Journal of the Nigerian Mathematical Society
Vol. 36, Issue 2, pp. 369-397, 2017

DUFOUR AND CHEMICAL REACTION EFFECTS ON UNSTEADY MHD FLOW OF A VISCOUS FLUID IN A PARALLEL POROUS PLATE CHANNEL UNDER THE INFLUENCE OF SLIP CONDITION

M. VENKATESWARLU[1] AND D. VENKATA LAKSHMI

ABSTRACT. This article attempts to analyze the Dufour and chemical reaction effects on the unsteady MHD convection flow of a radiating fluid between two parallel porous plates. Based on the pulsatile flow nature, exact solution of the governing equations for the fluid velocity, temperature and concentration are obtained. The expressions of skin friction, Nusselt number and Sherwood number are also derived. The numerical values of fluid velocity, temperature and concentration are displayed graphically whereas those of shear stress, rate of heat transfer and rate of mass transfer at the plate are presented in tabular form for various values of pertinent flow parameters.

Keywords and phrases: Dufour effect, Heat source, MHD fluid, Slip condition.
2010 Mathematical Subject Classification: 76D05, 74K20, 76S05, 76E06.

1. INTRODUCTION

The research on MHD flows has been continuously attracting more attention owing to its intriguing physical applications in science and engineering concerning the energy problems. In particular, the MHD property is used in MHD power generators, petroleum industry, solar physics, polymer technology, accelerators and aerodynamics. Magnetofluid dynamics for engineers and applied physicists was documented by Cramer and Pai [1]. The effect of slip condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi [2]. Raptis and Kafoussias [3] presented Magnetohydrodynamic free convection flow and mass transfer through porous medium bounded by an infinite vertical porous plate with constant heat flux, due to the importance

Received by the editors October 27, 2016; Revised: December 16, 2016; Accepted: August 09, 2017
www.nigerianmathematicalsociety.org
[1]Corresponding author: M. VENKATESWARLU

of mass transfer and that of applied magnetic field in the study of star and planets. Adesanya and Makinde [4] investigated on MHD oscillatory slip flow and heat transfer in a channel filled with porous media. The relevance of mass diffusion and thermal diffusion specifically in chemical engineering is well known.

Chemically reacting fluids offer many technological applications ranging from the formation of thin films for electronics, combustion reactions, catalysis, biological systems, energy transfer in a wet cooling tower, generating electric power, manufacturing of ceramics or glassware, food processing etc. Chemical reactions can be classified as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface and homogeneous if it takes place in the solution. A reaction is said to be of the first order, if the rate of reaction is directly proportional to the concentration itself, Cussler [5]. Ahmed [6] investigated the effects of chemical reaction on the transient MHD free convective flow in a slip flow regime. The effect of chemical reaction on heat and mass transfer in a laminar boundary layer flow has been studied under different conditions by several authors [7-14].

The heat transfer enhancement is one of the most important technical aims for engineering systems due to its wide range applications in electronics, fire and combustion modelling, cooling systems, development of metal waste from spent nuclear fuel, heat exchangers technology, next-generation solar film collectors, applications in the field of nuclear energy and various thermal systems. Sparrow and Cess [15] and Cess et al. [16] were one of the initial investigators to consider temperature dependent heat absorption on steady stagnation point flow and heat transfer. The effects of thermal radiation were also investigated by the researchers; see for example [17-23].

The Dufour and Soret effects in the combined heat and mass transfer processes, due to the thermal energy flux resulting from concentration gradients and the thermal diffusion flux resulting from the temperature gradients, may be significant in the areas of geosciences and chemical engineering, Eckert and Drake [24]. Such physical effects were explored in the papers by Kafoussias and Williams [25], by Anghel et al. [26], by Postelnicu [27] and by Lin et al. [28], amongst others. Tai and Char [29] examined the Soret and Dufour effects on free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium with thermal radiation. Recently, Venkateswarlu et al. [30] presented Soret, hall current, rotation, chemical reaction and thermal radiation effects on unsteady MHD heat and mass transfer natural convection flow past an accelerated vertical plate.

The objective of the present paper is to study the Dufour, heat source and chemical reaction effects on an unsteady heat and mass transfer MHD flow of a viscous, incompressible and electrically conducting fluid between two parallel porous plates under the influence of slip condition. The behavior of velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number has been discussed in detail for variation of thermo physical parameters. The following strategy is pursued in the rest of the paper. Section two presents the formation of the problem. The analytical solutions are presented in section three. Results are discussed in section four and finally section five provides a conclusion of the paper.

2.FORMATION OF THE PROBLEM

We consider the unsteady laminar slip flow of an incompressible, viscous and electrically conducting fluid through a channel with non-uniform wall temperature bounded by two parallel plates separated by a distance a. The channel is assumed to be filled with a saturated porous medium. A uniform magnetic field of strength B_o is applied normal to the plates. The above plate is heated at a constant temperature and thermal radiation effect is also taken in to account. It is assumed that there exist a homogeneous chemical reaction of first order with constant rate K_r^* between the diffusing species and the fluid. All of the fluid properties are assumed to be independent of x except possibly the pressure. Geometry of the problem is presented in Fig. 1. We choose a Cartesian coordinate system (x, y) where x- lies along the centre of the channel, y- is the distance measured in the normal section such that $y = a$ is the channel's width as shown in the figure below. Under the assumptions made by Adesanya and Makinde [4], as well as of the usual Boussinesq's approximation, the governing equations the flow can be expressed as:

Continuity equation:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

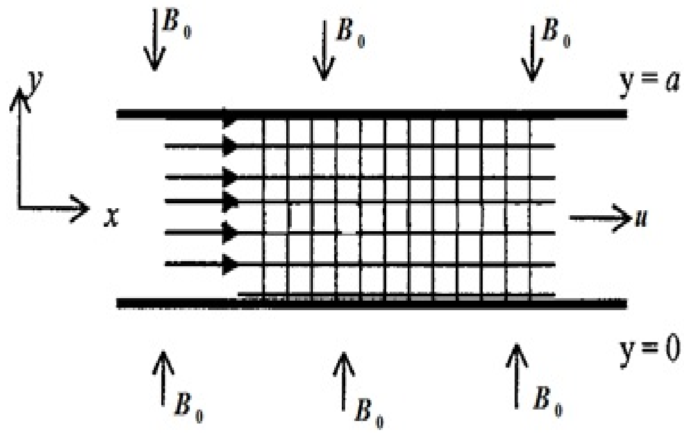

Fig. 1. Geometry of the problem

Momentum equation:

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e B_o^2}{\rho}u+g\beta_T(T-T_o)+g\beta_C(C-C_o)-\frac{\nu}{K}u \quad (2)$$

Energy equation:

$$\frac{\partial T}{\partial t}=\frac{K_T}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_s c_p}\frac{\partial^2 C}{\partial y^2}-\frac{Q_o}{\rho c_p}(T-T_o)-\frac{4\alpha^2}{\rho c_p}(T-T_o) \quad (3)$$

Diffusion equation:

$$\frac{\partial C}{\partial t}=D_m\frac{\partial^2 C}{\partial y^2}-K_r^*(C-C_o) \quad (4)$$

where u - fluid velocity in x- direction, v - fluid velocity in y- direction, p - fluid pressure, g - acceleration due to gravity, ρ - fluid density, β_T - coefficient of thermal expansion, β_C - coefficient of concentration volume expansion, t - time, K - permeability of porous medium, B_o - magnetic induction, T - fluid temperature, T_o - temperature at the cold wall, K_T - thermal diffusivity of the fluid, Q_o - dimensional heat source parameter, C - species concentration in the fluid, C_o - concentration at the cold wall, σ_e - fluid electrical conductivity, C_s - concentration susceptibility, c_p - specific heat at constant pressure, D_m - chemical molecular diffusivity, T_m - mean fluid temperature, ν - kinematic viscosity of the fluid and K_r^* - dimensional chemical reaction parameter respectively.

Assuming that slipping occurs between the plate and fluid, the corresponding initial and boundary conditions of the system of partial differential equations are given below

$$\left.\begin{array}{l} u = \phi_1 \, \frac{\partial u}{dy}, T = T_o, C = C_o \text{ at } y = 0 \\ u = \phi_2 \, \frac{\partial u}{dy}, \; T = T_1 + \epsilon \, (T - T_o) \; exp(int), \\ C = C_1 + \epsilon \, (C - C_o) \; exp(int) \text{ at } y = a \end{array}\right\} \tag{5}$$

where T_1 - fluid temperature at the heated plate, C_1 - species concentration at the heated plate, ϕ_1 - cold wall dimensional slip parameter, ϕ_2 - heated wall dimensional slip parameter, n - frequency of oscillation and $\epsilon << 1$ is a very small positive constant.

We introduce the following non-dimensional variables

$$\left.\begin{array}{l} \psi = \frac{x}{h}, \eta = \frac{y}{h}, U = \frac{h}{\nu}u, P = \frac{h^2}{\rho\nu^2}p, \gamma = \frac{\phi_1}{h}, \sigma = \frac{\phi_2}{h}, \\ \omega = \frac{h^2}{\nu}n, \tau = \frac{\nu}{h^2}t, \theta = \frac{T-T_o}{T_1-T_o}, \phi = \frac{C-C_o}{C_1-C_o} \end{array}\right\} \tag{6}$$

Equations (2), (3) and (4) reduce to the following non-dimensional form

$$\frac{\partial U}{\partial \tau} = -\frac{\partial P}{\partial \psi} + \frac{\partial^2 U}{\partial \eta^2} - [M + \frac{1}{Da}]U + Gr\theta + Gm\phi \tag{7}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} + Dr\frac{\partial^2 \phi}{\partial \eta^2} - [N + H]\theta \tag{8}$$

$$\frac{\partial \phi}{\partial \tau} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial \eta^2} - Kr\phi \tag{9}$$

Here $Gr = \frac{g\beta_T(T_1-T_o)h^3}{\nu^2}$ is the thermal buoyancy force, $Gm = \frac{g\beta_c(C_1-C_o)h^3}{\nu^2}$ is the concentration buoyancy force, $M = \frac{\sigma_e B_0^2 h^2}{\rho\nu}$ is the magnetic parameter, $Da = \frac{K}{h^2}$ is the Darcy parameter, $Pr = \frac{\rho c_p \nu}{K_T}$ is the Prandtl number, $H = \frac{Q_o h^2}{\rho c_p \upsilon}$ is the heat source parameter, $Dr = \frac{D_m K_T(C_1-C_o)}{C_s c_p \nu(T_1-T_o)}$ is the Dufour number, $N = \frac{4\alpha^2 h^2}{\rho c_p \nu}$ is the thermal radiation parameter, $Sc = \frac{\nu}{D_m}$ is the Schmidt number, $Kr = \frac{h^2}{\nu}k_r^*$ is the chemical reaction parameter respectively.

The corresponding initial and boundary conditions can be written as

$$\left.\begin{array}{l} U = \gamma\frac{\partial U}{d\eta}, \; \theta = 0, \phi = 0 \; at \; \eta = 0 \\ U = \sigma\frac{\partial U}{d\eta}, \; \theta = 1 + \epsilon \; exp(i\omega\tau), \\ \phi = 1 + \epsilon \; exp(i\omega\tau) \; at \; \eta = 1 \end{array}\right\} \tag{10}$$

For purely an oscillatory flow we take the pressure gradient of the form (see, Adesanya and Makinde [4])

$$\lambda = -\frac{dP}{d\psi} = \lambda_o + exp(i\omega\tau)\lambda_1 \tag{11}$$

where λ_o and λ_1 are constants and ω is the frequency of oscillation.

Given the velocity, temperature and concentration fields in the boundary layer, the shear stress τ_w, the heat flux q_w and mass flux j_w are obtained as

$$\tau_w = \mu\left[\frac{\partial u}{\partial y}\right] \tag{12}$$

$$q_w = -K_T\left[\frac{\partial T}{\partial y}\right] \tag{13}$$

$$j_w = -D_m\left[\frac{\partial C}{\partial y}\right] \tag{14}$$

In non-dimensional form the skin-friction coefficient Cf, heat transfer coefficient Nu and mass transfer coefficient Sh are defined as

$$Cf = \frac{\tau_w}{\rho(\frac{\nu}{h})^2} \tag{15}$$

$$Nu = \frac{hq_w}{K_T(T_1 - T_o)} \tag{16}$$

$$Sh = \frac{hj_w}{D_m(C_1 - C_o)} \tag{17}$$

Using non-dimensional variables in equation (6) and equations (12) to (14) into equations (15) to (17), we obtain the physical parameters

$$Cf = \left[\frac{\partial U}{\partial \eta}\right] \tag{18}$$

$$Nu = -\left[\frac{\partial \theta}{\partial \eta}\right] \tag{19}$$

$$Sh = -\left[\frac{\partial \phi}{\partial \eta}\right] \tag{20}$$

3. SOLUTION OF THE PROBLEM

Equations (7) to (9) are coupled non-linear partial differential equations and these cannot be solved in closed form. So, we reduce these non-linear partial differential equations into a set of ordinary differential equations, which can be solved analytically. This can be done by assuming the trial solutions for the velocity, temperature and concentration of the fluid as(see, Adesanya and Makinde [4], Malapati and Dasari [31], Siva Kumar et al [32])

$$U(\eta,\tau) = U_o(\eta) + \epsilon\ exp(i\omega\tau)U_1(\eta) + 0(\epsilon^2) \tag{21}$$

$$\theta(\eta,\tau) = \theta_o(\eta) + \epsilon\ exp(i\omega\tau)\theta_1(\eta) + 0(\epsilon^2) \tag{22}$$

$$\phi(\eta,\tau) = \phi_o(\eta) + \epsilon\ exp(i\omega\tau)\phi_1(\eta) + 0(\epsilon^2) \tag{23}$$

Substituting equations (21) to (23) into equations (7) to (9), then equating the harmonic and non-harmonic terms and neglecting the higher order terms of $0(\epsilon^2)$, we obtain

$$U_o'' - \Big[M + \frac{1}{Da}\Big]U_o = -[Gr\theta_o + Gm\phi_o + \lambda_o] \tag{24}$$

$$U_1'' - \Big[M + \frac{1}{Da} + i\omega\Big]U_1 = -[Gr\theta_1 + Gm\phi_1 + \lambda_1] \tag{25}$$

$$\theta_o'' - Pr[N+H]\ \theta_o = -PrDr\phi_o'' \tag{26}$$

$$\theta_1'' - Pr\ [N+H+i\omega]\theta_1 = -PrDr\phi_1'' \tag{27}$$

$$\phi_o'' - Sc\ Kr\ \phi_o = 0 \tag{28}$$

$$\phi_1'' - Sc\ [Kr + i\omega]\ \phi_1 = 0 \tag{29}$$

where the prime denotes the ordinary differentiation with respect to η .

The corresponding initial and boundary conditions can be written as

$$\left.\begin{array}{l} U_o = \gamma U_o',\ U_1 = \gamma U_1',\ \theta_0 = 0, \theta_1 = 0, \\ \phi_0 = 0,\ \phi_1 = 0,\ at\ \eta = 0 \\ U_o = \sigma U_o',\ U_1 = \sigma U_1',\ \theta_0 = 1,\ \theta_1 = 1, \\ \phi_0 = 1,\ \phi_1 = 1,\ at\ \eta = 1 \end{array}\right\} \tag{30}$$

The analytical solutions of equations (24) to (29) with the boundary conditions in equation (30), are given by

$$\begin{aligned} U_o = A_{22}\ exp(m_5\eta) + A_{21}\ exp(-m_5\eta) + A_8 \\ - \frac{A_6 \sinh(m_3\eta)}{\sinh(m_3)} + \frac{A_7 \sinh(m_1\eta)}{\sinh(m_1)} \end{aligned} \tag{31}$$

$$\begin{aligned} U_1 = A_{39}\ exp(m_6\eta) + A_{38}\ exp(-m_6\eta) + A_{25} \\ - \frac{A_{23} \sinh(m_4\eta)}{\sinh(m_4)} + \frac{A_{24} \sinh(m_2\eta)}{\sinh(m_2)} \end{aligned} \tag{32}$$

$$\theta_o = \frac{A_3 \sinh(m_3\eta)}{\sinh(m_3)} - \frac{A_2 \sinh(m_1\eta)}{\sinh(m_1)} \tag{33}$$

$$\theta_1 = \frac{A_5 \sinh(m_4\eta)}{\sinh(m_4)} - \frac{A_4 \sinh(m_2\eta)}{\sinh(m_2)} \tag{34}$$

$$\phi_o = \frac{\sinh(m_1\eta)}{\sinh(m_1)} \tag{35}$$

$$\phi_1 = \frac{\sinh(m_2\eta)}{\sinh(m_2)} \tag{36}$$

By substituting equations (31) to (36) into equations (21) to (23), we obtained solutions for the fluid velocity, temperature and concentration and are presented in the following form

$$\begin{aligned} U(\eta,\tau) = \Bigg[& A_{22}\ exp(m_5\eta) + A_{21}\ exp(-m_5\eta) + \\ & A_8 - \frac{A_6 \sinh(m_3\eta)}{\sinh(m_3)} + \frac{A_7 \sinh(m_1\eta)}{\sinh(m_1)}\Bigg] + \\ \epsilon\ exp(i\omega\tau)\Bigg[& A_{39}\ exp(m_6\eta) + A_{38}\ exp(-m_6\eta) + A_{25} - \\ & \frac{A_{23} \sinh(m_4\eta)}{\sinh(m_4)} + \frac{A_{24} \sinh(m_2\eta)}{\sinh(m_2)}\Bigg] \end{aligned} \tag{37}$$

$$\begin{aligned} \theta(\eta,\tau) = & \left[\frac{A_3 \sinh(m_3\eta)}{\sinh(m_3)} - \frac{A_2 \sinh(m_1\eta)}{\sinh(m_1)}\right] + \\ & \epsilon\ exp(i\omega\tau)\left[\frac{A_5 \sinh(m_4\eta)}{\sinh(m_4)} - \frac{A_4 \sinh(m_2\eta)}{\sinh(m_2)}\right] \end{aligned} \tag{38}$$

$$\phi(\eta,\tau) = \left[\frac{\sinh(m_1\eta)}{\sinh(m_1)}\right] + \epsilon\ exp(i\omega\tau)\left[\frac{\sinh(m_2\eta)}{\sinh(m_2)}\right] \tag{39}$$

3.1 Skin friction : From the velocity field, the skin friction at the plate can be obtained, which is given in non dimensional form as

$$Cf = \Bigg[A_{22}m_5\ exp(m_5\eta) - A_{21}m_5\ exp(-m_5\eta) - \frac{A_6m_3\cosh(m_3\eta)}{\sinh(m_3)} + \frac{A_7m_1\cosh(m_1\eta)}{\sinh(m_1)}\Bigg] + \epsilon\ exp(i\omega\tau)\Bigg[A_{39}m_6\ exp(m_6\eta) - A_{38}m_6\ exp(-m_6\eta) - \frac{A_{23}m_4\cosh(m_4\eta)}{\sinh(m_4)} + \frac{A_{24}m_2\cosh(m_4\eta)}{\sinh(m_2)}\Bigg] \tag{40}$$

3.2 Nusselt number : From temperature field, we obtained heat transfer coefficient which is given in non-dimensional form as

$$Nu = -\left[\frac{A_3m_3\cosh(m_3\eta)}{\sinh(m_3)} - \frac{A_2m_1\cosh(m_1\eta)}{\sinh(m_1)}\right] - \epsilon\ exp(i\omega\tau)\left[\frac{A_5m_4\cosh(m_4\eta)}{\sinh(m_4)} - \frac{A_4m_2\cosh(m_2\eta)}{\sinh(m_2)}\right] \tag{41}$$

3.3 Sherwood number : From concentration field, we obtained mass transfer coefficient which is given in non-dimensional form as

$$Sh = -\left[\frac{m_1\cosh(m_1\eta)}{\sinh(m_1)}\right] - \epsilon\ exp(i\omega\tau)\left[\frac{m_2\cosh(m_2\eta)}{\sinh(m_2)}\right] \tag{42}$$

3. RESULTS AND DISCUSSION

A series of computations has been carried out for the effects of the following parameters: cold wall slip parameter γ , heated wall slip parameter σ , thermal Grashof number Gr , solutal Grashof number Gm, magnetic parameter M , pressure gradient λ , Darcy parameter Da , Prandtl number Pr , Dufour number Dr, radiation parameter N , heat source parameter H , Schmidt number Sc, chemical reaction parameter Kr and time τ on the velocity U, temperature θ, concentration ϕ, skin friction Cf, Nusselt number Nu as well as Sherwood number Sh. The analysis of the fluid flow for the distribution profiles with the following default values for the parameters, $Gr = Gm = M = Da = N = \lambda = Dr = H = Kr = \omega = \varepsilon = 1$, $\tau = 0$, $\gamma = \sigma = 0.1$, $Pr = 0.71$ and $Sc = 0.78$. Therefore all the graphs and tables are corresponding to these values unless specifically indicated on the appropriate graph or table.

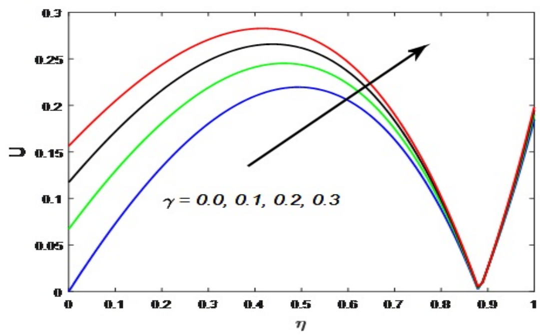

Fig. 2. Influence of cold wall slip parameter on velocity profiles.

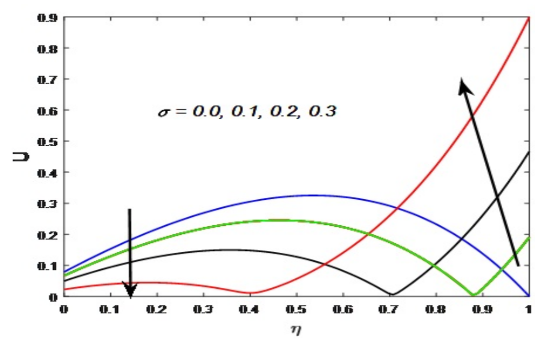

Fig. 3. Influence of heated wall slip parameter on velocity profiles.

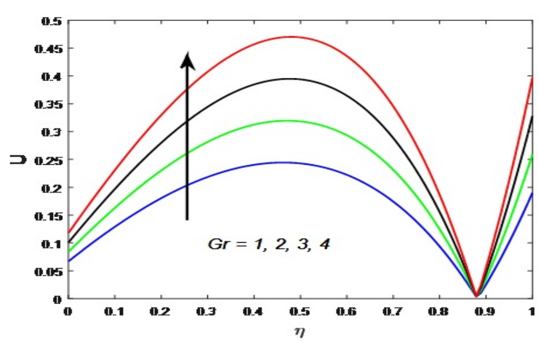

Fig. 4. Influence of Grashof number on velocity profiles.

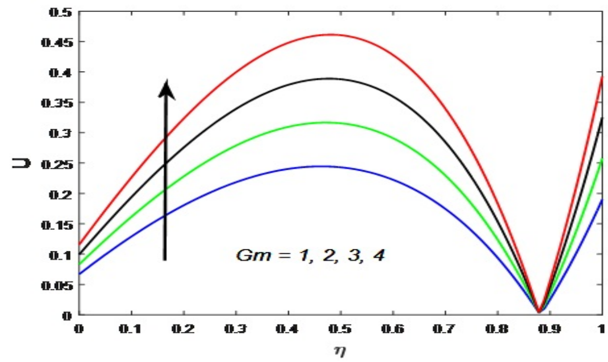

Fig. 5. Influence of solutal Grashof number on velocity profiles.

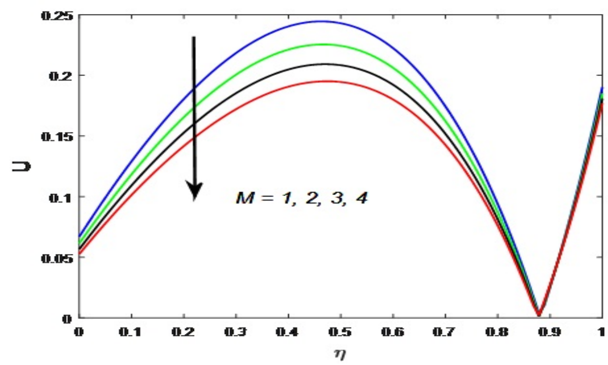

Fig. 6. Influence of magnetic parameter on velocity profiles.

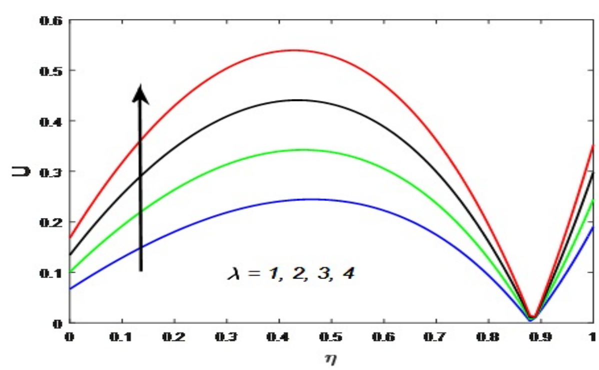

Fig. 7. Influence of pressure gradient on velocity profiles.

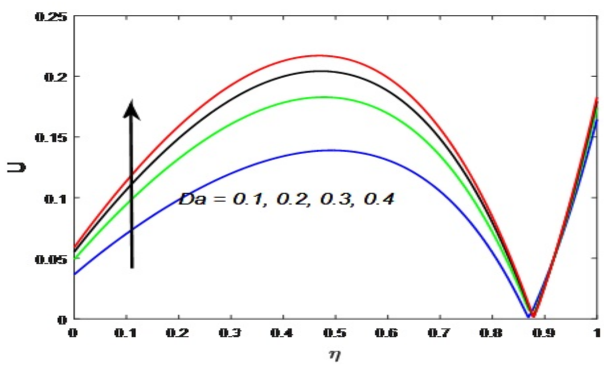

Fig. 8. Influence of Darcy parameter on velocity profiles.

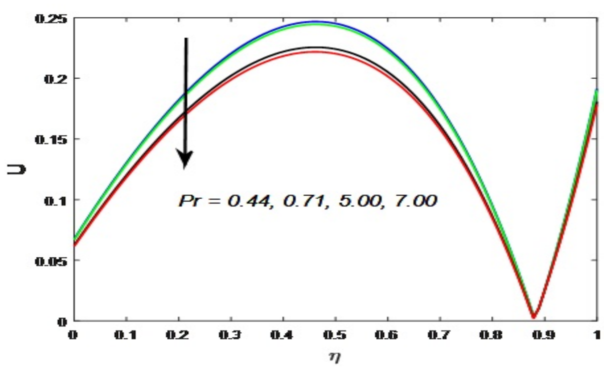

Fig. 9. Influence of Prandtl number on velocity profiles.

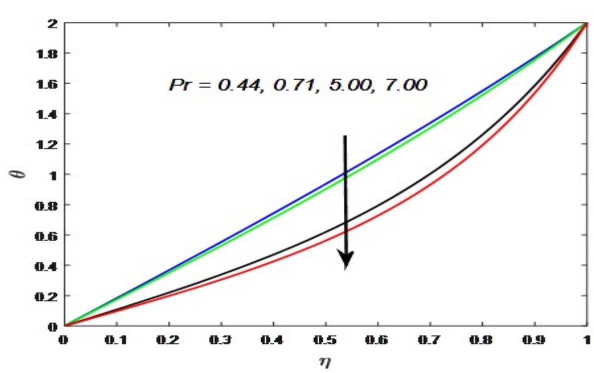

Fig. 10. Influence of Prandtl number on temperature profiles.

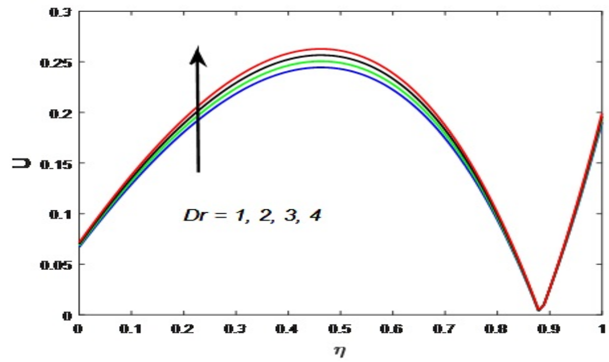

Fig.11. Influence of Dufour number on velocity profiles.

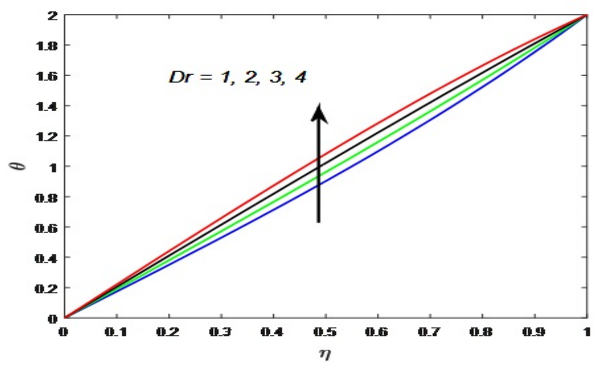

Fig. 12. Influence of Dufour number on temperature profiles.

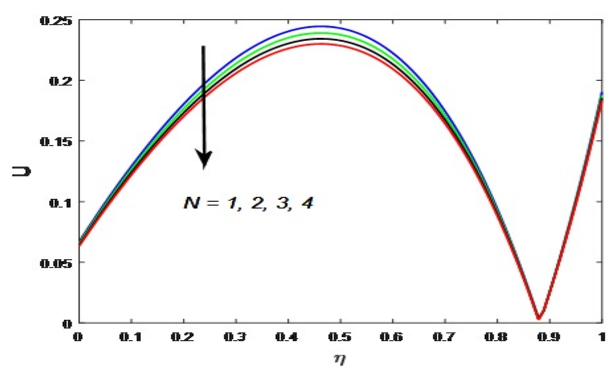

Fig. 13. Influence of radiation parameter on velocity profiles.

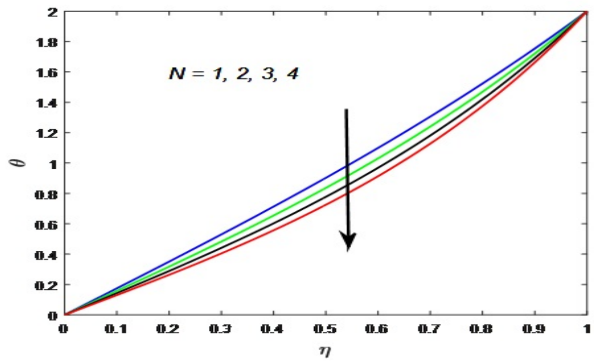

Fig. 14. Influence of radiation parameter on temperature profiles.

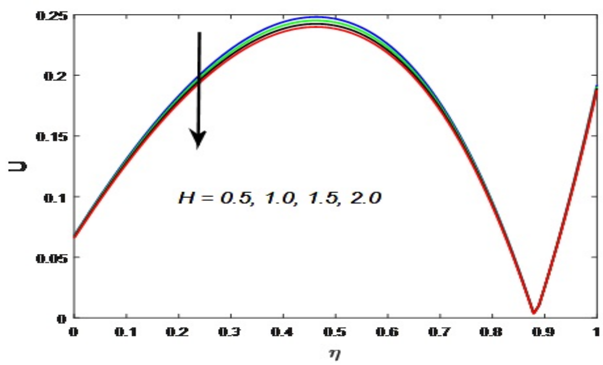

Fig. 15. Influence of heat source parameter on velocity profiles.

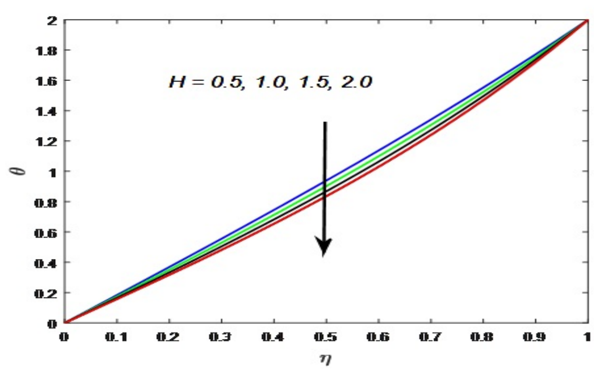

Fig. 16. Influence of heat source parameter on temperature profiles.

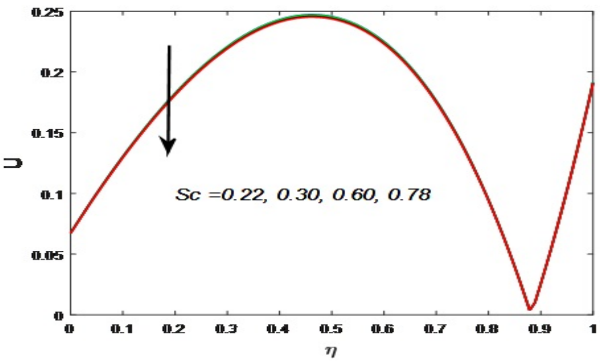

Fig. 17. Influence of Schmidt number on velocity profiles.

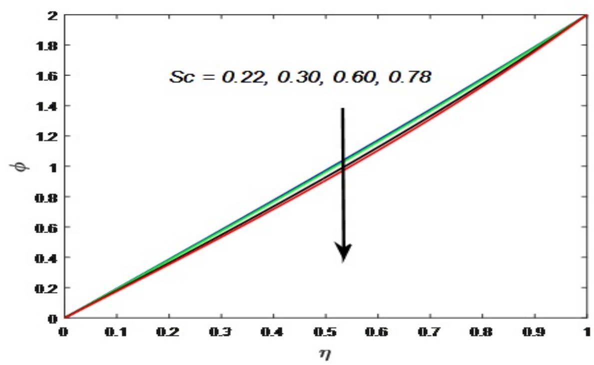

Fig. 18. Influence of Schmidt number on concentration profiles.

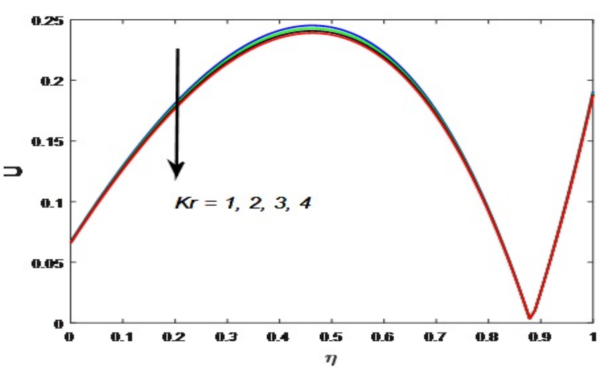

Fig. 19. Influence of chemical reaction parameter on velocity profiles.

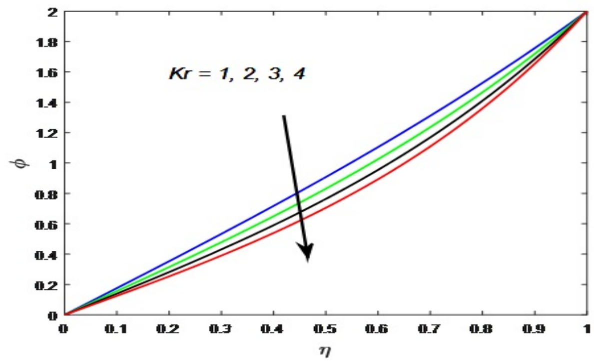

Fig. 20. Influence of chemical reaction parameter on concentration profiles.

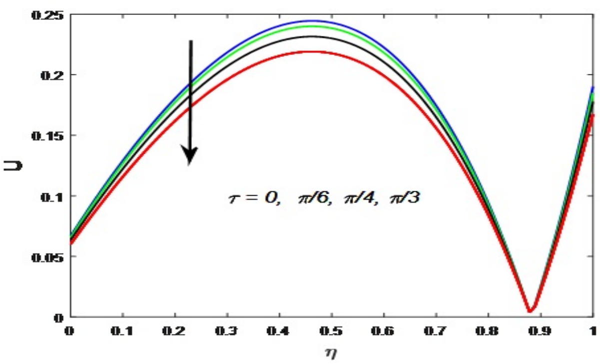

Fig. 21. Influence of time on velocity profiles.

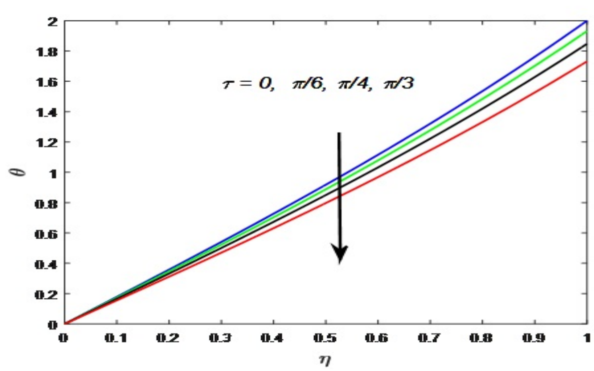

Fig. 22. Influence of time on temperature profiles.

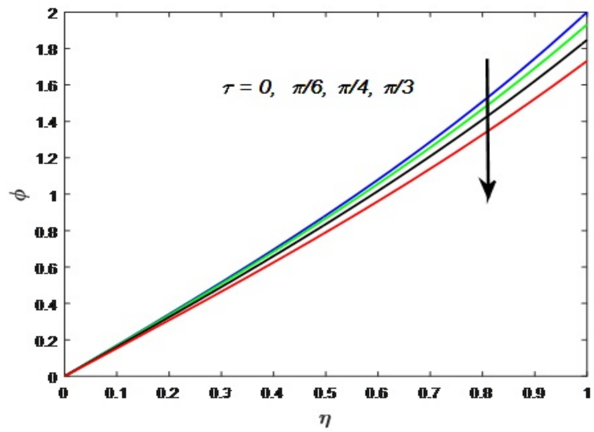

Fig. 23. Influence of time on concentration profiles.

Figs. 2 and 3 demonstrate the fluid velocity profile variations with the cold wall slip parameter γ and the heated wall slip parameter σ. It is observed that, the fluid velocity U increases on increasing the cold wall slip parameter γ thus enhancing the fluid flow. The cold wall slip parameter γ did not cause any appreciable effect on the heated wall. An increase in the heated wall slip parameter σ decreases the fluid velocity U minimally at the cold wall and increasing the heated wall slip parameter σ causes a flow reversal towards the heated wall. It is observed that in Fig.3, $\sigma = 0$ corresponds to the pulsatile case with no slip condition at the heated wall.

The effects of thermal Grashof number Gr and solutal Grashof number Gm on the velocity U of the flow field are presented in Figs.4 and 5. A study of the curves shows that thermal Grashof number Gr and solutal Grashof numberGm accelerates the velocity U of the flow field at all points. This is due to the reason that there is an enhancement in thermal buoyancy force and concentration buoyancy force.

The influence of magnetic parameter M on the fluid velocity U is shown in the Fig.6. It is noticed that, an increase in the magnetic parameter M decreases the fluid velocity U due to the resistive action of the Lorenz forces except at the heated wall where the reversed flow induced by wall slip caused an increase in the fluid velocity. This implies that magnetic field tends to decelerate fluid flow. Fig.7 demonstrates the influence of pressure gradient λ on the fluid velocity U. It is observed that, the fluid velocity U increases on increasing the pressure gradient λ. Fig.8 shows the variation of fluid velocity U with the Darcy parameter Da . The graph shows that an increase in the Darcy parameter increases the fluid flow except at the flow reversal point at the heated wall.

Figs. 9 and 10, shows the plot of velocity U and temperature θ of the flow field against different values of Prandtl number Pr taking other parameters are constant. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. It is evident from Figs. 9 and 10, velocity U and temperature θ decreases on increasing Prandtl number Pr. Thus higher Prandtl number leads to faster cooling of the plate.

The influence of Dufour number Dr on the fluid velocity and temperature profiles is plotted in Figs. 11 and 12 respectively. The Dufour number signifies the contribution of the concentration gradients to the thermal energy flux in the flow. It is found that an increase in Dufour number Dr causes a rise in the velocity U and temperature θ.

It is observed that, from Figs.13 and 14 both the fluid velocity U and temperature θ decreases on increasing the radiation parameter N . Figs. 15 and 16, demonstrate the plot of fluid velocity U and temperature θ for a variety of heat source parameter H . It is seen in figures that, the fluid velocity U and temperature θ decrease on increasing the heat source parameter H. This implies radiation and heat source have tendency to reduce fluid temperature.

The nature of fluid velocity U and concentration ϕ in presence of foreign species such as Hydrogen ($Sc = 0.22$), Helium ($Sc = 0.30$), Water vapour ($Sc = 0.60$) and Ammonia ($Sc = 0.78$) is shown in Figs.17 and 18. Physically, Schmidt number signifies the relative strength of viscosity to chemical molecular diffusivity. It is observed that velocity U and concentration ϕ decreases on increasing Schmidt number Sc in Figs. 17 and 18.

Figs. 19 and 20 demonstrate the influence of chemical reaction parameter Kr on the velocity U and species concentration ϕ. It is observed that, both velocity U and species concentration ϕ decreases on increasing the chemical reaction parameter Kr. This implies that, chemical reaction tends to reduce the fluid velocity and species concentration. It may be noted from Figs.21 to 23 that fluid velocity U, temperature θ and concentration ϕ are decreases with the progress of time τ.

From tables 1 to 3, it is clear that the skin friction Cf increases on increasing the thermal Grashof number Gr , solutal Grashof number Gm , Darcy parameter Da , pressure gradient λ and Dufour number Dr whereas it decreases on increasing the magnetic parameter M , Prandtl number Pr , radiation parameter N , heat source parameter H ,Schmidt number Sc , chemical reaction parameter Kr and time τ at both cold and heated walls. The skin friction coefficient Cf decreases at the cold wall and increases at the heated wall on increasing the cold wall slip parameter γ and heated wall slip parameter σ.

From table 4, it is clear that the mass transfer coefficient Sh increases at the cold wall and decreases at the heated wall on increasing the Schmidt number Sc and chemical reaction parameter Kr. Mass transfer coefficient Sh increases at both cold and heated plates with the progress of time τ.

From table 5, it is clear that the heat transfer coefficient Nu increases at the cold wall and decreases at the heated wall on increasing the Prandtl number Pr , radiation parameter N and heat source parameter H . Nusselt number Nu decreases at the cold and increases at the heated wall on increasing the Dufour number Dr. Heat transfer coefficient Nu increases at both cold and heated plates with the progress of time τ .

Table 1. Influence of Gr, Gm, M, Da and λ on the skin friction coefficient.

Gr	Gm	M	Da	λ	Skin friction Cf Cold wall	Heated wall
1.0	1.0	1.0	1.0	1.0	0.6708	1.9069
2.0	1.0	1.0	1.0	1.0	0.8377	2.5895
3.0	1.0	1.0	1.0	1.0	1.0047	3.2720
4.0	1.0	1.0	1.0	1.0	1.1716	3.9545
1.0	1.0	1.0	1.0	1.0	0.6708	1.9069
1.0	2.0	1.0	1.0	1.0	0.8383	2.5909
1.0	3.0	1.0	1.0	1.0	1.0059	3.2749
1.0	4.0	1.0	1.0	1.0	1.1735	3.9589
1.0	1.0	1.0	1.0	1.0	0.6708	1.9069
1.0	1.0	2.0	1.0	1.0	0.6154	1.8547
1.0	1.0	3.0	1.0	1.0	0.5681	1.8111
1.0	1.0	4.0	1.0	1.0	0.5274	1.7745
1.0	1.0	1.0	0.1	1.0	0.3670	1.6483
1.0	1.0	1.0	0.2	1.0	0.4919	1.7437
1.0	1.0	1.0	0.3	1.0	0.5539	1.7982
1.0	1.0	1.0	0.4	1.0	0.5908	1.8320
1.0	1.0	1.0	1.0	1.0	0.6708	1.9069
1.0	1.0	1.0	1.0	2.0	1.0072	2.4473
1.0	1.0	1.0	1.0	3.0	1.3436	2.9877
1.0	1.0	1.0	1.0	4.0	1.6800	3.5281

Table 2. Influence of Pr, N, Dr and H on the skin friction coefficient.

Pr	N	Dr	H	Skin friction Cf Cold wall	Heated wall
0.44	1.0	1.0	1.0	0.6775	1.9198
0.71	1.0	1.0	1.0	0.6708	1.9069
5.00	1.0	1.0	1.0	0.6166	1.7912
7.00	1.0	1.0	1.0	0.6062	1.7641
0.71	1.0	1.0	1.0	0.6708	1.9069
0.71	2.0	1.0	1.0	0.6578	1.8815
0.71	3.0	1.0	1.0	0.6464	1.8587
0.71	4.0	1.0	1.0	0.6363	1.8381
0.71	1.0	1.0	1.0	0.6708	1.9069
0.71	1.0	2.0	1.0	0.6826	1.9295
0.71	1.0	3.0	1.0	0.6945	1.9522
0.71	1.0	4.0	1.0	0.7063	1.9749
0.71	1.0	1.0	0.5	0.6780	1.9208
0.71	1.0	1.0	1.0	0.6708	1.9069
0.71	1.0	1.0	1.5	0.6641	1.8938
0.71	1.0	1.0	2.0	0.6578	1.8815

Table 3. Influence of σ, γ, Sc, Kr and τ on the skin friction coefficient.

σ	γ	Sc	Kr	τ	Skin friction Cf Cold wall	Skin friction Cf Heated wall
0.0	0.1	0.78	1.0	0.0	0.7899	1.6102
0.1	0.1	0.78	1.0	0.0	0.6708	1.9069
0.2	0.1	0.78	1.0	0.0	0.4975	2.3344
0.3	0.1	0.78	1.0	0.0	0.2249	2.9985
0.1	0.0	0.78	1.0	0.0	0.7827	1.8492
0.1	0.1	0.78	1.0	0.0	0.6708	1.9069
0.1	0.2	0.78	1.0	0.0	0.5867	1.9500
0.1	0.3	0.78	1.0	0.0	0.5213	1.9834
0.1	0.1	0.22	1.0	0.0	0.6757	1.9159
0.1	0.1	0.30	1.0	0.0	0.6750	1.9146
0.1	0.1	0.60	1.0	0.0	0.6723	1.9098
0.1	0.1	0.78	1.0	0.0	0.6708	1.9069
0.1	0.1	0.78	1.0	0.0	0.6708	1.9069
0.1	0.1	0.78	2.0	0.0	0.6649	1.8959
0.1	0.1	0.78	3.0	0.0	0.6599	1.8862
0.1	0.1	0.78	4.0	0.0	0.6554	1.8776
0.1	0.1	0.78	1.0	0.0	0.6708	1.9069
0.1	0.1	0.78	1.0	$\frac{\pi}{6}$	0.6584	1.8544
0.1	0.1	0.78	1.0	$\frac{\pi}{4}$	0.6352	1.7802
0.1	0.1	0.78	1.0	$\frac{\pi}{3}$	0.6011	1.6755

Table 4. Influence of Sc, Kr and τ on the mass transfer coefficient.

Sc	Kr	τ	Sherwood number Sh Cool wall	Sherwood number Sh Heated wall
0.22	1.0	0.0	-1.9279	-2.1468
0.30	1.0	0.0	-1.9024	-2.2000
0.60	1.0	0.0	-1.8095	-2.3988
0.78	1.0	0.0	-1.7559	-2.5167
0.78	1.0	0.0	-1.7559	-2.5167
0.78	2.0	0.0	-1.5565	-2.9625
0.78	3.0	0.0	-1.3868	-3.3739
0.78	4.0	0.0	-1.2411	-3.7563
0.78	1.0	0.0	-1.7559	-2.5167
0.78	1.0	$\frac{\pi}{6}$	-1.7242	-2.3707
0.78	1.0	$\frac{\pi}{4}$	-1.6638	-2.2360
0.78	1.0	$\frac{\pi}{3}$	-1.5750	-2.0631

Table 5. Influence of Pr, N,H, Dr and τ on the heat transfer coefficient.

Pr	N	H	Dr	τ	Nusselt number Nu Cold wall	Nusselt number Nu Heated wall
0.44	1.0	1.0	1.0	0.0	-1.8320	-2.3456
0.71	1.0	1.0	1.0	0.0	-1.7434	-2.5410
5.00	1.0	1.0	1.0	0.0	-1.0759	-4.6975
7.00	1.0	1.0	1.0	0.0	-0.9682	-5.3966
0.71	1.0	1.0	1.0	0.0	-1.7434	-2.5410
0.71	2.0	1.0	1.0	0.0	-1.5726	-2.9369
0.71	3.0	1.0	1.0	0.0	-1.4242	-3.3057
0.71	4.0	1.0	1.0	0.0	-1.2944	-3.6513
0.71	1.0	0.5	1.0	0.0	-1.8386	-2.3314
0.71	1.0	1.0	1.0	0.0	-1.7434	-2.5410
0.71	1.0	1.5	1.0	0.0	-1.6549	-2.7426
0.71	1.0	2.0	1.0	0.0	-1.5726	-2.9369
0.71	1.0	1.0	1.0	0.0	-1.7434	-2.5410
0.71	1.0	1.0	2.0	0.0	-1.8982	-2.2075
0.71	1.0	1.0	3.0	0.0	-2.0547	-1.8860
0.71	1.0	1.0	4.0	0.0	-2.2126	-1.5837
0.71	1.0	1.0	1.0	0.0	-1.7434	-2.5410
0.71	1.0	1.0	1.0	$\frac{\pi}{6}$	-1.6908	-2.4408
0.71	1.0	1.0	1.0	$\frac{\pi}{4}$	-1.6208	-2.3274
0.71	1.0	1.0	1.0	$\frac{\pi}{3}$	-1.5230	-2.1742

4. CONCLUSIONS

In this paper we have studied analytically the influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in presence of Dufour, heat source and chemical reaction effects. From the present investigation the following conclusions can be drawn:

- An increase in the heated wall slip parameter decreases the fluid velocity minimally at the cold wall and increasing in the heated wall slip parameter causes a flow reversal towards the heated wall. The cold wall slip parameter did not cause any appreciable effect on the heated wall.
- Skin friction coefficient decreases on increasing the cold wall slip parameter and heated wall slip parameter at the cold wall whereas it has a reverse effect at the heated wall.

- Both velocity and temperature profiles are increases on increasing the Dufour number. Nusselt number decreases at the cold wall and increases at the heated wall on increasing the Dufour number.
- Prandtl number, radiation parameter and heat source parameter have tendency to retard the fluid temperature. Heat transfer coefficient increases at the cold wall and decreases at the heated wall on increasing the Prandtl number, radiation parameter and heat source parameter.
- Schmidt number and chemical reaction parameter have tendency to decelerate the species concentration. Mass transfer coefficient increases at the cold wall and decreases at the heated wall on increasing the Schmidt number and chemical reaction parameter.

ACKNOWLEDGEMENTS

Authors are thankful for the suggestions and comments of the referees, which have led to improvement of the paper.

NOMENCLATURE

a	distance between two parallel plates
B_o	uniform magnetic field
C	species concentration
C_f	skin-friction coefficient
C_1	species concentration at the heated wall
C_o	species concentration at the cold wall
c_p	specic heat at constant pressure
C_s	concentration susceptibility
D_m	chemical molecular diffusivity
Dr	Dufour number
Gm	Solutal Grashof number
Gr	thermal Grashof number
g	acceleration due to gravity
H	non- dimensional heat absorption parameter
j_w	mass flux
K	permeability of porous medium
k_r^*	dimensional chemical reaction parameter
kr	non-dimensional chemical reaction parameter
Da	Darcy parameter

K_T thermal conductivity of the fluid
M Magnatic parameter
Nu Nusselt number
n frequency of oscillation
Pr Prandtl number
Q_o dimensional heat absorption parameter
q_w heat flux
Sc Schmidt number
Sh Sherwood number
T fluid temperature
T_m mean temperature of the fluid
T_1 fluid temperature at the heated wall
T_0 fluid temperature at the cold wall
t dimensional time
U A scaled velocity
u fluid velocity in x-direction
v fluid velocity in y-direction

Greek Symbols

β_c coefficient expansion for species concentration
β_T coefficient of thermal expansion
ν kinematic coefficient of viscosity
ω A scaled frequency
ϕ A scaled concentration
ϕ_1 dimensional cold wall slip parameter
ϕ_2 dimensional heated wall slip parameter
ρ fluid density
σ_e electrical conductivity
τ non dimensional time
τ_w shear stress
ψ A scaled coordinate
η A scaled coordinate
θ A scaled temperature
γ non-dimensional cold wall slip parameter
σ non-dimensional heated wall slip parameter

Appendix

$$m_1 = \sqrt{ScKr}, m_2 = \sqrt{Sc(Kr + i\omega)}, m_3 = \sqrt{Pr(N + H)},$$

$$m_4 = \sqrt{Pr(N + H + i\omega)}, m_5 = \sqrt{M + \frac{1}{Da}},$$

$$m_6 = \sqrt{M + \frac{1}{Da} + i\omega}, A_1 = PrDr, A_2 = \frac{m_1^2 A_1}{m_1^2 - m_3^2}, A_3 = 1 + A_2,$$
$$A_4 = \frac{m_2^2 A_1}{m_2^2 - m_4^2}, A_5 = 1 + A_4, A_6 = \frac{GrA_3}{m_3^2 - m_5^2}, A_7 = \frac{GrA_2 - Gm}{m_1^2 - m_5^2},$$

$$A_8 = \frac{\lambda_o}{m_5^2}, A_9 = \frac{\gamma m_3 A_6}{\sinh(m_3)}, A_{10} = \frac{\gamma m_1 A_7}{\sinh(m_1)}, A_{11} = A_{10} - (A_8 + A_9),$$

$$A_{12} = 1 - \gamma m_5, A_{13} = 1 + \gamma m_5, A_{14} = 1 - \sigma m_5, A_{15} = 1 + \sigma m_5,$$
$$A_{16} = A_6[\sigma m_3 \coth(m_3) - 1], A_{17} = A_7[\sigma m_1 \coth(m_1) - 1],$$

$$A_{18} = A_{17} - (A_8 + A_{16}), A_{19} = A_{12}A_{15}\, exp(-m_5) - A_{13}A_{14}\, exp(m_5),$$

$$A_{20} = A_{12}A_{18} - A_{11}A_{14}\, exp(m_5), A_{21} = \frac{A_{20}}{A_{19}}, A_{22} = \frac{A_{11} - A_{13}A_{21}}{A_{12}},$$
$$A_{23} = \frac{GrA_5}{m_4^2 - m_6^2}, A_{24} = \frac{GrA_4 - Gm}{m_2^2 - m_6^2}, A_{25} = \frac{\lambda_1}{m_6^2}, A_{26} = \frac{\gamma m_4 A_{23}}{\sinh(m_4)},$$

$$A_{27} = \frac{\gamma m_2 A_{24}}{\sinh(m_2)} A_{28} = A_{27} - (A_{25} + A_{26}), A_{29} = 1 - \gamma m_6,$$

$$A_{30} = 1 + \gamma m_6, A_{31} = 1 - \sigma m_6, A_{32} = 1 + \sigma m_6,$$
$$A_{33} = A_{24}[\sigma m_2 \coth(m_2) - 1], A_{34} = A_{23}[\sigma m_4 \coth(m_4) - 1],$$

$$A_{35} = A_{33} - (A_{25} + A_{34}), A_{36} = A_{29}A_{32} exp(-m_6) - A_{31}A_{30} exp(m_6),$$
$$A_{37} = A_{29}A_{35} - A_{28}A_{31} exp(m_6), A_{38} = \frac{A_{37}}{A_{36}}, A_{39} = \frac{A_{28} - A_{30}A_{38}}{A_{29}}$$

REFERENCES

[1] K. R. Cramer and S. I. Pai: *Magnetofluid dynamics for engineers and applied physicists*, McGraw Hill Book Company, New York, 1973.

[2] O. D. Makinde and E. Osalusi: *MHD steady flow in a channel with slip at the permeable boundaries*, Romanian Journal of Physics **51**, pp.319-328, 2006.

[3] A. Raptis and N. G. Kafoussias:*Magnetohydrodynamic free convection flow and mass transfer through porous medium bounded by an infinite vertical porous plate with constant heat flux*, Can. J. Phys., **60**, pp. 1725-1729, 1982.

[4] S. O. Adesanya and O. D. Makinde:*MHD oscillatory slip flow and heat transfer in a channel filled with porous media*, U.P.B. Sci. Bull. Series A, **76** pp. 197-204, 2014.

[5] E. L. Cussler:*Diffusion Mass Transfer in Fluid Systems*, Cambridge University Press, 1998.

[6] S. Ahmed:*Influence of chemical reaction on transient MHD free convective flow over a vertical plate in slip flow regime*, Emirates J Eng Res, **15** (1), pp. 534.2010.

[7] A. J. Chamkha:*MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction*, Int. Commun. Heat and mass transfer,**30** (3), pp. 413-422, 2003.

[8] H. S. Takhar, A. J. Chamkha and G. Nath: *Flow and Mass Transfer on a stretching sheet with a magnetic field and chemically reactive species*, Int. J. Eng. Sci., **38** (12), pp.1303-1314, 2000.

[9] M. Venkateswarlu and P. Padma:*Unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction*, Procedia Engineering,**127**, pp. 791-799, 2015.

[10] R. Muthucumarswamy and P. Ganesan:*Effect of chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate*, J. Appl. Mech. Tech. Phys., **42** (2), pp.665-671, 2001.

[11] R. Muthucumarswamy: *Effects of chemical reaction on a moving isothermal vertical surface with suction*, Acta Mech. **155**, pp. 65-70, 2002.

[12] K. Manivannan , R. Muthucumarswamy and V. Thangaraj: *Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion*, Thermal science,**51** (1) vol.13, no.2, pp.155-162, 2009.

[13] A. A. Bakr: *Effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference*, Commun. Nonlinear Sci. Numer. Simulation, **16**, pp. 698710, 2011.

[14] J. Manjula, P. Padma, M. Gnaneswara Reddy and M. Venakateswarlu: *Influence of thermal radiation and chemical reaction on MHD flow, heat and mass transfer over a stretching surface*, Procedia Engineering, **127**, pp. 1315-1322, 2015.

[15] E. M. Sparrow and R. D. Cess:*Temperature dependent heat sources or sinks in a stagnation point flow*, Appl. Sci. Res., **10**,pp. 185-197, 1961.

[16] R. D. Cess, T.F. Irvine and J.P. Hartnett: *The interaction of thermal radiation with conduction and convection heat transfer*, Adv. Heat Transfer, Academic Press, New York, **1** (1), pp. 1-50, 1964.

[17] M. Venkateswarlu, G. V. Ramana Reddy and D. V. Lakshmi: *Radiation effects on MHD boundary layer flow of liquid metal over a porous stretching surface in porous medium with heat generation*, J. Korean Soc. Ind. Appl. Math., **19**(1), pp.83-102, 2015.

[18] A. Ishak, *Mixed convection boundary layer flow over a horizontal plate with thermal radiation, Heat and Mass Transfer*, **46** (1), pp.147151, 2009.

[19] O. D. Makinde and P. Y. Mhone *Heat transfer to MHD oscillatory flow in a channel filled with porous medium*, Rom. Journ. Phys,**50**, pp. 931-938, 2005.

[20] A. Mehmood and A. Ali:*The effect of slip condition on unsteady MHD Oscillatory flow of a viscous fluid in a planer channel*, Rom. Journ. Phys, **52**, pp. 85-91, 2007.

[21] J. C. Umavathi, A. J. Chamkha, A. Mateen and A. Al-Mudhaf: *Unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel*, Nonlinear analysis: Modelling and control, **14**, pp. 397-415, 2009.

[22] M. Venkateswarlu, G. V. Ramana Reddy and D. V. Lakshmi: *Thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer*

flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion, J. Korean Soc. Ind. Appl. Math., **18**(3), pp. 257-268, 2014.

[23] M. Venkateswarlu, D. Venkata Lakshmi and G. Darmaiah: *Influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in presence of heat absorption and chemical reaction* , J. Korean Soc. Ind. Appl. Math., **20** (4), pp. 333 -354, 2016.

[24] E. R. G. Eckert and R. M. Drake: *Analysis of Heat and Mass Transfer*, McGraw-Hill, New York, 1972.

[25] N.G. Kafoussias and E.W. Williams: *Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity*, Int. J. Eng. Sci.**33** (9), pp. 1369-1384, 1995.

[26] M. Anghel, H. S. Takhar and I. Pop:*Dufour and Soret effects on free-convection boundary layer over a vertical surface embedded in a porous medium*, Studia Universitatis Babes Bolyai, Mathematica, **XLV**, pp. 11-21, 2000.

[27] A. Postelnicu: *Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects*, Heat Mass Transfer,**43**, pp. 595-602, 2007.

[28] H. C. Lin, M. I. Char and W. J. Chang: *Soret effects on non-Fourier heat and non- Fickianmass diffusion transfer in a slab*, Numer. Heat Transfer Part A,**55** (1), pp. 1096-1115, 2009.

[29] Bo-C. Tai and M.-I. Char: *Soret and Dufour effects on free convection flow of non- Newtonian fluids along a vertical plate embedded in a porous medium with thermal radiation*, Int. Commun. Heat and Mass Transfer, **37**, pp. 480483, 2010.

[30] M. Venkateswarlu, D. Venkata Lakshmi and K. Nagamalleswara Rao: *Soret, hall current, rotation, chemical reaction and thermal radiation effects on unsteady MHD heat and mass transfer natural convection flow past an accelerated vertical plate*, J. Korean Soc. Ind. Appl. Math., **20** (3), pp. 203 -224, 2016.

[31] V. Malapati and V. L. Dasari: *Soret and chemical reaction effects on the radiative MHD flow from an infinite vertical plate*, J. Korean Soc. Ind. Appl. Math., **21**(1), pp. 39 - 61, 2017.

[32] N. Siva Kumar, Rushi Kumar and A. G. Vijaya Kumar: *Thermal diffusion and chemical reaction effects on unsteady flow past a vertical porous plate with heat source dependent in slip flow regime*, Journal of Naval Architecture and Marine Engineering, **13**, pp. 51-62, 2016.

DEPARTMENT OF MATHEMATICS, V. R. SIDDHARTHA ENGINEERING COLLEGE, VIJAYAWADA, KRISHNA (DIST), ANDHRA PRADESH, INDIA, PIN: 520 007

E-mail address: `mvsr2010@gmail.com`

DEPARTMENT OF MATHEMATICS, BAPATLA WOMEN'S ENGINEERING COLLEGE, BAPATLA, GUNTUR (DIST), ANDHRA PRADESH, INDIA, PIN: 522 102

E-mail address: `himaja96@gmail.com`