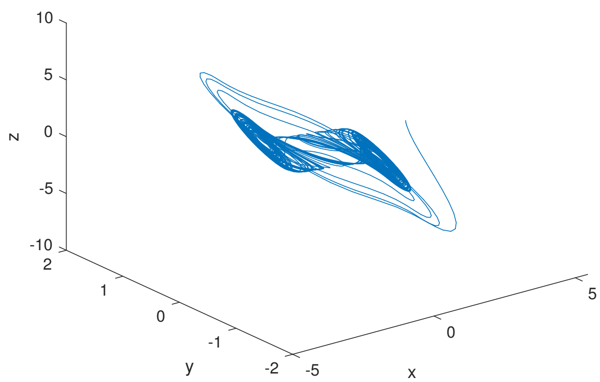

FIGURE 3. Chaotic Chua's System for $\mathbb{A}_{0.9}$

the fuzzy chaotic system illustrated in Figure 3 is controlled as shown in Figure 4.

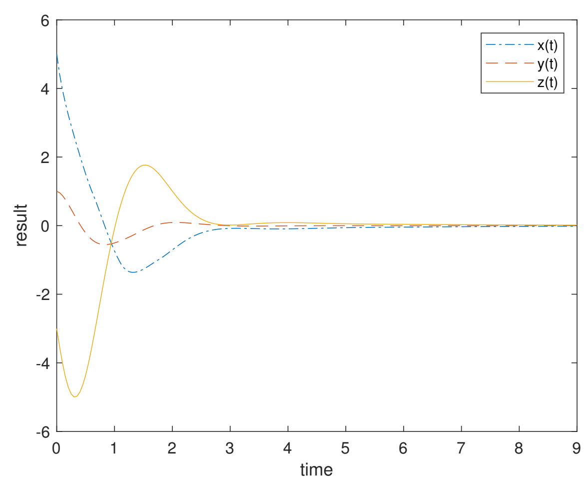

FIGURE 4. Controlled Chua's System for $\mathbb{C}_{0.8}$, $\mathbb{C}_{0.9}$ and $\mathbb{A}_{0.9}$

It should be noted that, while [20] and [22] have not been able to make matrix $\mathbb{A}$ fuzzy, this paper has achieved that.

7. COMPARISON OF RESULTS

It is worthy of note that it took about 12 units of time to control in [3] and 5 units of time in [20] but the time taken in this new method is about 4 unit of time. The LMI terms are reduced. In addition, there is more flexibility for $\mathbb{A}, \mathbb{C}_1$ and $\mathbb{C}_2$ in the new method of this paper.

8. CONCLUSION

For fuzzy parameters $\mathbb{A}_{\alpha_3}$ of the system and the fuzzy control matrices $\mathbb{C}_{\alpha_1}$ and $\mathbb{C}_{\alpha_2}$ introduced into the system, we are able to obtain a suitable positive definite symmetric matrix $\mathbb{P}_{(\alpha_1,\alpha_2,\alpha_3)}$ which depends on the uncertainties of the system. The control matrices obtained tend to control the system faster with less energy than the control in [3] and [20]. Besides, the fizziness introduced into the system afford the opportunity for the system to be naturally flexible. This method could also be tried in some other systems such as biological and financial systems in which uncertainties do occur.

ACKNOWLEDGEMENTS

We sincerely thank the our Chief Host, Director-General/Chief Executive of the National Mathematical Centre (NMC), Prof. Promise Mebine, the Deputy Director, Prof. B. O. Oyelami, the Director/Co-Ordinator of the Mathematics Programme, Prof. James Ajie, and all other members of staff of the NMC. In particular, we express our deep appreciation to the National Mathematical Centre for its commitment to advancing research and education in the field of Mathematics. Our appreciation also goes to the coordinators of the research groups, Professor Okhuonguae Daniel and Dr B. O. Onasanya. We also cannot forget our supportive friends, Prof. Yuming Feng, Cheng Gia, Shisian Zhu, for their contributions to the success of this research.

REFERENCES

[1] S. O. Kim, *Thermal stability of a reactive non-Newtonian flow in a sphere*, Int. Commu. Heat and Mass Transfer **60** (1) 70-81, 2009.

[2] L. C. Evans and P. A. Alli, *Partial Differential Equations*, American Mathematical Society, Providence, Rhode Island, 1998.

[3] H. A. Okoye, *How to define reasonably weighted Sobolev spaces*, Ph.D. thesis, University of Uyo, Uyo, Akwa Ibom State, Nigeria, 2009.

[1] S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, Philadelphia: SIAM, 1994.

[2] E. H. Connell, *Elements of Abstract and Linear Algebra*, Florida: Orange Grove Texts Plus, 2009.
[3] Y. Feng, C. Li, T. Huang and W. Zhao, *Alternate control systems*, Advances in Differential Equations, **305** 1–9, 2014.
[4] Y. Feng, Z. Wang, X. Liu and W. Zhang, *A nonlinear impulsive control system with impulsive time windows and un-fixed coefficient of impulsive intensity*, 2019 6th International Conference on Information, Cybernetics, and Computational Social Systems (ICCSS), Chongqing, China, 2019, 67-71, doi: 10.1109/ICCSS48103.2019.9115475.
[5] C. Hu, H. Jiang and Z. Teng, *Fuzzy impulsive control and synchronization of general chaotic system*, Acta Appl. Math., **109**, 463–485, 2010.
[6] J. Huang, C. Li and Q. Han, *Quasi-stabilization of chaotic neural networks with parameter mismatch by periodically intermittent control*, Proceeding of CSIE 2009, Los Angeles, California, USA, March 31- April 2 2009.
[7] J. Huang, C. Li and X. He, *Stabilization of memristor-based chaotic system by intermittent control and fuzzy processing*, Int. J. Control Autom. Syst. vol., **11** (3) 643–647, 2013.
[8] T. Huang, C. Li and X. Liu, *Synchronization of chaotic systems with delay using intermittent linear state feedback*, Chaos. **18** (3) 643–647, 2008.
[9] N. Li and J. Cao, *Periodically intermittent control on robust exponential synchronization for switched interval coupled networks*, Neurocomputing, **131** (11) 52–58, 2014.
[10] C. Li, G. Feng and X. Liao, *Stabilization of nonlinear systems via intermittent control*, IEEE Trans. Circuits Syst II, Express Briefs, **54** (11) 1019–1023, 2007.
[11] E. N. Sanchez and J. P. Perez, *Input-to-state stability (ISS) analysis for dynamic Neural Networks*, IEEE Trans. Circuit Syst I, Fundam. Theory Appl., **46** (11) 1395–1398, 1999.
[12] L. Shilnikov, *Chua's circuit: rigorous results and future problems*, Int. J. of Bifur. Chaos., **4** 489–519, 1994.
[13] Z.-P. Wang and H.-N. Wu, *Fuzzy impulsive control for uncertain nonlinear systems with guaranteed cost*, Fuzzy Sets and Systems, **302** 143–162, 2016.
[14] Y. Wang and H. Yu, *Fuzzy synchronization of chaotic system via intamittent control*, Chaos, Soliton & Fractals, **106** 154–160, 2018.
[15] T. Yang, *Impulsive Control Theory*, Berlin: Springer-Verlag, 2001.
[16] T. Yang, *Impulsive control*, IEEE Trans. Autom. Control, **44** (5) 1081–1083, 1999.
[17] T. Yang and L. O. Chua, *Impulsive stabilization for control and synchronization of chaotic systems: theory and application to secure communication*, IEEE Trans. Circuit Syst. I, Fundam. Theory Appl., **44** (10) 976–988, 1997.
[18] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** 338–353, 1965.
[19] M. Zochowski, *Intermittent Dynamical Control*, Physica D., 145 (3–4) 181–190, 2000.
[20] B. O. Onasanya, S. Wen, Y. Feng, W. Zhang and N. Tang, *On fuzzy alternate control Systems*, 2021 13th International Conference on Andvanced Computational Intelligence (ICACI), Wanzhou, China, 2022, pp. 14-16.
[21] B. O. Onasanya, S. Wen, Y. Feng, W. Zhang and J. Xiong, *Fuzzy coefficient of impulsive intensity in a nonlinear impulsive control system*, Nueral Processing Letters, **53** (6) 4639–4657, 2021.
[22] N. Tang, Y. Feng, B. O. Onasanya and W. Zhang, *Fuzzy Alternate Control Delayed Systems*, 2022 8th International Conference on Information, Conference on Computer and Communication (ICCC), Tsinghua University, China, 2022, pp. 67-71, doi: 10.1109/ICCC56324.2022.10065957

[1]DEPARTMENT OF MATHEMATICS, UNIVERSITY OF IBADAN, IBADAN, NIGERIA
E-mail address: babtu2001@yahoo.com
[2]DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ABUJA, ABUJA, NIGERIA
E-mail addresses: kemo4xrist@gmail.com
[3]DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ABUJA, ABUJA, NIGERIA
E-mail addresses: juliusfelix1254@gmail.com
[4]DEPARTMENT OF MATHEMATICS, NASARAWA STATE UNIVERSITY, KEFFI, NIGERIA
E-mail addresses: adodaniel580@gmail.com

[5]DEPARTMENT OF MATHEMATICS, FEDERAL UNIVERSITY OF LAFIYA, NASARAWA, NIGERIA
E-mail addresses: akere194@gmail.com