

## HEAT TRANSFER ENHANCEMENT OF $Cu - Al_2O_3/Water$ HYBRID NANOFLUID FLOW OVER A STRETCHING SHEET

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ABSTRACT. A new concept of Hybrid Nanofluid has been triggered to the enhancement of heat transfer in the boundary layer flow. New modeled thermophysical properties have been proposed. Experimental values of thermal conductivity is compared with our proposed model. Two different kinds of fluids namely Hybrid nanofluid ( $Cu - Al_2O_3/Water$ ) and Nanofluid ( $Cu/Water$ ) are used to investigate the flow past a Stretching sheet. A parametric study has been carried out to explore the effects of physical parameters involved in the problem. From this study it is observed that, the Nusselt number of Hybrid nanofluid enhances upto 17.3% than pure water and 11.2% than nanofluid for 0.06 vol. concentration ( $\phi_2$ ). The heat transfer rate of Hybrid nanofluid ( $Cu - Al_2O_3/Water$ ) is higher than that of Nanofluid ( $Cu/Water$ ). By choosing different and appropriate nanoparticle combinations in Hybrid nanofluid, the desired heat transfer rate can be achieved.

**Keywords and phrases:** Hybrid Nanofluid, Boundary layer flow, Stretching sheet.

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### 1. INTRODUCTION

In recent era, to accomplish the striking improvement in thermal efficiency, new types of fluids such as nanofluids which are defined as colloidal solvent containing dispersed nanometer sized particles are introduced. These types of fluids have seen unprecedented growth in practical applications including refrigeration & air-conditioning, transportation, solar thermal, microelectronics, processors of mobile computers and also in high-capacity military communication devices and so on.

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Even though nanofluids succour the thirst of Engineers/Scientists in the thermal efficiency, still a better type of fluid is in search even till today. In coping of these, higher level type of nanofluids such as 'Hybrid nanofluids' have come into existence which are having high thermal conductivity than that of nanofluids. Hence the current work is chiefly dealt with Hybrid nanofluids which is the ultimate aim of the authors for enhancing the heat transfer rate.

The low cost, good fluidity and long-term stability are three most significant preconditions for the nanofluids toward practical applications in heat transfer field. Recently, many researchers use oxide nanoparticles, which have realized mass production as substitutions for metal nanoparticles and carbon nano materials. Unfortunately, the thermal conductivity of oxide nanoparticles is less compared to metal nanoparticles and in order to overcome that high volume fraction suspension of oxide particles ( $> 5.0\%vol.$ ) is required to achieve desired enhancement of thermal conductivity.

To maintain good fluidity, the only way is to decrease the concentration of the particle. In other words, oxide particles cannot meet the current criteria and finding new nano particle with low cost and high performance is still the most important challenge in the area of nanofluids.

In order to make it economical, a new type of nanofluid is introduced namely 'Hybrid Nanofluid'. The incorporation of small amount of metal nanoparticles / nano tubes into an oxide / metal nanoparticles which is already suspended in a base fluid can significantly improve the thermal properties. The benefits of 'Hybrid Nanofluid' are the high effective thermal conductivity, improved heat transfer, stability, advantages of individual suspension, attributed to good aspect ratio and synergistic effect of nano materials. The high thermal conductivity of Hybrid nanofluids translates into higher energy efficiency, better performance and lower operating costs.

Application areas of Hybrid nanofluids are varied widely in almost all the fields of heat transfer such as electronic cooling, engine cooling/vehicle thermal management, generator cooling, coolant in machining, welding, nuclear system cooling, lubrication, thermal storage, solar heating, cooling and heating in buildings, transformer cooling, biomedical, drug reduction, heat pipe, refrigeration, defence, space aircrafts and ships with better efficiency than that of nanofluids applicability.

These applicable characteristics attracted the researchers to work towards Hybrid nanofluid in real world heat transfer problems. Many experimental studies have made remarkable results from the usage of these types of Hybrid nanofluid systems. Niihara [1] demonstrated that the nano composites significantly improved mechanical and thermal properties by a new material design concept. Jana [2] observed the enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives. Suresh et al. [3] carried out a method to synthesis of  $Al_2O_3 - Cu/Water$  hybrid nanofluid. Nano composites proposed by them had a new designed concept of nano material and significantly it improved thermal and mechanical properties.

Experimental investigation of mixed convection with Hybrid nanofluid in inclined tube for laminar flow was examined by Momin [4]. Later, the effect of  $Al_2O_3 - Cu/Water$  hybrid nano fluid in heat transfer was reported by Suresh et. al. [5]. Synthesis of spherical silica/multiwall carbon nanotubes hybrid nanostructures and investigation of thermal conductivity of related nanofluids was analysed by Baghbanzadeh et. al. [6].

Numerically, only very few articles were published in Hybrid nanofluid. Augmentation of the heat transfer performance of a sinusoidal corrugated enclosure by employing Hybrid nanofluid was portrayed by Takabi and Salehi [7]. Numerical investigation on effect of base fluids and hybrid nanofluid in forced convective heat transfer was demonstrated by Nuim Labib et. al. [8]. Takabi and Shokouhmand [9] conveyed the effects of  $Al_2O_3 - Cu/water$  hybrid nanofluid on heat transfer and flow characteristics in turbulent regime. Recently, Finite element simulation of forced convection in a flat plate solar collector: influence of nanofluid with double nanoparticles was examined by Nasrin and Alim [10]. These are some great progress in Hybrid nanofluid research but still not enough.

The research topic of stretching sheet is often encountered in real life problems which attracted tremendous interest from researchers due to their numerous importance in the fields such as metal extrusion, glass fiber production, micro fluidics, transportation, manufacturing, hot rolling, paper production, space, acoustics, avionics, glass blowing and so on. Initially, Crane [11] studied the flow past a stretching plate. Later, Andersson et. al. [12] observed the flow of a power-law fluid over a stretching sheet.

An exact solution for self-similar boundary-layer flows induced by permeable stretching walls was investigated by Magyari and Keller [13]. Andersson [14] analysed slip flow past a stretching surface. In the literature, the concept of Nanofluid was introduced by Choi [15]. Further Khan and Pop [16] explored the boundary-layer flow of a nanofluid past a stretching sheet. Boundary-layer flow of nanofluids over a moving surface in a flowing fluid was studied by Bachok et. al. [17]. Anjali Devi and Julie Andrews [18] investigated about laminar boundary layer flow of nanofluid over a flat plate. Later, Makinde and Aziz [19] examined boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Several research investigations were carried out later in this field for various and different situation and physical conditions.

Very recently, Kalidas Das [20] extended the work on Nanofluid flow over a non-linear permeable stretching sheet with partial slip. The boundary layer flow of nanofluid over a nonlinearly stretching sheet with convective boundary condition was examined by Mustafa et. al. [21]. Numerical solution for hydromagnetic boundary layer flow and heat transfer past a stretching surface embedded in non-Darcy porous medium with fluid-particle suspension was explored by Gireesha et. al. [22]. Effect of Lorentz force on forced-convection nanofluid flow over a stretched surface was studied by Mohsen Sheikholeslami et. al. [23]. The magnetohydrodynamic stagnation point flow of a nanofluid over a stretching/shrinking sheet with suction was investigated by Mansur et. al. [24]. Dulal Pal and Gopinath Mandal [25] portrayed about the mixed convection-radiation on stagnation-point flow of nanofluids over a stretching/shrinking sheet in a porous medium with heat generation and viscous dissipation. Thermal radiation and slip effect on MHD stagnation point flow of nanofluid over a stretching sheet was demonstrated by Rizwan Ul Haq et. al. [26].

Nanoparticles effects on MHD fluid flow over a stretching sheet with solar radiation was numerically studied by Ghasemi et. al. [27]. Pourmehran et. al. [28] analysed the heat transfer and flow analysis of nanofluid flow induced by a stretching sheet in the presence of an external magnetic field. Sheikholeslami et. al. [29] examined the effect of Lorentz forces on forced-convection nanofluid flow over a stretched surface.

As far as author's knowledge is concerned, so far no work has been done on "Hybrid nanofluid flow over a stretching sheet" and the present work is the one to initiate the influence of Hybrid nanofluid

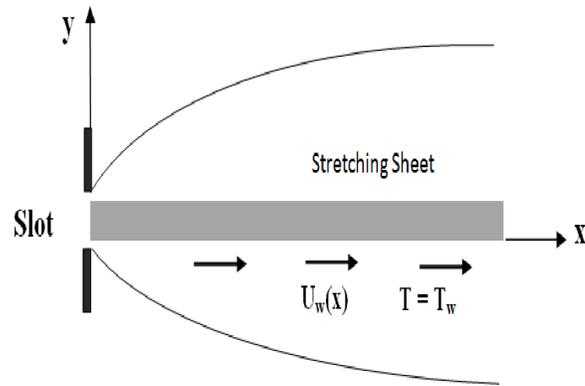


FIGURE 1. Schematic diagram of the problem

over a stretching sheet. This new type of study will attract more researchers due to its enormous industrial applications which motivated us to analyse the present work.

## 2. FORMULATION OF THE PROBLEM

Steady, two dimensional, nonlinear, laminar boundary layer flow of an incompressible, viscous and Hybrid Nanofluid flow past a stretching sheet is considered. The  $x$ -axis is chosen in the direction of the sheet motion and the  $y$ -axis is perpendicular to it. The wall is impermeable (with  $v_w = 0$ ) as shown in Fig. [1]. The viscous and Joule dissipation are considered to be negligible. Under the above assumptions, the steady boundary layer equations of the Hybrid nanofluid flow problem are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{hnf} \frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

with the boundary conditions

$$\begin{cases} u(x, 0) = U_w(x) = cx, v(x, 0) = 0, T(x, 0) = T_w, \\ u(x, \infty) = 0, T(x, \infty) = T_\infty. \end{cases} \quad (4)$$

## 2.1 MODEL AND THERMO-PHYSICAL PROPERTIES

A special form of thermo physical properties are introduced in the present study to analyze the boundary layer equations for Hybrid Nanofluid. Hybrid Nanofluid is considered by taking the mixture of *Cu* nanoparticles into 0.1vol. of  $Al_2O_3/water$  to form the required Hybrid nanofluid.

In this model, initially the nanoparticle of  $Al_2O_3$  ( $\phi_1$ ) is added to the base fluid with 0.1 vol. solid volume fraction (*i.e.*,  $\phi_1 = 0.1$  which is fixed throughout the problem hereafter) and consequently *Cu* ( $\phi_2$ ) is added with various solid volume fractions to form the Hybrid nanofluid namely  $Cu - Al_2O_3/Water$ . To make it clear, the final form of the effective thermo-physical properties of nanofluid and Hybrid nanofluid are given in Table 1.

Properties	Nanofluid	Hybrid Nanofluid
Density	$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$	$\rho_{hnf} = \{(1 - \phi_2)[(1 - \phi_1)\rho_f + \phi_1\rho_{s1}]\} + \phi_2\rho_{s2}$
Heat capacity	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$	$(\rho C_p)_{hnf} = \{(1 - \phi_2)[(1 - \phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_{s1}]\} + \phi_2(\rho C_p)_{s2}$
Viscosity	$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$	$\mu_{hnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}}$
Thermal Conductivity	$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f - (n-1)\phi(k_f - k_s)}{k_s + (n-1)k_f + \phi(k_f - k_s)}$	$\frac{k_{hnf}}{k_{bf}} = \frac{k_{s2} + (n-1)k_{bf} - (n-1)\phi_2(k_{bf} - k_{s2})}{k_{s2} + (n-1)k_{bf} + \phi_2(k_{bf} - k_{s2})}$ where $\frac{k_{bf}}{k_f} = \frac{k_{s1} + (n-1)k_f - (n-1)\phi_1(k_f - k_{s1})}{k_{s1} + (n-1)k_f + \phi_1(k_f - k_{s1})}$

TABLE 1. Thermo Physical Properties

Where  $n = 3$  is for spherical nano particles. The basic thermo-physical properties of nanofluid are taken from the standard literatures. The thermo physical properties of fluid at  $25^\circ C$  and particles are provided in Table 2.

Properties	Water ( <i>f</i> )	$Al_2O_3$	<b>Cu</b>
$\rho$ ( $kg/m^3$ )	997.0	3970	8933
$C_p$ ( $J/kgK$ )	4180	765	385
$k$ ( $W/mK$ )	0.6071	40	400

TABLE 2. Thermophysical properties of fluid and nanoparticles

From Fig. [2] it is noted that our thermophysical property is compared with the experimental results of Suresh et al. [3]. In particular, the proposed model of thermal conductivity of Hybrid nanofluid ( $Al_2O_3 - Cu/Water$  with 90:10 ratio concentrations) is plotted for various volume fractions such as 0.1%, 0.33%, 0.75%,

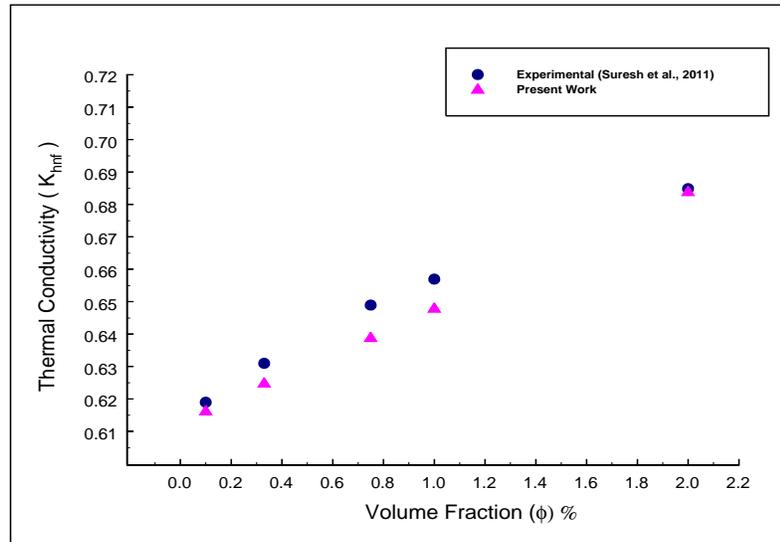


FIGURE 2. Thermal Conductivity Experimental vs Numerical model

1% and 2% gives an excellent correlation with the existing experimental data and it sets an advancement to handle the physical problem using these new thermophysical properties.

### 3. SIMILARITY TRANSFORMATIONS

The stream function and similarity variables are introduced to solve the equations (1)-(3) subject to (4). They are given by,

$$u = c x f'(\eta), \quad (5)$$

$$v = -\sqrt{c \nu_f} f(\eta), \quad (6)$$

$$\eta = \sqrt{\frac{c}{\nu_f}} y. \quad (7)$$

Considering the similarity transformation  $\theta$  as:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}. \quad (8)$$

Equations (5)-(8) are proposed based on the standard practice for similarity transformation of partial differential equations.

Equation of continuity (1) is automatically satisfied. Using the similarity transformations (5)-(8), the nonlinear partial differential

equations (2) and (3) with boundary conditions (4) are reduced to the following nonlinear ordinary differential equations:

$$f''' = (1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} \left\{ (1 - \phi_2) \left[ (1 - \phi_1) + \phi_1 \left( \frac{\rho_{s1}}{\rho_f} \right) \right] + \phi_2 \left( \frac{\rho_{s2}}{\rho_f} \right) \right\} \cdot [(f')^2 - f f''] , \quad (9)$$

$$\theta'' = - \left( \frac{k_f}{k_{hnf}} \right) \left\{ (1 - \phi_2) \left[ (1 - \phi_1) + \phi_1 \frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right] + \phi_2 \frac{(\rho C_p)_{s2}}{(\rho C_p)_f} \right\} Pr f \theta' \quad (10)$$

with the boundary conditions

$$\begin{aligned} f(0) &= 0, \quad f'(0) = 1, \quad \theta(0) = 1, \\ f'(\infty) &= 0, \quad \theta(\infty) = 0 \end{aligned} \quad (11)$$

where the prime indicates differentiation with respect to  $\eta$ . The Physical quantities of interest are Skin friction coefficient and reduced Nusselt number which are given by

$$C_f = \frac{\mu_{hnf} \left( \frac{\partial u}{\partial y} \right)_{y=0}}{\rho_f u_w^2}, \quad Nu_x = - \frac{x k_{hnf} \left( \frac{\partial T}{\partial y} \right)_{y=0}}{k_f (T_w - T_\infty)}.$$

Hence

$$Cf_x Re_x^{\frac{1}{2}} = \frac{f''(0)}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5}}, \quad Nu_x Re_x^{-\frac{1}{2}} = - \frac{k_{hnf}}{k_f} \theta'(0). \quad (12)$$

#### 4. NUMERICAL SOLUTION

Numerical solution of the problem is obtained as equations (9) and (10) are highly nonlinear and along with equation (11), they form a nonlinear boundary value problem which is difficult to solve. An efficient Nachtsheim-Swigert shooting iteration technique for the satisfaction of the asymptotic boundary conditions along with Runge-Kutta-Fehlberg Integration method has been employed to solve. The appropriate initial guess for the unknowns  $f''(0)$  and  $\theta'(0)$  are taken in order to satisfy the asymptotic boundary conditions. The above shooting process is repeated until the converged results are

Pr	Khan and Pop[16]	Wang[30]	Gorla and Sidawi[31]	Present study when $\phi_1, \phi_2 = 0$
2.0	0.9113	0.9114	0.9114	0.91135
6.13	-	-	-	1.75968
7.0	1.8954	1.8954	1.8954	1.89540
20.0	3.3539	3.3539	3.3539	3.35390

TABLE 3. Comparison of numerical values of  $-\theta'(0)$ 

obtained within a tolerance limit of  $10^{-5}$  level. The numerical solutions are represented graphically and salient features of the problem are analyzed.

## 5. RESULTS AND DISCUSSION

In order to get the clear insight of the physical problem, the paramount features of the flow and heat transfer characteristics are obtained using Hybrid nanofluid ( $Cu - Al_2O_3/Water$ ) and nanofluid ( $Cu/Water$ ) over a stretching sheet. The numerical solutions are obtained for several values of physical parameters.

Validation of numerical code is obtained by comparing the numerical results of present study with the comparison results of Khan and Pop [16] which are portrayed in Table [3]. In the absence of solid volume fraction ( $\phi_1 = 0$  and  $\phi_2 = 0$ ) and for different values of Prandtl number, it is noted that our results are found to be in an excellent agreement with them.

Numerical solution are obtained especially for Solid volume fraction ( $0.005 \leq \phi_2 \leq 0.06$ ), Prandtl number,  $Pr = 6.135$  which is calculated and fixed throughout this problem respectively. The numerical results are presented graphically through Fig. [3] and Fig. [4].

Fig. [3] it is observed from this study of Hybrid nanofluids that for increasing solid volume fraction, the velocity  $f'(\eta)$  gets decreased for both  $Cu - Al_2O_3/Water$  and  $Cu/Water$ . The flow motion gets decelerated due to adding up of more nanoparticles to the flow. Simultaneously the momentum boundary layer also get thinner.

Fig. [4] depicts the trend of temperature distribution for solid volume fraction of  $Cu - Al_2O_3/Water$  and  $Cu/Water$ . Physically, the nanoparticles dissipate energy in the form of heat. Simultaneously adding up of more nanoparticles may exert more energy which enhances the temperature for both Hybrid nanofluid and nanofluid.

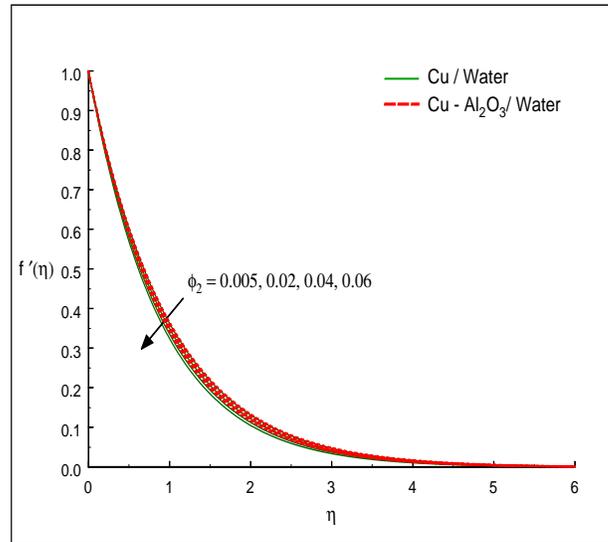


FIGURE 3. Velocity distribution for various values of  $\phi_2$

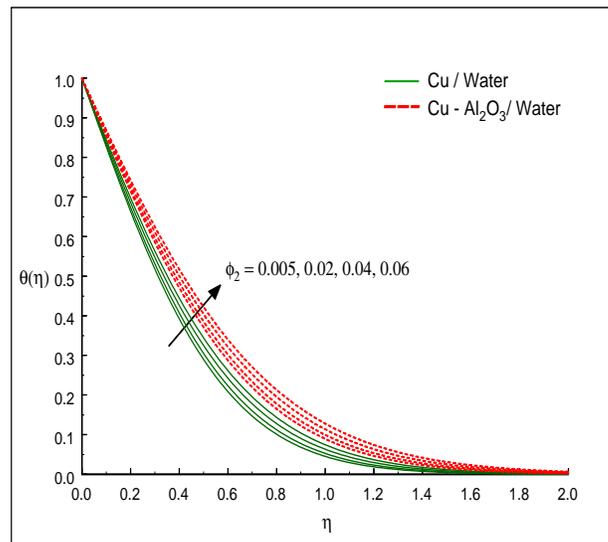


FIGURE 4. Temperature distribution for various values of  $\phi_2$

$\phi_2$	Skin Friction Coefficient		Nusselt number	
	$Cu$ /Water	$Cu - Al_2O_3$ /Water	$Cu$ /Water	$Cu - Al_2O_3$ /Water
0.005	-1.026272	-1.327310	1.768118	1.961686
0.02	-1.104327	-1.409683	1.791406	1.989226
0.04	-1.208432	-1.520894	1.822984	2.026368
0.06	-1.313402	-1.634279	1.855179	2.064075

TABLE 4. Skin friction Coefficient and Nusselt Number for  $Cu - water$  and  $Cu - Al_2O_3/Water$

Hence the thermal boundary layer thickens for increasing solid volume fraction.

Since increase in solid volume fraction decelerates the flow velocity, the skin friction Coefficient is obviously decreased for both Nanofluid and Hybrid nanofluid which is depicted in Table [4]. It is explored that, efficient heat transfer rate can exert with increasing Hybrid nanofluid due to the higher thermal conductivity. From Table [4], it is important to note that the heat transfer rate of Hybrid nanofluid enhances upto 17.3% than pure water for 0.06 vol. of Solid volume fraction ( $\phi_2$ ). In particular, heat transfer rate for Hybrid nanofluid is enhanced upto 11.2% than nanofluid for 0.06 vol. of Solid volume fraction ( $\phi_2$ ). In view of these, it is obvious that the heat transfer rate for Hybrid nanofluid ( $Cu - Al_2O_3/Water$ ) is higher than that of nanofluid ( $Cu/Water$ ). It could achieve more and desired heat transfer rate by combining appropriate proportion of the nano composite.

## 6. CONCLUSION

The present study has addressed the Hybrid nanofluid flow over a stretching sheet. A parametric study has been made in order to get clear physical insight of the problem. A new type of fluid namely Hybrid nanofluid ( $Cu - Al_2O_3/Water$ ) and Nanofluid ( $Cu/Water$ ) are used for the investigation. The main advantage of using this next generation Hybrid nanofluid is to improve its thermal characteristics. By choosing an appropriate combination of the nanoparticles one can manipulate to enhance the positive features of each other and cover their disadvantages of using them separately.

In practical situations, the stretching sheet is considered as an important problem in many industrial application like metal extrusion, hot rolling, glass blowing, glass-fibre production, wire drawing, stretching fins and so on. These manufacturing environments posses enormous heat which may influence the products. Instead of using nanofluid to cool such heat environments, those cooling process can be advanced by adapting Hybrid nanofluids which make cooling process more efficient as evidenced through our present study that the heat transfer rate of Hybrid nanofluid ( $Cu - Al_2O_3/Water$ ) is higher than that of Nanofluid ( $Cu/Water$ ). By using different and appropriate nanoparticle proportions in Hybrid nanofluid, the desired heat transfer rate can be achieved at low cost. This numerical investigation using present proposed model for thermo physical property sets a new scope for researchers in the field of heat transfer analysis.

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#### NOMENCLATURE

$T$	Fluid temperature
$T_w$	Wall temperature
$T_\infty$	Temperature far away from the sheet
$u$	Velocity component in the $x$ directions
$v$	Velocity component in the $y$ directions
$\eta$	Plate surface
$h$	Heat transfer coefficient
$\phi_{s_1}$	Nanoparticle volume fraction of alumina
$\phi_{s_2}$	Nanoparticle volume fraction of copper
$(C_p)_f$	Specific heat capacity of the fluid
$(C_p)_{nf}$	Specific heat capacity of the nanofluid
$(C_p)_{hnf}$	Specific heat capacity of the hybrid nanofluid
$(C_p)_{s_1}, (C_p)_{s_2}$	Specific heat capacity of the solid nanoparticles
$k_f$	Thermal conductivity of the fluid
$k_{nf}$	Thermal conductivity of the nanofluid
$k_{hnf}$	Thermal conductivity of the hybrid nanofluid
$k_{s_1}, k_{s_2}$	Thermal conductivity of the solid nanoparticles
$\alpha_{hnf}$	Thermal diffusivity of Hybrid nanofluid
$\nu_f$	Kinematic viscosity of fluid
$\nu_{nf}$	Kinematic viscosity of nanofluid

$\nu_{hnf}$	Kinematic viscosity of hybrid nanofluid
$\nu_{s_1}, \nu_{s_2}$	Kinematic viscosity of the solid nanoparticles
$\rho_f$	Density of the fluid
$\rho_{nf}$	Density of the nanofluid
$\rho_{hnf}$	Density of the hybrid nanofluid
$\rho_{s_1}, \rho_{s_2}$	Density of the solid nanoparticles
$Pr$	Prandtl number, $Pr = \frac{\mu_f C_{p_f}}{k_f}$
$Re_x$	Local Reynolds number, $Re_x = \frac{U_w x}{\nu_f}$ .

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