

A REVIEW OF SOME RESEARCH PUBLICATIONS OF PROFESSOR ADEWALE ROLAND TUNDE SOLARIN

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1. INTRODUCTION

When the council of the Nigerian Mathematical Society (NMS) approved the request of the author that an issue of the Journal of the Society be published to commemorate the 70th Birthday anniversary of Professor A.R.T. Solarin (Prof. ARTS). I knew that the herculean task of writing this article will definitely fall on me. This is due to the fact that we have not been able to get in touch with the first successful Ph.D. candidate (Prof. Victoria Ohanuma Chiboka, PhD (Ife), 1990) of Prof. ARTS. I being the second successful Ph.D. candidate of Prof. ARTS have to accept my "fate" and confront the task head long.

Prof. ARTS has his hands in all major area of research in Mathematical Sciences, namely Mathematics, Statistics and Computer Science with Mathematics being his major area of interest. Hence, this paper will be restricted to only the research publications of Prof ARTS research in Mathematics.

Prof ARTS was born at Jericho Nursing Home, Ibadan, on Monday, April 5, 1954. He was at the Methodist School, Sagamu for his Elementary Education from 1961 to 1966 after which he attended the famous Mayflower School, Ikenne from 1966 to 1970 where he made Division 1 in the West African School Certificate Examination. After spending a year in the Higher School Certificate Class of the Mayflower School, he was admitted to the University of Ife (Now Obafemi Awolowo University) to study for a Bachelor of Science Degree in Mathematics in 1971 which he completed in Second Class (Upper Division) in June 1975

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with the Federal Government of Nigeria Scholarship Award . He studied and completed his M.Sc. and Ph.D. in 1978 and 1986 respectively at University of Ife (Now Obafemi Awolowo University)utilising the University of Ife Staff Development Scholarship.

Prof. ARTS began his Academic Career at University of Ife (Now Obafemi Awolowo University) as a Graduate Assistant in September 1976 and rose through the rank to become a Reader in Mathematics. He was appointed a Professor at the University of Agriculture, Abeokuta in September 1997 where he eventually transferred his service. He served as Head of Department (1997-2000), Director of Academic Planning (2000-2004), Deputy Vice-Chancellor (2004-2006) and Member of Governing Council (2000 – 2006).

In 2007, Prof ARTS joined services of the National Mathematical Centre (NMC), Abuja as a Professor and Co-ordinator of the Mathematics Programme. Under his leadership, the Nigerian Universities Undergraduate Mathematics Competition was introduced. He also came up with the strategies for the training of the Nigerian Olympiad Team to the Annual International Mathematical Union's International Mathematics Olympiad (IMO). This resulted into the Nigerian Team obtaining her first IMO Medal in 2010. Prof. ARTS was appointed the Director and Chief Executive of the NMC in 2012 first in acting capacity and he became the substantive Director in 2013. Though his tenure as the Director was truncated in September 2016 due to the change of political leadership of the country, he was able to reactivate the cordial relationship which had existed between the NMS and NMC which seems to have been truncated by his predecessor. This led the NMC to take over the responsibility of the payment of the International Mathematical Union (IMU) annual dues on behalf of the Nigerian Mathematical Community which was then a big burden for the NMS. He also reintroduced the Graduate Foundation Programmes and the Research Oriented Courses which had been neglected for some time by the NMC.

In the course of his career, Prof. ARTS has had the privilege of visiting many institutes either to teach at undergraduate and postgraduate levels or to conduct research in algebra. These includes visits to University of Cortona, Italy, Technical University Darmstadt, Germany, University of Erlangen, Germany, University of Iowa, USA, Stellenbosch University, South Africa, University of Kyoto, Japan, University of Cape Town, South Africa, The Hong Kong University of Science and Technology, University of Goroka, Papua New Guinea, University Dar Es Salam, Tanzania, University of Makerere, Uganda and a host of

others. Furthermore, he also won a lot of Research Fellowships, including the Italian Foreign Affairs Fellowship (1980-1983), International Mathematical Union Fellowship (1990), the Deutscher Akademischer Austauschdienst (DAAD) (1996), African Mathematical Union Fellowship (2005), The Association of African Universities Fellowship (2006), African Millenium Mathematics Initiative Fellowship (2006), The UNESCO Mathematics Chair of Mathematics (2014).

Prof ARTS have served as assessor of academic colleagues at profes-sorial levels within and outside the shores of the country. He has also served as external examiners at various times for postgraduate students in Nigeria and abroad. He has successfully supervised many students at Masters and Doctoral levels. Eight of those that he supervised at doctoral level are now full Professors serving within and outside Nigeria.

Prof ARTS research activities in Mathematics is majorly in Construc-tion and Characterisation of various loop structures. The outputs of his research has been eye-openers as method of construction in other struc-tures in the field of Non-associative Algebra. He began his career at the University of Ife (Now Obafemi Awolowo University) in 1976 and retired from the National Mathematical Centre, Abuja in 2024 having reached the mandatory retirement age of 70 after a 48 years of merito-rious service to the Mathematics community globally.

2. PROF ARTS RESEARCH

The research of Prof ARTS which has been published in several rep-utable journals explored the properties and constructions of Bol loops, including their classifications and relationship with other algebraic struc-tures. Specifically, Prof ARTS investigations include

- (1) Bol loops
- (2) G-Loops
- (3) Bol-Moufang Type Identities
- (4) Osborn loops
- (5) Bol-Moufang Type Algebras

2.1. Bol loops. In abstract algebra, a Bol loop is an algebraic structure generalizing the notion of group. Bol loops are named after the Dutch mathematician Gerrit Bol [5] who introduced them in 1937 when he established the relationship between Bol loops and Moufang loops, the latter which was discovered earlier in 1935 by Ruth Moufang [18]

Thereafter, a theory of Bol loops was developed through the Ph.D. Thesis of Daniel Robinson [24] in 1964 where an algebraic study of Bol loops, Moufang loops, isotopy of Bol loops and other notions on Bol loops were conducted.

In the 1980s the study of construction of finite Bol caught the attention of Prof ARTS and other researchers. In [31] Prof ARTS constructed Bol loops of order $2n^2$ and Bol loops of order $4n$ which are shown to be non-Moufang given that $n > 2$ is a positive integer. It was also shown that if H is a non abelian subgroup of a group G and $A = H \times G$ then for $(h_1, g_1), (h_2, g_2) \in A$ if

$$(h_1, g_1) \circ (h_2, g_2) = (h_1 h_2, h_2 g_1 h_2^{-1} g_2)$$

then $(A, \circ)$ is a Bol loop (MR0739560).

Furthermore, in 1984 [32] Prof. ARTS constructed non-associative right Bol loops of order $p^2 q$ and $4k$, where p, q are distinct primes and q is a divisor of $p - 1$ and $k \geq 2$ is any integer. A corrected form of Theorem 3 of Bol[5] was also presented. In Niederreiter & Robinson (1981)[23], it was stated that Bol loops of orders 18, 35, 50, 70, 77, 85 and 98 can not be determine but Prof ART was able to obtain Bol loops of orders 18, 50 and 98 in [32] (MR0777019).

Employing the work of Burn, 1978[6], it was proved in [33] that up to isomorphism there exists exactly two Bol loops order 12 which are not Moufang. It was noted that the results obtained here are generally true for Bol loops of order $4p$ where p is any odd prime number [7]. It was also proved in [34] that up to isomorphism there are exactly 21 different non-Moufang Bol loops of order 16, each of which contains at least one element of order 8. These 21 loops were explicitly exhibited. In [35], 8 methods of construction of Bol loops of order $4n$ with 2 generators in terms cyclic groups and other parameters were provided where n is a suitable positive integer. When $n = 4$ non-associativity (consequently Moufang) is the result of the given construction. This proved that Bol loops of order 16 given by the constructions are non-isomorphic. Hence they are all unique.

In [37], the classification of the Bol loops of certain prescribed order for each odd prime p . It was shown that up to isomorphism there exists two non-associative Bol loops of order $2p^2$. Each of these subloops was shown to have

- (1) $p, p^2 - 1, p(p - 1)$, elements of order 2, p and p^2 respectively
- (2) a normal associative subloop of order p^2
- (3) center of order p

Also, in [36] it was established that there exists 65 non-isomorphic Bol loops of order 16, each of which contains 2 of order 4 and 13 elements of order 2 which are neither Moufang loops or groups. It was also established that there exists 26 non-isomorphic Bol loops of order 16 with 14 elements of order 4 and 1 element of order 2. It was

found out that all the loops obtained satisfied the Lagrange's theorem and the Sylow's main theorems. In [40], It was proved that a finite G-loop of prime order must be a group, and showed that for each even integer $n \geq 5$ there is a G-loop of order $n > 5$ which is not a group. He offered the following conjecture: For each composite integer $n > 5$ there exists a G-loop of order n which is not a group. This work addressed the open question of Wilson [41]. Appealing to results of [23] and obtained the following result: If $(L, .)$ is a finite Bol loop (that is, $((x.y).z).y = x.((y.z).y) \forall x, y, z \in L$) of order pq where p and q are primes such that $2 < p < q$ and $q^2 \equiv 1 \pmod{p}$, then $(L, .)$ is a G-loop. In [40] it was shown that

- (1) there are $(p+7)/6$ non-isomorphic (non-associative) Bol loops of order $3p$, when 3 does not divide $p-1$, and $(p+5)/6$ non-isomorphic (non-associative) Bol loops of order $3p$, when 3 divides p ;
- (2) the Bol loops of order $3p$ are isomorphic to their loop isotopes.

These results invalidated some of the results obtained in [24].

2.1.1. Solarin's Method of Construction of Bol-Quasigroup in Generalised Groups. In [3], 'generalized group' means a semigroup $(G, .)$ with the following conditions: for any $x \in G$ there exists a unique $e(x) \in G$ such that $x.e(x) = e(x).x = x$; for any $x \in G$ there exists $x^{-1} \in G$ such that $x.x^{-1} = x^{-1}.x = e(x)$. A groupoid $(G, .)$ with the identity $((xy)z)y = x((yz)y)$ is called a Bol groupoid. Properties of generalized groups, including homomorphisms, subgroups and direct products, were studied. Using generalized groups, Bol groupoids were constructed. The following theorem is proved: If H is a subgroup of a non abelian generalized group G and let $A = H \times G$. then for $(h_1, g_1), (h_2, g_2) \in A$ if

$$(h_1, g_1) \circ (h_2, g_2) = (h_1 h_2, h_2 g_1 h_2^{-1} g_2)$$

then $(A, \circ)$ is a Bol loop (MR0739560). Therefore, $(A, \circ)$ is a Bol groupoid. If H is abelian and G satisfies the cancellation law, then $(A, \circ)$ is a Bol quasigroup with a left identity element. It is known that any Bol quasigroup has a left identity element [24], page 57. The construction given in [3] generalized the method of construction given by Robinson [22] for using Bol loops

2.2. G-Loops. A loop is said to be a G-Loop if and only if it is isomorphic to all its loop isotopes. Goodaire and Robinson [12] undertook the study of conjugacy closed loops and showed that are G-loops.

If A is a group of automorphisms of a loop $(G, .)$, let $H = A \times G$. For all $X = (\alpha, x)$ and $Y = (\beta, y)$ in H define $X \circ Y = (\alpha\beta, x\beta.y)$. Then

$(H, \circ)$ is a loop extension of $(G, \cdot)$. In [10], Prof ARTS adopted the techniques of the of Robinson[20], MR0308312 to obtain necessary and sufficient conditions for this A -holomorph $(H, \circ)$ of an inverse property loop $(G, \cdot)$ to be conjugacy closed, namely, that $(G, \cdot)$ be conjugacy closed and $g^{-1}.g\alpha$ be in the nucleus of $(G, \cdot)$ whenever g is in G and α is in A . It was also proved that a loop $(G, \cdot)$ is a G-loop if and only if there are two permutations ϕ and ν of G such that for all $a \in G$ the triples $(\phi, \phi L(a)-1, \phi)$ and $(\nu R(a)-1, \nu, \nu)$ are both autotopisms of $(G, \cdot)$. As an application of this result, it was shown that, for a subloop $(H, \cdot)$ of $(G, \cdot)$:

- (1) $(H, \cdot)$ is also a G-loop;
- (2) if $(H, \cdot)$ is normal, then $(G/H, \cdot)$ is a G-loop

In [39], utilizing the work of Goodaire and Robinson[12] it was proved that extra loops and central loops are isomorphic to their loop isotopes. Hence, they are G-loops.

2.3. Bol Moufang Type Identities. In [25] the loops that occupy the less familiar branches of this treeâ€”C, LC, and RC loopsâ€”were examined and generalized in a manner reminiscent of the techniques and concerns present in some earlier papers of Orlik-Pflugfelder[19] and Chein & Robinson[8]. In the paper of Robinson[21] concerning right Bol loops); a di-associative loop G is a C-loop if and only if x^2 is in the nucleus of G for all $x \in G$; a relationship between the supernucleus of a loop G and its nuclei was explored when G does not satisfy the inverse property; a pair of non isomorphic RC loops of order 16 neither of which is a Bol loop was given.

2.3.1. Bol-Moufang Type Loops and Gyrogroups. Gyrogroup theory in [17] provides a link between diverse aspects of loops, thereby revealing its significance in loop theory. A groupoid is a nonempty set with a binary operation. The groupoid $(L, \cdot)$ is a quasigroup if each of the two equations in the groupoid, $a.x = b$ and $x.a = b$, has a unique solution for the unknown x . Finally, a quasigroup $(L, \cdot)$ is a loop if there exists in L a unique identity element. Among non-group loops, gyrogroups are special in the sense that the non-associativity of a gyrogroup operation is controlled by special automorphisms called gyroautomorphisms or, in short, gyrations. Gyrations are also called Thomas gyrations since they are extended by abstraction from the special relativistic effect known as Thomas precession. Thus, for instance, $gyr[x, y]$ is the Thomas gyration generated by $x, y \in G$ in a gyrogroup $(G, \oplus)$. In [17] the gyrogroup theory centrally placed in the study of some loops of the Bol-Moufang

type. Following a presentation of a list of thirteen loops of the Bol-Moufang type, gyrogroup theory was briefly described. The definition of each of these loops was presented, along with its characterization in terms of a Thomas gyration condition. Thus, for instance,

- (1) the three loops called EL, CL and RCL were found to be equivalent, and were characterized by the Thomas gyration condition $\text{gyr}[x, y]Lx\text{gyr}[x, y] = Lx$, Lx being a left gyrotranslation by x ;
- (2) the two loops called RBL and ML were found to be equivalent, and were characterized by the Thomas gyration condition $\text{gyr}[x, y] = Lx\text{gyr}[x, y]L - 1x$.

Additionally, novel properties of the Thomas gyration of gyrogroups and gyrocommutative gyrogroups that underlie some loops of Bol-Moufang type were deduced.

2.4. Osborn Loops. A loop $(L, \cdot)$ is called an Osborn loop if it satisfies the identity

$$x^\lambda y \cdot zx = x(yz \cdot x)$$

for all $x, y, z \in L$. The term Osborn loops first appeared in a work of Huthnance Jr.[13] on generalized Moufang loops. However, the above definition is that of Basarab and Belioglo[4]. Prof ARTS studied this type of loops and came up with new properties of the loop. He also constructed finite examples using modified methods which he earlier used in the construction finite Bol loops [10, 32–37]. In [15], several necessary and sufficient conditions for a loop to be universal Osborn loop were given. It was also proved that Kinyon's conjecture, which roughly states that every conjugacy closed quasi-group is isotopic to a universal Osborn loop, is true if and only if every conjugacy closed quasi-group satisfies some given identities. In [16], the answer to the following open problem posed by Kinyon was provided: Does there exist a proper Osborn loop with a trivial nucleus? In this connection, this problem was expressed in terms of the orders of the nucleus, 2nd Bryant-Schneider and automorphism groups of the loop, and obtained some necessary conditions for the existence of a universal (left or right) Osborn loop with trivial nucleus (based on the orders of the 1st Bryant-Schneider, 2nd Bryant-Schneider and automorphism groups) and also some sufficient conditions for the non-existence of a universal (left or right) Osborn loop with trivial nucleus (again, based on the orders of the 1st Bryant-Schneider, 2nd Bryant-Schneider and automorphism groups). Since every Moufang loop is an Osborn loop, it is believed that the results of this paper can help to answer the following famous problems: Phillips' Problem: Is there a Moufang loop of odd order with trivial nucleus? and

Doro's Conjecture: Does a Moufang loop with trivial nucleus necessarily have normal commutant?

[14] provided an answer to the Kinyon's question: "is every Osborn loop is universal?" by providing four distinct constructions for non-universal Osborn loops. It was then shown that non-universal Osborn loops do not have many nice properties of loops. Among other properties, it was shown that non-universal Osborn loops are not CIP, WIPL, AIPL, AAIP, LIP, RIP, flexible, power associative, diassociative or CC.

2.5. Bol-Moufang Type Algebras. Bol-Moufang algebras, in the context of loops and quasigroups, refer to algebraic structures defined by identities of a specific type, including Bol loops, Moufang loops, and C-loops, with a focus on the relationships between these structures and their defining identities. Prof. ARTS on Algebras satisfying these identities was able to link these algebras with Jordan and Lie Algebras.

In [26] A Bol algebra over a field F is defined to be an algebra B in which $(xy.z)y = x(yz.y)$ for all $x, y, z \in B$. For an element $a \in B$ $R_a : A \rightarrow A$ [$L_a : A \rightarrow A$] by $x \rightarrow xa$ [$x \rightarrow ax$]. A derivation on an algebra B is a linear operator D on B such that $(xy)D = (xD)y + x(yD)$ for all $x, y \in B$. It was proved if B is a Bol algebra. $R_x R_y - R_y R_x$ is a derivation of B if and only if x and y commute in B . Also, if B is a Bol algebra, $x.yz^2 = xy.z^2$ for all $x, y, z \in B$, then B is a noncommutative Jordan algebra. Let A be a nonassociative algebra over a field K . Using the operations of A , several quadrilinear operations (Bol bracket, extra bracket, modified extra bracket, flexible bracket) were defined and establishes relations between these operations and the commutator operation $[x, y] = xy - yx$ as well as the associator operation $(x, y, z) = (xy)z - x(yz)$ of A [27]. Traditional techniques of the field - linearization, substitution, manipulation of identities - the author studies algebras for which the multiplicative structure satisfies identities of the Bol-Moufang type were used in [28]. The results here may be summarized by saying that if an algebra satisfies certain of these identities, then it satisfies a variety of other identities as well. Furthermore, If the underlying algebra also has a tangent algebra, then a number of identities are also shown to hold for this tangent algebra.

An algebra E is an Aktivis algebra if it has a binary and ternary multiplication and satisfies the identity

$$\Sigma \text{sgn}(\sigma) \sigma \langle x, y, z \rangle : \sigma \in S_3 = [[x, y], z] + [[y, z], x] + [[z, x], y].$$

In [29] the notion of a modified Aktivis algebra was defined and it was proved that an anti-commutative algebra E , i.e. $x^2 = 0$ for all $x \in E$, is a modified Aktivis algebra. It follows that an algebra which satisfies one

of the identities $x^2.yz = x^2y.z; yx^2.z = y.x^2z; yz.x^2 = y.zx^2; (yx.x)z = y.x^2z$ or $y(x.xz) = yx^2.z$ is a modified Aktivis algebra. In addition, it was proved that a loop L which satisfies the identities $(yx.x)z = y.x^2z$ and $y(x.xz) = yx^2.z$ is conjugacy closed. Invoking a result in [12] it was concluded that L is a G-loop. In [30], Algebras satisfying identities of Bol-Moufang type was studied. For each identity studied, an algebraic relation similar to the Jacobian relation for a Lie algebra was obtained. It has been shown that algebras satisfying the Bol-Moufang type identities are anti-commutative algebras except for Bol and Moufang type identities [28]. If in these latter two cases the supplementary condition that the square of every element belongs to the right nucleus [left or middle nucleus] is satisfied, then an anti-commutative algebra was obtained.

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3. WHAT SOME PEOPLE SAY OF SOLARIN

Nkechi Agwu¹

Professor Adewale Solarin is indeed a leader in mathematical sciences for development and relevance. His impact in shaping the landscape of mathematical sciences in Africa knows no bounds.

When he was Director of the National Mathematical Centre (NMC), under his leadership many programs and initiatives were birthed at the NMC. Two significant ones among these were the 1st Pan-African Mathematics Olympiad for Girls. He brought successful Nigerian women professors to serve as mentors for the female participants. This mentoring component by women is extremely important as research shows that mentoring is an important aspect of women's success as leaders in mathematics. He is always recommending young female mathematicians to me to mentor. Also, the Ethnomathematics Research Group (ERG) birthed in 2015 and still ongoing at the NMC. The NMC ERG was born in partnership with me as a Carnegie African Diaspora Fellow. He learned about my work as a Carnegie Fellow to the Federal University of Technology, Akure, Centre for Gender Studies in Science and Technology and saw the need for replication at the NMC. Today the ERG stands as a pillar for promoting Ethnomathematics Research on Nigerian Cultural Groups. Their research and co-curriculum development have resulted in teaching modules on various concepts in the mathematical sciences from a Nigerian Cultural context. Special thanks go to Professor Adewale Solarin for opening the door to Ethnomathematics Research at the NMC and to my professional growth as a development leader in the mathematical sciences. May he continue in this path of leadership raising fishers of (wo)men in the mathematical sciences that will change the landscape in unimaginable ways. Amen.

M. I. Modebei²

It was just like yesterday when my classmates and I first met you one-on-one in Advance Algebra class and how you taught us to flip pages of textbooks carefully, I knew there were countless things to learn from you.

Your search and reward for excellence in young mathematician earned me my job and by extension my doctorate degree in Mathematics.

Prof., you have been a silent mentor and I thank God the very day my path crossed yours. As you walk through the phase of retirement, I

¹Carnegie African Diaspora Fellow, Founder of CHI STEM TOYS Foundation and Professor of Mathematics at the City University of New York

²Senior Research Fellow, National Mathematical Centre, Abuja, Nigeria

know the light you have created in others with lighten and brighten your path through.

Thank you, Prof, for being a rare father in academics!

T.O. Obilade³

Professor ART Solarin is an enigma of international repute having created a distinctive line of research in Bol loops. He attended Mayflower school where the founder, Dr Tai Solarin, made maths students first among all, and arrived at university of Ife fully charged to receive lectures from Egyptians, Americans, Russians, Indians and some devoted Nigerian pioneers. Prof Solarin ended up inspiring a third of his class to make first class in maths. He combined his knowledge of maths and statistics to self- teach himself in computer and open his career in Bol loops. This was at a time when theorems emerged less from existing theorems, which were scarce, but more from deep suspicion and inspiration leading to revelation of what was true in general and conditions that had to be imposed for particular generalisation. Prof ART Solarin became a professor, committedly taking his allocated courses selflessly and sponsoring himself for overseas training and conferences. He naturally emerged a scholar with the dignity of a Prof and the humanity of a child of God from a good home. He was a mentor and builder who helped his friends and colleagues to reach the zenith of their career. His students will tell you he used international standard questions to groom them. On a personal level he once released his spare tyre for me for days when he was in Aserifa estate because he found I lost two tyres on a convocation day at Ile-Ife with my children sweating profusely from the heat from sun inside a modest car. Other people have greater testimonies on his humanity and forgiving spirit. Many people have spoken of his working with integrity. He would not take any toxic special allocations as a CEO that would mean paying hefty amount to those making the offer. He has absolute trust in God. Little wonder the Lord is his refuge and shield. My prayer is that the Lord will continue to increase him and children even in retirement.

Samuel Segun Okoya⁴

It was some time in 1984 when I first met Professor A. R. T. Solarin at a close range in White House, University of Ife (now Obafemi Awolowo University - OAU), Ile â€“ Ife, Nigeria. I was then on my primary assignment under the NYSC scheme and later became a staff in the Department of Mathematics. I was then a young mathematician working

³Department of Mathematics, Obafemi Awolowo University, Ile-Ife.

⁴Department of Mathematics, Obafemi Awolowo University, Ileâ€“Ife, Nigeria

in the area not directly related to his. Both of us were very active in the National Association of Mathematics Students of Nigeria â€“ NAMSM (now National Association of Mathematical Sciences Students of Nigeria â€“ NAMSSN). Years later, Professor Solarin invited me to the Quality Control Society of Nigeria which he pioneered and in 1990, I became the Secretary, Local Organising Committee, Quality Control Society of Nigeria. We toured many industries in Lagos. I still treasure the exposure of linking our research to the industry.

Professor ART Solarinâ€™s career has a significant service component, including a member of the Board of Trustees of the Pastor Enoch Adejare Adeboye Endowed Professorial Chair in Mathematics, University of Lagos (UNILAG), Nigeria when I occupied as the first occupier in 2013. Professor Solarin was physically active at my First Annual Lecture of Pastor E. A. Adeboye Professorial Chair (2015) as well as my Second Annual and Valedictory Lecture at the end of my four-year tenure at UNILAG (2017). There is no doubt that his constructive comments reflected in my Inaugural Lecture Series 329 at Obafemi Awolowo University, Ile â€“ Ife, Nigeria in 2018.

I am happy that our paths crossed and the memories will last forever. Wishing you happy 70th birthday and longevity as well as fruitful and prosperous retirement from active service.

M. O. Omeike⁵

I have known Prof. A.R.T Solarin for a longtime to be a generous man to everyone since I joined the services of the Federal University of Agriculture, Abeokuta (FUNAAB) in 2001.

He was once my boss at the Academic Planning Unit of the University when I was the Academic Planning Officer in which he extended his helping hands towards me in times of need which caused me to record achievements during my tenure. He was also part of my PhD supervisory committee when he also showed and proved to be a generous man by giving me all round assistance in areas like academic and sometimes financial which gave me an indelible notion of his kindness in my heart which would never fade away.

I would like to end this by encouraging him to continue his undying attribute of generosity.

⁵Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria

I. A. Osinuga⁶

I got to know Prof. as the Head, Department of Mathematics, FUNAAB. However, I worked directly with him as the Secretary of Directorate of Education of a socio-political group in Remo. He taught me the rudiments behind meeting up with deadlines as he will not wait for you and will not complain. Be rest assured that the job will be delivered if you don't meet up. In a nut shell, Professor is indeed a very approachable academic father to all. Enjoy the rest of your days, months and years in good health and sound mind. Congratulations sir!

Abimbola Shonubi⁷

I remember in 1998 when I was in 100 level in the then UNAAB, now FUNAAB, it was this compulsory course MTS 101... a complete DUGBE CLASS.

Quietly, he walked in and started teaching. We were like...who is this? Because his face was not the common type.

After the class, someone asked... your name, Sir? He wrote ART. We were like ART? He just gave a funny smile and walked out. Then the big question began. Who is this new face by the name ART? Finally, someone found the answer... He's a Prof.

This was how my journey with Prof. ART began.

Fast forward to 2003 when I was planning for my wedding, I invited him to be the chairman of the occasion and he humbly accepted. He graduated to become my Spiritual Dad and his wife, Pastor Mrs Titilayo Solarin My Mom.

In 2014, Prof. ART Solarin was the Director General/CEO of National Mathematical Centre FCT Abuja, and I was opportuned to be gainfully employed.

My Daddy, Prof. Adewale Roland Tunde Solarin is retiring from active service today but definitely not tired! Moyinoluwa. Oreoluwa. Iseoluwa and Ayanfeoluwa see in him a grandfather they never had. Thanks for being my Daddy.

T. G. Jaiyeola⁸

I got to know Prof. Solarin as an undergraduate student at the Department of Mathematical Sciences (now Department of Mathematics), University of Agriculture, Abeokuta-UNAAB (now Federal University

⁶Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria

⁷Adopted Daughter

⁸Pastor Enoch A. Adeboye Professorial Chair of Mathematics, Department of Mathematics, University of Lagos, Nigeria

of Agriculture, Abeokuta-FUNAAB) when he became the Head of Department in the 1997/1998 Session. I received two letters of commendation from him for my excellent performance during the 2nd Semester 1997/1998 Examination and 1st Semester 1998/1999 Examination.

During my Master's Degree Programme between 2003 and 2005 at UNAAB (FUNAAB), Prof. taught me the course 'Latin Square and its Applications' which further prepared me for my research work under the supervision of Prof. Adeniran in Quasigroup and Loop Theory. In My M.Sc. Research work which was based on the 'Isotopic Invariance of Central Loops', I was in search of a central square loops. A visit to Prof Solarin's office aided my search. I can still remember the dialogue very well. **Me:** Good day sir. **Prof:** You are welcome. **Me:** I am the M.Sc. student working with Dr. Adeniran (now Prof). **Prof:** What are you working on? **Me:** Central loops. **Prof:** go to the board (he gave me a marker) and define a central loop. **Prof:** What problem(s) are you facing? **Me:** I am looking for central square loops. **Prof:** Consider Moufang loops of small orders.

I ran back to Prof. Adeniran who searched through his archive of articles for Moufang loops of small orders. We found many of them.

Prof. Solarin was my co-supervisor during my Ph.D. Research work which was based on 'The Universality of Osborn Loops' under the supervision of Prof. Adeniran. He called my attention to the importance of my work to the Phillips' open problem and Doro's conjecture. The latter has been reportedly solved for Moufang loops by S. M. Gagola III (2012) while we (Jaiyeola, Adeniran and Solarin) attempted the solution to the general form in 2011.

The exploits of Prof. Solarin and Prof. Sharma (his Ph.D. supervisor) while they were at the Department of Mathematics, University of Ife (now Obafemi Awolowo University-OAU) in the 80s has stimulated my interest in the study of generalized Bol loops which was originally initiated by Prof. Sharma. One of their methods of construction of Bol loops from groups has been found useful in the construction of generalized Bol loops of order in one of the M.Sc. Mathematics theses which I have supervised at OAU. Prof was very excited when I got appointment at the Department of Mathematics, OAU in 2006. Often, whenever I met with Prof (especially at the NMC), he shared the ordeals he went through at OAU during his promotion to the professorial rank(s). I used to reject such unpalatable experience, not knowing that I would have experience something similar. Thank God that I was able to weather the storm through grace and the words of encourage of some other senior academic colleagues (including Prof).

B-Big:: Prof is blessed with a big body statue (geometric property).

O-Open:: Prof is open minded in matters and issues (algebraic property).

L-Loving:: Prof operates in love as required by God's standard of loving one's neighbour (topological property) irrespective of your status.

Bol Theorem:: Prof. Solarin is a Bol Human Being. My best wishes in your retirement.

Shola ADEYEMI⁹

In the annals of academia, few figures shine as brightly as Professor A.R.T. Solarin, a distinguished luminary in the field of mathematics, particularly in the realm of algebra and its interconnectedness with statistics. His towering intellect, unwavering dedication to scholarship, and profound contributions to mathematical theory and education have left an indelible mark on the discipline and inspired countless generations of mathematicians including myself.

Our path crossed through another "mathematical phenomenon", late Professor Uyi Afuwape. As at the time I was joining the Department of Mathematics, Obafemi Awolowo University, Ile-Ife, he was on his way to UNAAB. I got to learn about his story, his insatiable thirst for knowledge, which led him to excel academically and earn numerous accolades in the realm of mathematics. I took a keen interest and lo and behold, my Ife story became almost exactly as his.

Throughout his illustrious career, Professor Solarin's pioneering research and groundbreaking discoveries illuminated the pathways of algebra, pushing the boundaries of mathematical understanding to new heights. His seminal works on abstract algebra, group theory, and algebraic geometry have not only advanced the frontiers of mathematical knowledge but have also laid the groundwork for countless future inquiries and developments in the field.

However, Professor Solarin's contributions extend far beyond the confines of his research laboratory. As a devoted educator, he devoted himself wholeheartedly to nurturing the next generation of mathematical minds, instilling in his students a deep appreciation for the beauty and elegance of mathematical reasoning. His mentorship was characterized by patience, kindness, and an unwavering commitment to fostering intellectual curiosity and academic excellence. No surprises his footprints are everywhere with many Professors, grand professors and

⁹Chief Innovations and Data Strategy Officer, BOHEMIAN SMARTLYTICS LIMITED, Guildhall, Cambridge, United Kingdom

great grand professors he's mentored as well as his exploits at National Mathematical Centre, Abuja, African Mathematical Union, Mathematics Olympiad, etc.

Beyond his academic pursuits, Professor Solarin was revered for his humility, integrity, and generosity of spirit, which culminated in being ordained a Bishop. He embodied the values of intellectual honesty and scholarly rigor, serving as a beacon of inspiration and a role model for aspiring mathematicians around the world.

In recognition of his outstanding contributions to the field of mathematics, Professor A.R.T. Solarin was awarded numerous prestigious honors and accolades, cementing his legacy as one of the preeminent figures in the history of mathematical scholarship. Yet perhaps his greatest legacy lies not in the accolades he received or the equations he solved, but in the lives, he touched and the minds he inspired.

As we reflect on the life and legacy of Professor A.R.T. Solarin, let us pay tribute to a true titan of mathematics, whose brilliance illuminated the darkest corners of mathematical theory and whose passion ignited the flames of curiosity in countless hearts. May he be blessed with long life, good health and may his legacy continue to inspire generations of mathematicians to come, as we endeavor to build upon the foundation he so meticulously laid.

S. A. Akinleye¹⁰

Professor Solarin is an amiable, hardworking and indefatigable Mathematician. He is one person I find who rarely sleep more than four hours in a day. Yet, he never lost weight or suffer sleep disorders.

Quite versatile, he has his hands in various areas of Mathematics like Statistics, Quality Control and Algebra, qualifying as the Father of Loop Theory in Nigeria. He is always ready to challenge and encourage research, while also fully focused on the rewards from their applications.

Ever smiling, he makes friends easily and is always reachable. Above everything, he is God fearing, loves people and hardly show his anger on people. Indeed, he is a Minister of the Word and surely a Bishop over His congregation. Congratulations, Sir, on your 70th birthday.

Abednego O. Isere¹¹ "The Work of a Librarian". "You are a strong man. You want to do the work of a librarian". These were the words of Prof. ART Solarin, in one of my visits to him at the early stage of

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¹¹Department of Mathematics, Ambrose Alli University, Ekpoma, Edo State, Nigeria

my Ph.D. program in 2011. I was startled, not getting the sense of those words. Perhaps, he observed that I was struggling to understand the meaning of what he said. He calmly explained: "It is a librarian who determines the number of books of the same discipline puts them on a shelf, and labels them, indicating the exact number of books on the shelf". Similarly, the construction and classification of algebraic structures determines the exact number of algebraic structures of particular orders up to isomorphism.

Those words of an academic sage certainly helped me to see the enormity of what I embarked on. He compared the work of classification to the work of a librarian and made me assess the pros and cons of what I intended to study. I thank him so much for prudently going through my work, for confirming what I was doing, and for his invaluable contributions.

Prof. ART Solarin did the work of a librarian on Bol loops of orders 12, 16, 32, and generally of order $3p(p > 3)$, where p is prime. He found that there were six Bol loops of order 12 and labeled them, and so many shelves of Bol loops of order 16, depending on the order of elements present. For example, he labeled that there were twenty-six Bol loops of order 16, with at least one element of order 8, and so on. These works, he has labeled and archived in the world libraries all over the continents.

My gratitude, once again goes to you sir, the mathematical librarian, the sage, and my academic grandpa, under whose tutelage my supervisor (Prof O. J. Adeniran) was groomed. Perhaps, there is no way I would have come to know my supervisor if not for your ingenuity in organizing an international workshop on Geometry and Algebra, titled: African Network for Geometry and Algebra Applied to Development (ANGAAD-NMC-2009), held 30th November 2009 to 8th December 2009, with resource persons from far and near Africa and beyond.

That workshop was a turning point in my academic trajectory. The workshop also allowed me to meet my friend, my research partner, and a brother, Prof. T. G. Jaiyeola who was my roommate during the workshop. He would make me sleep in loops and wake me up in quasigroups.

I pray God almighty to give you long life and sound health to enjoy your retirement and witness the success and progress of all your children.

Y. T. Oyebo¹²

I got to know him when he was appointed a professor and Head of Department of Mathematics, University of Agriculture, Abeokuta (UN-AAB), I was an undergraduate in year 2, he was already a big figure in Mathematics. The first regular staff with that high rank, the next to him was a lecturer I, many assistant lecturers and graduate assistants.

I started to take interest in his mentoring prowess, when he encouraged us to take a carrier in Mathematics and ensure that, commendation letters are issued to us, whenever we attain a Grade Point Aggregate of 4.00 and above.

Also, within a space of time, most of our lecturers who were hitherto a very low ranked staff in the faculty got new appointment with higher ranks. Not only that, in a very quick succession, Master of Science (MSc) and Doctor of Philosophy (PhD) programs were introduced, and the first PhD graduate was recorded 2001.

Being a mentor of mentors, he trained our teachers who later trained some of us who are now professors and directors in different callings both home and abroad. Professor Solarin is a leader, who does not stop at training his mentees alone; he also ensures that you are making progress in your chosen areas of interest.

Where necessary, he calls our attentions and influences opportunities in our ways and in some case he leads the ways and makes them accessible for us to tread and trends. All these are evidence from number of journal publications and books he has co-authored with his numerous students both locally and international.

He is a mediator per excellence; he builds bridges that connect the old and new generation of Mathematicians, both in Nigeria and the Diaspora. He is a pacesetter, setting pace for the new generation of mathematicians from elementary classes to advanced classes; this is also evidenced from his involvement in PAMO.

I can go on and on, but I will stop here and join millions of Nigerian home and abroad to wish Professor ARTS a happy retirement, though not tired. May what he has worked for in his mathematical education sojourn in Nigerian be sources of joy for him throughout his life.

Olusola John Adeniran¹³

I don't know how to start this short tribute in honour my teacher, mentor, and adviser per excellence in every ramification. I kept on wondering from which of the infinitely countable many dimensions that our

¹²Department of Mathematics, Lagos State University Ojo, Lagos State, Nigeria

¹³Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.

relationship grown to be I should write about. The fact is that this tribute will not be enough no matter no matter how many words I am able to write.

My first encounter with Professor Adewale Roland Solarin was with then Dr. ARTS (that's what my classmates and I used to call him; behind him of course) at the Ajoye Lecture Theatre, Obafemi Awolowo University, Ile-Ife during which he taught us Trigonometry for just three weeks. His lectures which were spiced with relevant diagrams were found to be pedagogic. It was as if I had never been taught the same concept before being admitted to the University. It would only take someone to be a moron not understand any concept in details when it is taught by Professor ARTS. Due to the fact that my classmates appointed me the Class Representative, I had the special privilege of being in charge of collection of assignment from classmates and delivering same to Dr. ARTS' office I was able to be in close contact with him more than many of my classmates. I found him to be very accommodating contrary to what we've been made to believe about lecturers in the Department. He later taught me two other courses in parts 3 and 4 both of which I had the best grade possible. On a particular day I was studying at the Hezekiah Oluwasanmi Library, I took time to flip through the University's Postgraduate Prospectus checking on the research areas of Lecturers, a particular area of research - Loop Theory caught my attention. The Lecturers in this area as provided in the prospectus were Professor B. L. Sharma and Dr. A. R. T. Solarin. I checked what Loop Theory is and found out that it's very close Group Theory, a concept that we were taught just the semester before that I found very interesting. Hence I decided then that my undergraduate honour's project would be in Loop Theory. When the Head of Department request that we should indicate our area of interest for our undergraduate Honours Project, I quickly submitted Loop Theory. That is how my foray into research in Loop Theory and Non-associative Algebra research was kick started under the supervision of Professor ARTS.

Professor ARTS obtained his Bachelor of Science Degree (Second Upper Division) from the University of Ife in 1975, followed by Master of Science in Mathematics as a member of the pioneer set postgraduate students (of which only two were successful out about 10 candidates) in the Department of Mathematics of the same University. He is also the first successful Doctoral Degree in Mathematics candidate of the University of Ife (Now Obafemi Awolowo University) who also had the privilege of supervising the first successful Female Ph.D Mathematics

candidate of Obafemi Awolowo University. His academic career which began in forty-eight years ago as been excellent in all ramifications.

When I assumed duty as a Graduate Assistant at the Obafemi Awolowo University, Ile-Ife, the next Monday after Youth Service, I reported to Professor ARTS office for supervision of my graduate studies. A couple of other lecturers in the Department including the then Head of the Department (may the Lord rest his soul) wanted to be my supervisor but I stood my ground, Professor ARTS, it must be and no other person in the Department.

When a person is being supervised by Professor ARTS at graduate level during my time as his student, all the research papers needed to make a success of the problem(s) identified for or by the candidate would be provided even if some them are thousands of kilometres from the reach of such student. The internet then was almost non-existent. I was a 'spoilt' postgraduate student then (at least to my colleagues being supervised by other lecturers in the same Department) just because the department had just a monochrome computer without hard drive but Prof ARTS brought one with hard drive for my research from Germany with the state of the art software that was available then for computational algebra - GAP.

After completion of a Doctoral Degree, most supervisors would immediately wean their students. This is not so with Prof ARTS. Every visit to him is an attraction of the question "What is the current research line of in Loop Theory?". From there you will discover that you are in for a research retreat. I personally ensured that I create time for such retreat as much as time and responsibilities would permit. This has helped a lot in projecting many of us who have been fortunate to pass through the tutelage of Professor ARTS either directly as his mentee or mentees of his mentees to at least three generations as at today. The Mathematics genealogy project showed that Professor ARTS has more than 30 Mathematical descendants. This number is huge because it unusual to have Pure Mathematicians off African descent who have been able to mentor this number of PhD candidates in any area of Pure Mathematics.

I need to add at this point that of all the PhD candidates trained by Professor ARTS, I have the singular honour of being trained by him writing my undergraduate honour project together with Masters and Doctoral Theses under his supervision. I am eternally grateful to him for the opportunity to stand on his shoulders.

Professor ARTS is one the very few researchers in Mathematics in Africa who can impact Mathematical Knowledge pupils at the basic level. He was a time the Team Leader/Head Coach of the Nigerian

Mathematics Olympiad Team and at a time a Member of of the UNESCO committee on Basics Sciences. The Nigerian Olympiad Team was able to will a lot of medals both at the Continental and International Levels during his tenure. He is current the President of the African Mathematical Union's Mathematical Olympiad Commission. He is the prime mover of the campaign for the use of Chinese Abacus for the teaching of Mathematics and reintroduction of Mental Arithmetic to the pupils at Basic levels of Education. These have been found to be tools that have put the Chinese pupils ahead of their contemporaries all over of the world.

Professor ARTS, a man of many parts is also an Ordained Bishop in His vineyard. As his celebrating his seventieth Birthday on this occasion. I wish him many happy returns in good health, sound mind, better relevance in service to humanity and greater usefulness the service of the most high.

Adio Akinwale¹⁴

Prof A. R. T. Solarin employed me to Department of Mathematical Sciences, Federal University of Agriculture, Abeokuta, Nigeria in 1998. Prof has been leading us and is still leading us by example. He is a very good teacher especially in the area of algebra and a sound researcher of highest order. He is a gentleman, soft spoken, honest and extremely humble human being. Thank you so much Sir for your encouraging words which are full of wisdom.

Promise MEBINE¹⁵

Prof Adewale Roland Tunde (A. R. T.) Solarin, (FMAN, FPNGMS, FNMS) - The Algebraic Loop Specialist, worked at Mathematics Programme, National Mathematical Centre (NMC), Abuja, FCT, Nigeria from 7th August 2008 to 5th April 2024. At the Centre, he was the Coordinator of the Mathematics Programme. He Championed International Olympiad Competitions, Identification and nurturing of gifted children, Mental Arithmetic and capacity pieces of training of lecturers in Mathematics. He organized the National and International Conferences in his field of study. Professor A.R.T. Solarin made tremendous contributions to the many accomplishments of the Centre before he was appointed as Director/Chief Executive of the Centre, where

¹⁴Dean, College of Physical Sciences, Federal University of Agriculture, Abeokuta, Nigeria

¹⁵Director/Chief Executive, National Mathematical Centre, Abuja

he displayed his wealth of Administrative experiences towards the development of the Centre. He was the Director/Chief Executive Officer, National Mathematical Centre, Abuja, from 31st March, 2013 to 1st August, 2016. He retired from the Centre on 5th April 2024. Professor Solarin does research in Algebra, Geography and Cartography. He was the President of African Mathematical Union (AMU) in July, 2013 to July, 2017. He was the President, African Mathematical Union Pan African Mathematics Olympiads Commission (AMU-PAMOC). He was appointed COMSATS Distinguished Professor of Mathematics in 2014. He was appointed a member of the Scientific Board of UNESCO's International Basic Science Programme, Paris, France on 28th January, 2015 and UNESCO Chair of Mathematics the same year. He has many scholarships, awards and Prizes. Professor A.R.T. Solarin's contributions, no doubt, have had overwhelming impacts on the development of Mathematics Education and other sectors in Nigeria and beyond. Three specifics of his legacies are examined briefly concerning his Initials: A. R. T.

A: Advancements in Loop Theory: His research led to significant advancements in loop theory, including developing new algebraic structures and solving long-standing problems. Consequently, he supervised students' theses (BSc, MSc, PhD, PGD, MBA, and MBF) in topics in Mathematics, Statistics, Quality Control, Computer Science, and Business Management.

R: Reliable Inspiration to African Mathematicians: Solarin's work inspired generations of African mathematicians, demonstrating the potential for African scholars to make significant contributions to mathematics.

T: Total Quality Advocate in Mathematics Education in Nigeria: Solarin played a key role in shaping mathematics education in Nigeria, helping to ensure total quality using Mathematics as a Panacea in National Productivity Improvement. The history of the National Mathematical Centre, Abuja, cannot be completed without the mention of Professor A.R.T. Solarin because of his immense contributions to the development of the Centre. We wish him a happy retirement life.

Godwin C. E. Mbah¹⁶

I sincerely congratulate you on your 70th birthday celebration by all of us who knows and cherish you. In my early days of development as a Mathematician, there were three persons that I felt being like, Prof. Bestman of Uniport, Prof ART Solarin of OAU, Ile-Ife and Prof.

¹⁶President, Nigerin Mathematical Society,
Department of Mathematics, University of Nigeria, Nsukka, Nigeria

Charles Nwachukwu of Uniben. When Bestman died and Nwachukwu travelled abroad, I was left with ART Solarin as a proposed mentor. However, by the time I was released by my then institution (Uniagric Makurdi), Prof. Solarin had left OAU, Ile-Ife. I was so attracted to Professor Solarin because of his gentle and humane disposition in addition to his area of Mathematics which is a general way out for every complex mathematical problem. I so wished to be like him then until my attention was diverted to Mathematical Modelling. Professor ART Solarin as the Director of National Mathematical Centre returned the Centre to the core purpose for which it was set up after many years of derailment by those who have nothing to do with the Mathematics Research based nature of the Centre. He is a kind, honest, dedicated and great mathematician of our time. On behalf of the Mathematicians in Nigeria (NMS, MAN, NSMB, NAMP, etc), my family and well-wishers, I congratulate you and wish you the best for reaching this God approved age. I wish you good health, longevity and God's favour to witness many more fruitful years.