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ON NUMERICAL COMPUTATIONAL SOLUTION OF SEVENTH ORDER BOUNDARY VALUE PROBLEMS

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ABSTRACT: In this paper, we present and applied Exponentially Fitted Collocation Approximate Method for the numerical solution of seventh order boundary value problems. The approximate solution of the problem is computed using Maple 18 software after the problem was slightly perturbed and collocated. Three examples have been considered to illustrate the efficiency and implementation of the method and the results are compared with the exact solution and some existing work in literature.

Keywords and phrases: Exponentially Fitted Collocation Approximate Method, Seventh Order Boundary Value Problems, Maple 18 Software and Analytical solution

2019 Mathematical Subject Classification: A80

1. INTRODUCTION

The boundary value problems of higher order have been discussed due to their mathematical role as well as their potential for applications in hydromagnetic and hydrodynamic stability, modeling of induction motor behaviors and other fields of science and engineering. The application of seventh order BVPs available in engineering sciences. These problems arise in Mathematical modeling of induction motors with two rotor circuits [1]. Generally, this problem is difficult to solve analytically. Many authors have developed numerical techniques for the numerical solution of this form of boundary values problems. For instance, [2] contains theorems which detail the conditions for existence and uniqueness of solutions of higher boundary value problems. [3] applied Haar Wavelet approach for the solution of seventh order ordinary differential equations, moreover, [4] applied Differential transformation method to

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solve seventh-order boundary value problem and [5] proposed an efficient numerical solution for seventh order differential equation by using Septic B-Spline collocation method with non-uniform length proposed and just to mention a few. This paper is to employ an easy, fast and accurate numerical technique to obtain numerical solution of higher order boundary value problems and comparing results with other methods available in literature[6, 7, 8].

We consider general seventh order non-homogenous differential equation with mixed boundary conditions of the form:

$$\frac{d^7y(t)}{dt^7} + a(t)y(t) = g(t) \quad ; \qquad t \in [a,b] \tag{1}$$

subject to the boundary conditions;

$$\begin{aligned} y(t_0) &= \beta_1 \\ y'(t_0) &= \beta_2 \\ y''(t_0) &= \beta_3 \\ y'''(t_0) &= \beta_4 \\ y'(t_1) &= \beta_5 \\ y''(t_1) &= \beta_6 \\ y'''(t_1) &= \beta_7 \end{aligned} \tag{2}$$

where $a(t)$ and $g(t)$ are smooth functions and β_1, β_2, β_3, β_4, β_5, β_6, β_7 are constants.

In this paper, we obtain numerical solution of equation (1) using exponentially fitted collocation approximate method. The whole idea of the method is to use power series as a basis function and its derivative substituted into a slightly perturbed equation which eventually collocate. However, use of Chebyshev Polynomials during the perturbation of the equations, which established in literature that Chebyshev Polynomials of the first kind possess properties that minimize errors within the region of consideration [9].The advantage of one of these properties called minimax property is in this study harnessed in such a way that the derived formula is automated in symbolic algebraic program using MAPLE 18 software.

2. CONSTRUCTION OF COMPUTATIONAL TECHNIQUE

We formulate and employ exponentially fitted collocation approximate method to obtain numerical solution for seven order boundary value problem (1).
Suppose we have power series of the form;

$$y_N(t) = \sum_{k=0}^{N} s_k t^k \tag{3}$$

And Exponentially fitted approximate solution proposed [10].

$$y_N(t) \approx \sum_{k=0}^{N} s_k t^k + \tau_n e^t \tag{4}$$

where t represents the dependent variables in the problem, n represents the order of the dependent variable and $s_k y_N(t)$ $(k \geq 0)$ are the unknown constants to be determined, N is the length of computation and degree of chebyshev polynomials defined [11].
Obtain seventh derivative of equation(3), we have;

$$y_N^7(t) = \sum_{k=7}^{N} k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)s_k t^{k-7} \tag{5}$$

substitute equations (3) and (5) into equation (1);

$$[\sum_{k=7}^{N} k(k-1)(k-2)(k-4)(k-5)(k-6)s_k t^{k-7}] + a(t)[\sum_{k=0}^{N} s_k t^k] = g(t)$$

$$\begin{aligned} &5040s_7 + 40320ts_8 + 181440t^2 s_9 + \ldots \\ &\quad + N(N-1)(N-2)(N-3)(N-4)(N-5)(N-6)t^{N-7}s_N \\ &\quad + a(t)[s_0 + ts_1 + t^2 s_2 + t^3 s_3 + t^4 s_4 + t^5 s_5 + t^6 s_6 + \cdots + s_N t^N] \\ &\quad = g(t) \end{aligned} \tag{6}$$

Collect the likes terms in equation (6), we have;

$$\begin{aligned} &a(t)s_0 + a(t)ts_1 + a(t)t^2 s_2 + a(t)t^3 s_3 + a(t)t^4 s_4 + a(t)t^5 s_5 + \\ &\quad a(t)t^6 s_6 + [5040 + a(t)t^7]s^7 + [40320t + a(t)t^8]s_8 + [181440t^2 + \\ &\quad a(t)t^9]s_9 + \\ &\qquad \vdots \\ &\quad + N(N-1)(N-2)(N-3)(N-4)(N-5)(N-6)t^{N-7}s_N \\ &\quad + a(t)t^N = g(t) \end{aligned} \tag{7}$$

We perturbed and collocate equation 7, we have

$$\begin{aligned}
&a(t)s_0 + a(t)ts_1 + a(t)t^2 s_2 + a(t)t^3 s_3 + a(t_q)t_q^4 s_4 + a(t_q)t_q^5 s_5 + \\
&\quad a(t_q)t_q^6 s_6 + [5040 + a(t_q)t_q^7]s^7 + [40320t_q + a(t_q)t_q^8]s_8 + [181440t_q^2 + \\
&\quad a(t_q)t_q^9]s_9 + \\
&\qquad\quad \vdots \\
&\quad + [N(N-1)(N-2)(N-3)(N-4)(N-5)(N-6)t_q^{N-7} \\
&\quad + a(t_q)t_q^N]s_N = g(t_q) + H(t_q) \qquad (8)
\end{aligned}$$

where $t_q = a + \frac{(b-a)q}{N+2}$; $\qquad q = 1, 2, \ldots, N+1$

$$H(t_q) = \tau_1 T_N(t_q) + \tau_2 T_{N-1}(t_q) + \tau_3 T_{N-2}(t_q) + \tau_4 T_{N-3}(t_q) + \tau_5 T_{N-4}(t_q)$$

$$+\tau_6 T_{N-5}(t_q) + \tau_7 T_{N-6}(t_q)$$

Here τ_1,τ_2,τ_3,τ_4,τ_5,τ_6,τ_7 are free tau parameters to be determined and $T_N(t_q)$, $T_{N-1}(t_q)$, $T_{N-2}(t_q)$, $T_{N-3}(t_q)$, $T_{N-4}(t_q)$, $T_{N-5}(t_q)$, $T_{N-6}(t_q)$ are the Chebyshev Polynomials of degrees $N, N-1, N-2, N-3, \ldots, N-6$ define in [11].
Equation (8) is called perturbed collocated seventh order equation. Coupled with boundary conditions (2) and using approximate solution (5), we have

$$\begin{aligned}
y(t_0) &= s_0 + \tau_7 e^0 = \beta_1 \\
y'(t_0) &= s_1 + \tau_7 e^0 = \beta_2 \\
y''(t_0) &= 2s_2 + \tau_7 e^0 = \beta_3 \\
y'''(t_0) &= 6s_3 + \tau_7 e^0 = \beta_4 \qquad (9) \\
y'(t_1) &= s_0 + s_1 + s_2 + s_3 + s_4 + \cdots + s_N + \tau_7 e^1 = \beta_5 \\
y''(t_1) &= s_1 + 2s_2 + 3s_3 + 4s_4 + \cdots + Ns_N + \tau_7 e^1 = \beta_6 \\
y'''(t_1) &= 2s_2 + 6s_3 + 12s_4 + \cdots + N(N-1)s_N + \tau_7 e^1 = \beta_7
\end{aligned}$$

where t_0 and $t_1 = 1$ Altogether, we obtained $(N+8)$ algebraic linear equations in $(N+8)$ unknown constants. Thus, we put the $(N+8)$ algebraic equations in Matrix form as:

$$YS = G \qquad (10)$$

Here

$$
Y = \begin{bmatrix}
y_{11} & y_{12} & y_{13} & \cdots & \cdots & \cdots & \cdots & y_{1N} & T_N(t-1) & T_{N-1}(t_1) & \cdots & \cdots & \cdots & \cdots & T_{N-6}(t_1) \\
y_{21} & y_{22} & y_{23} & \cdots & \cdots & \cdots & \cdots & y_{2N} & T_N(t-1) & T_{N-1}(t_1) & \cdots & \cdots & \cdots & \cdots & T_{N-6}(t_2) \\
y_{31} & y_{32} & y_{33} & \cdots & \cdots & \cdots & \cdots & y_{3N} & T_N(t-1) & T_{N-1}(t_1) & \cdots & \cdots & \cdots & \cdots & T_{N-6}(t_3) \\
y_{41} & y_{42} & y_{43} & \cdots & \cdots & \cdots & \cdots & y_{2N} & T_N(t-1) & T_{N-1}(t_1) & \cdots & \cdots & \cdots & \cdots & T_{N-6}(t_4) \\
y_{51} & y_{52} & y_{53} & \cdots & \cdots & \cdots & \cdots & y_{2N} & T_N(t-1) & T_{N-1}(t_1) & \cdots & \cdots & \cdots & \cdots & T_{N-6}(t_5) \\
y_{61} & y_{62} & y_{63} & \cdots & \cdots & \cdots & \cdots & y_{2N} & T_N(t-1) & T_{N-1}(t_1) & \cdots & \cdots & \cdots & \cdots & T_{N-6}(t_6) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
y_{1N} & y_{2N} & y_{3N} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \ell^1 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \cdots & \cdots & \cdots & \cdots & N & \ell^1 \\
0 & 0 & 2 & 6 & 12 & 20 & 30 & 42 & 56 & \cdots & \cdots & \cdots & \cdots & N(N-1) & \ell^1
\end{bmatrix}
$$

$$S = [s_0, s_1, s_2, \ldots, s_N, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7]^T$$

$$G = [g(t_0), g(t_1), g(t_2), \ldots, g(t_N), \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7]^T$$

The unknown constants $s_0, s_1, s_2, s_3, s_4, \ldots, s_N, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6,$ and τ_7 are obtain using MAPLE 18 software which eventually substitute into the approximate solution (4).

3. NUMERICAL APPLICATION

In this section, we consider three numerical test problems to demonstrate the efficiency of the propose method. Their numerical solutions and absolute error were presented. The numerical results for each problem are presented in tabular forms and compared with the exact solutions and existing work in the literature [6, 7, 8].

Example 1

Consider non-homogeneous seventh order boundary value problem with constant coefficient[6].

$$\frac{d^7 y(t)}{dt^7} - ty(t) = e^t(t^2 - 2t - 6); \qquad 0 \le t \le 1 \tag{11}$$

subject to boundary conditions;

$$\begin{aligned}
y(0) &= 1, & y(1) &= 0 \\
y'(0) &= 0, & y'(1) &= -e \\
y''(0) &= -1, & y''(1) &= -2e \\
y'''(0) &= -2
\end{aligned} \tag{12}$$

The exact solution is

$$y(t) = (1-t)e^t \tag{13}$$

EFCAM Technique

Compare equation (11) with equation (8), we have the following; $a(t) = t$, $g(t) = e^t(t^2 - 2t - 6)$ and taking computational length $N = 12$. We obtain;

$$
\begin{aligned}
&-t_q s_0 - t_q^2 s_1 - t_q^3 s_2 - t_q^4 s_3 - t_q^5 s_4 - t_q^6 s_5 - t_q^7 s_6 + \\
&\quad (-t_q^8 + 5040)s_7 + (-t_q^9 + 40320t_q)s_8 + (-t_q^{10} + 181440t_q^2)s_9 \\
&\quad + (-t_q^{11} + 604800t_q^3)s_{10} + (-t_q^{12} + 1663200t_q^4)s_{11} + (-t_q^{13} \\
&\quad + 3991680t_q^5)s_{12} - (8388608t_q^{12} - 50331648t_q^{11} + 132120576t_q^{10} \\
&\quad - 199229440t_q^9 + 190513152t_q^8 - 120324096t_q^7 + 50692096t_q^6 \\
&\quad - 14057472t_q^5 + 2471040t_q^4 - 25626t_q^3 + 13728t - q^2 - 288t_q + 1)\tau_1 \\
&\quad - (2097152t - q^{11} - 11534336t - q^{10} + 27394048t_q^9 - 36765696t_q^8 \\
&\quad + 30638080t_q^7 - 16400384t_q^6 + 5637632t_q^5 + 1208064t_q^4 + 151008t_q^3 \\
&\quad - 9680t_q^2 + 242t_q - 1)\tau_2 - (52488t_q^1 0 - 2621440t_q^9 + 5570560t_q^8 \\
&\quad - 6553600t_q^7 + 4659200t_q^6 - 2050048t_q^5 + 549120t_q^4 - 84480t_q^3 \\
&\quad + 6600t_q^2 - 200t_q + 1)\tau_3 - (131072t_q^9 - 589824t_q^8 + 1105920t_q^7 \\
&\quad - 1118208t_q^6 + 658944t_q^5 - 228096t_q^4 + 44352t_q^3 - 4320t_q^2 \\
&\quad + 162t_q - 1)\tau_4 - (32768t_q^8 - 131072t_q^7 + 212992t_q^6 - 180224t_q^5 \\
&\quad - 84480t_q^4 - 21504t_q^3 + 2688t_q^2 - 128t_q + 1)\tau_5 \\
&\quad - (8172t_q^7 - 28672t_q^6 + 39424t_q^5 - 26880t - q^4 \\
&\quad + 9408t_q^3 - 1568t_q^2 + 98t_q - 1)\tau_6 \\
&\quad - (2048t_q^6 - 6144t_q^5 + 6912t_q^4 \\
&\quad - 3584t_q^3 + 640t_q^2 - 72t_q + 1)\tau_7 \\
&\quad = e^{t_q}(t_q^2 - 2t_q - 6) \qquad (14)
\end{aligned}
$$

Collocate equation (14) as follows; $t_q = a + \frac{(b-a)q}{N+2}$; $\quad q = 1, 2, \ldots, N+1$, where $a = 0$, $b = 1$, $N = 12$

$$
\begin{gathered}
t_1 = \tfrac{1}{14},\ t_2 = \tfrac{2}{14},\ t_3 = \tfrac{3}{14},\ t_4 = \tfrac{4}{14},\ t_5 = \tfrac{5}{14}, \\
t_6 = \tfrac{6}{14},\ t_7 = \tfrac{7}{14},\ t_8 = \tfrac{8}{14},\ t_9 = \tfrac{9}{14},\ t_{10} = \tfrac{10}{14}, \\
t_{11} = \tfrac{11}{14},\ t_{12} = \tfrac{12}{14},\ t_{13} = \tfrac{13}{14},
\end{gathered}
$$

Couple with boundary conditions (12) and equation (10), thus, we use MAPLE 18 software to obtain unknown constants of equations (14), we have

$$
\begin{gathered}
S_0 = 0.9999996125\ ,\ S_1 = -0.00001551955341, \\
S_2 = -0.5000077598,
\end{gathered}
$$

$$S_3 = -0.33333359199\ ,\ S_4 = -0.12500006494\ ,$$
$$S_5 = -0.0333333345403,$$
$$S_6 = -0.006944474212,\ S_7 = -0.001190474683,$$
$$S_8 = -0.000173633177,$$
$$S_9 = -0.00002198698021\ ,\ S_{10} = -0.00000255519516,$$
$$S_{11} = -0.00000019988483,$$
$$S_{12} = -0.00000004008267738,\ \tau_1 = 6.09759720510^{-8},$$
$$\tau_2 = 1.68735200510^{-9},$$
$$\tau_3 = 2.86328811910^{-7},\ \tau_4 = -2.95003739310^{-9},$$
$$\tau_5 = 7.62366093610^{-7},\ \tau_6 = 5.34546785410^{-7},$$
$$\tau_7 = 0.00001551955341$$

Substitute the above constants into approximate solution (4), we have;

$$\begin{aligned} y_{12}(t) \approx\ & 0.9999996125 - 0.00001551955341t - 0.5000077598t^2 \\ & - 0.33333359199t^3 - 0.12500006494t^4 - 0.0333333345403t^5 \\ & - 0.006944474212t^6 - 0.001190474683t^7 - 0.000173633177t^8 \\ & - 0.00002198698021t^9 - 0.00000255519516t^{10} \\ & - 0.00000019988483t^{11} - 0.00000004008267738t^{12} \\ & + 0.00001551955341e^t \end{aligned} \tag{15}$$

Table 1: Numerical result for seventh order BVP Example 1

t	**Exact Solution**	**Absolute Error EFCAM**	**Absolute Error** [8]
0.0	1.0000000000	0.00	0.00
0.1	0.9946538262	1.10 E —19	9.00 E —10
0.2	0.9771222064	2.10 E —10	1.41 E —19
0.3	0.9449011656	4.10 E —10	4.87 E —09
0.4	0.8950948188	4.10 E —10	3.94 E —09
0.5	0.8243606355	3.10 E —10	9.28 E —10
0.6	0.7288475200	0.00	5.71 E —09
0.7	0.6041258121	0.00	6.24 E—08
0.8	0.4451081857	1.10 E —10	1.29 E —08
0.9	0.2459603111	0.00	1.05 E —08
1.0	0.0000000000	0.00	0.00

Example 2

Consider non-homogeneous seventh order boundary value problem with constant coefficient [7].

$$\frac{d^7y(t)}{dt^7} + y(t) = -e^t(35 + 12t + t^2); \qquad 0 \leq t \leq 1 \tag{16}$$

subject to boundary conditions;

$$\begin{aligned} y(0) &= 1, \qquad & y(1) &= 0 \\ y'(0) &= 1, \qquad & y'(1) &= -e \\ y''(0) &= 0, \qquad & y''(1) &= -4e \\ y'''(0) &= -3 & & \end{aligned} \tag{17}$$

The exact solution is

$$y(t) = t(1-t)e^t \tag{18}$$

EFCAM Technique

Compare equation (16) with equation (8), we have the following; $a(t) = 1$, $g(t) = -e^t(35+12t+t^2)$ and taking computational length $N = 14$. We obtain;

$$\begin{aligned} & s_0 + t_q s_1 + t_q^2 s_2 + t_q^3 s_3 + t_q^4 s_4 + t_q^5 s_5 + t_q^6 s_6 + \\ & \quad (t_q^7 + 5040)s_7 + (t_q^8 + 40320t_q)s_8 + (t_q^9 + 181440t_q^2)s_9 + \\ & \quad + (t_q^{10} + 604800t_q^3)s_{10} + (t_q^{11} + 1663200t_q^4)s_{11} + (t_q^{12} + 3991680t_q^5)s_{12} \\ & \quad + (t_q^{13} + 8648640t_q^6)s_{13} + (t_q^{14} + 17297280t_q^7)s_{14} \\ & \quad - (134217728t_q^{14} - 939524096t_q^{13} + 2936012800t_q^{12} - 5402263552t_q^{11} + \\ & \quad 6499598336t_q^{10} - 5369233408t_q^9 + 3111714816t_q^8 - 12710087680t_q^7 \\ & \quad + 361181184t_q^6 - 69701632t_q^5 + 8712704t_q^4 \\ & \quad - 65228t_q^3 + 25480t_q^2 - 392t_q + 1)\tau_1 \\ & \quad - (33554432t_q^{13} - 218103808t_q^{12} + 627048448t_q^{11} - 1049624576t_q^{10} + \\ & \quad 113311744t_q^9 - 825556992t_q^8 + 412778496t_q^7 - 141213696t_q^6 \\ & \quad + 32361472t_q^5 - 4759040t_q^4 + 416416t_q^3 \\ & \quad - (2097152t_q^{11} - 11534336t_q^{10} + 27394048t_q^9 - 36765696t_q^8 + \\ & \quad 30638080t_q^7 - 16400384t_q^6 + 5637632t_q^5 - 1208064t_q^4 \\ & \quad + 151008t_q^3 - 9680t_q^2 + 242t_q - 1)\tau_4 \\ & \quad - (52488t_q^{10} - 2621440t_q^9 + 5570560t_q^8 - 6553600t_q^7 + 4659200t_q^6 \\ & \quad - 2050048t_q^5 + 549120t_q^4 - 84480t_q^3 + 6600t_q^2 - 200t_q + 1)\tau_5 \\ & \quad - (131072t_q^9 - 589824t_q^8 + 1105920t_q^7 - 1118208t_q^6 + \\ & \quad 658944t_q^5 - 228096t_q^4 + 44352t_q^3 - 4320t_q^2 + 162t_q - 1)\tau_6 \\ & \quad - (32768t_q^8 - 131072t_q^7 + 212992t_q^6 - 180224t_q^5 \\ & \quad + 84480t_q^4 - 21504t_q^3 + 2688t_q^2 - 128t_q + 1)\tau_7 \\ & \quad = e^{t_q}(35 + 12t_q + t_q^2) \end{aligned} \tag{19}$$

Collocate equation (19) as follows; $t_q = a+\frac{(b-a)q}{N+2}; \quad q = 1, 2, \ldots, N+1$, where $a = 0$, $b = 1$, $N = 14$

$$t_1 = \tfrac{1}{16},\ t_2 = \tfrac{2}{16},\ t_3 = \tfrac{3}{16},\ t_4 = \tfrac{4}{16},\ t_5 = \tfrac{5}{16},$$
$$t_6 = \tfrac{6}{16},\ t_7 = \tfrac{7}{16},\ t_8 = \tfrac{8}{16},\ t_9 = \tfrac{9}{16},\ t_{10} = \tfrac{10}{16},$$
$$t_{11} = \tfrac{11}{16},\ t_{12} = \tfrac{12}{16},\ t_{13} = \tfrac{13}{16},\ t_{14} = \tfrac{14}{16}\ ,\ t_{15} = \tfrac{15}{16},$$

Couple with boundary conditions (17) and equation (10), thus, we use MAPLE 18 software to obtain unknown constants of equations (19), we have;

$$S_0 = 0.00000228395394\ ,\ S_1 = 1.000002284,$$
$$S_2 = 0.000001141976973,$$
$$S_3 = -0.4999996193\ ,\ S_4 = -0.3333332368\ ,$$
$$S_5 = -0.1249999847,,$$
$$S_6 = -0.03333332822,\ S_7 = 0.006944433316,$$
$$S_8 = -0.001190520128,$$
$$S_9 = -0.0001735127129\ ,\ S_{10} = -0.000022185107346,$$
$$S_{11} = -0.0000023531718333,\ S_{12} = -0.000000323875814,$$
$$S_{13} = 0.000000002083454712,\ S_{14} = -0.000000006118446529,$$
$$\tau_1 = -0.00000008483126116,\ \tau_2 = 0.0000002398392547,$$
$$\tau_3 = 0.0000004268330286,\ \tau_4 = 0.000001136805924,$$
$$\tau_5 = -0.000001236660853,\ \tau_6 = 0.000002957827944,$$
$$\tau_7 = -0.000002283953946$$

Substitute the above constants into approximate solution 4, we have;

$$\begin{aligned} y_{14}(t) \approx{} & 0.00000228395394 + 1.000002284t + 0.000001141976973t^2 \\ & - 0.4999996193t^3 - 0.3333332368t^4 - 0.1249999847t^5 \\ & - 0.03333332822t^6 + 0.006944433316t^7 - 0.001190520128t^8 \\ & - 0.0001735127129t^9 - 0.00002218510734t^{10} - 0.0000023531718t^{11} \\ & - 0.000000323875814t^{12} + 0.000000002083454712t^{13} \\ & - 0.000000006118446529t^{14} - 0.000002283953946e^t \end{aligned} \tag{20}$$

Table 2: Numerical result for seventh order BVP Example 2

t	Exact Solution	Absolute Error EFCAM	Absolute Error [7]
0.0	0.0000000000	0.00	0.00
0.1	0.0994653826	5.10 E —11	1.40 E —07
0.2	0.1954244413	1.10 E —10	6.40 E —07
0.3	0.2834703497	1.10 E —10	2.90 E —06
0.4	0.3580379275	0.00	4.40 E —06
0.5	0.41218031785	1.10 E —10	6.70 E —06
0.6	0.4373085120	2.10 E —10	6.40 E —06
0.7	0.4228880685	3.10 E —10	3.70 E —06
0.8	0.3560865485	2.10 E —10	3.30 E —07
0.9	0.2213642800	0.00	1.40 E —06
1.0	0.0000000000	2.10 E —10	....

Not available ....

Example 3

Consider non-homogeneous seventh order boundary value problem with constant coefficient [8].

$$\frac{d^7y(t)}{dt^7} - y(t) = -7e^t; \qquad 0 \le t \le 1 \tag{21}$$

subject to boundary conditions;

$$\begin{aligned} y(0) &= 1, & y(1) &= 0 \\ y'(0) &= 0, & y'(1) &= -e \\ y''(0) &= -1, & y''(1) &= -2e \\ y'''(0) &= -2 \end{aligned} \tag{22}$$

The exact solution is

$$y(t) = (1-t)e^t \tag{23}$$

EFCAM Technique

Compare equation (21) with equation (8), we have the following; $a(t) = -1$, $g(t) = -7e^t$ and taking computational length $N = 12$.

We obtain;

$$
\begin{aligned}
&-s_0 - t_q s_1 - t_q^2 s_2 - t_q^3 s_3 - t_q^4 s_4 - t_q^5 s_5 - t_q^6 s_6 + \\
&\quad (-t_q^7 + 5040)s_7 + (-t_q^8 + 40320 t_q)s_8 + (-t_q^9 + 181440 t_q^2)s_9 + \\
&\quad + (-t_q^{10} + 604800 t_q^3)s_{10} + (-t_q^{11} + 1663200 t_q^4)s_{11} + (-t_q^{12} s_{12} \\
&\quad + 3991680 t_q^5) - (8388608 t_q^{12} - 50331648 t_q^{11} + 132120576 t_q^{10} \\
&\quad - 199229440 t_q^9 + 190513152 t_q^8 - 120324096 t_q^7 + 50692096 t_q^6 \\
&\quad - 14057472 t_q^5 + 2471040 t_q^4 - 25626 t_q^3 + 13728 t - q^2 - 288 t_q \\
&\quad + 1)\tau_1 - (2097152 t - q^{11} - 11534336 t - q^{10} + 27394048 t_q^9 \\
&\quad - 36765696 t_q^8 + 30638080 t - q^7 - 16400384 t_q^6 + 5637632 t_q^5 \\
&\quad + 1208064 t_q^4 + 151008 t_q^3 - 9680 t_q^2 + 242 t_q - 1)\tau_2 \\
&\quad - (52488 t_q^1 0 - 2621440 t_q^9 + 5570560 t_q^8 - 6553600 t_q^7 + \\
&\quad 4659200 t_q^6 - 2050048 t_q^5 + 549120 t_q^4 - 84480 t_q^3 + \\
&\quad 6600 t_q^2 - 200 t_q + 1)\tau_3 \\
&\quad - (131072 t_q^9 - 589824 t_q^8 + 1105920 t_q^7 - 1118208 t_q^6 \\
&\quad + 658944 t_q^5 - 228096 t_q^4 + 44352 t_q^3 - 4320 t_q^2 + 162 t_q - 1)\tau_4 \\
&\quad - (32768 t_q^8 - 131072 t_q^7 + 212992 t_q^6 - 180224 t_q^5 \\
&\quad - 84480 t_q^4 - 21504 t_q^3 + 2688 t_q^2 - 128 t_q + 1)\tau_5 \\
&\quad - (8172 t_q^7 - 28672 t_q^6 + 39424 t_q^5 - 26880 t - q^4 \\
&\quad + 9408 t_q^3 - 1568 t_q^2 + 98 t_q - 1)\tau_6 \\
&\quad - (2048 t_q^6 - 6144 t_q^5 + 6912 t_q^4 \\
&\quad - 3584 t_q^3 + 640 t_q^2 - 72 t_q + 1)\tau_7 \\
&\quad = -7e^{t_q}
\end{aligned} \tag{24}
$$

Collocate equation (24) as follows; $t_q = a + \frac{(b-a)q}{N+2}$; $\quad q = 1, 2, \ldots, N+1$, where $a = 0$, $b = 1$, $N = 12$

$$
\begin{gathered}
t_1 = \tfrac{1}{14},\ t_2 = \tfrac{2}{14},\ t_3 = \tfrac{3}{14},\ t_4 = \tfrac{4}{14},\ t_5 = \tfrac{5}{14}, \\
t_6 = \tfrac{6}{14},\ t_7 = \tfrac{7}{14},\ t_8 = \tfrac{8}{14},\ t_9 = \tfrac{9}{14},\ t_{10} = \tfrac{10}{14}, \\
t_{11} = \tfrac{11}{14},\ t_{12} = \tfrac{12}{14},\ t_{13} = \tfrac{13}{14}
\end{gathered}
$$

Couple with boundary conditions (22) and equation (10), thus, we use MAPLE 18 software to obtain unknown constants of equations (23), we have;

$S_0 = 0.9999823812$, $S_1 = -0.00001761881384$, $S_2 = -0.5000088094$, $S_3 = -0.3333362698$, $S_4 = -0.1250007342$, $S_5 = -0.03333347987$, $S_6 = -0.006944468155$, $S_7 = -0.001190479545$,

$S_8 = -0.0001736311935$, $S_9 = -0.00002198834869$,
$S_{10} = -0.000002554945345$, $S_{11} = -0.0000001996845$,
$S_{12} = -0.00000004015877$, $\tau_1 = 1.541463958 \times 10^{-7}$,
$\tau_2 = 3.426494606 \times 10^{-8}$, $\tau_3 = 7.216480768 \times 10^{-7}$,
$\tau_4 = 1.327787170 \times 10^{-7}$, $\tau_5 = 0.000001890892661$,
$\tau_6 = 8.475452817 \times 10^{-7}$, $\tau_7 = 0.00001551955341$
Substitute the above constants into approximate solution(4), we have;

$$\begin{aligned} y_{12}(t) \approx\ & 0.9999823812 - 0.00001761881384t - 0.5000088094t^2 \\ & - 0.3333362698t^3 - 0.1250007342t^4 - 0.03333347987t^5 \\ & - 0.006944468155t^6 - 0.001190479545t^7 - 0.0001736311935t^8 \\ & - 0.00002198834869t^9 - 0.000002554945345t^{10} \\ & - 0.0000001996845t^{11} - 0.00000004015877t^{12} \\ & + 0.00001551955341e^t. \end{aligned} \tag{25}$$

Table 3: Numerical result for seventh order BVP Example 3

t	Exact Solution	Absolute Error EFCAM	Absolute Error [8]
0.0	1.0000000000	0.00	0.00
0.1	0.09946538262	1.10 E —10	1.10 E —16
0.2	0.9771222064	2.10 E —10	4.40 E —16
0.3	0.9449011656	2.10 E —10	2.20 E —14
0.4	0.8950948188	2.10 E —10	7.70 E —13
0.5	0.8243606355	0.00	1.10 E —11
0.6	0.7288475200	2.10 E —10	1.00 E —10
0.7	0.6041258121	1.10 E —10	6.40 E —10
0.8	0.4451081857	0.000	3.20 E –09
0.9	0.2459603111	1.10 E —10	1.30 E —09
1.0	0.0000000000	6.70 E —13	4.60 E —08

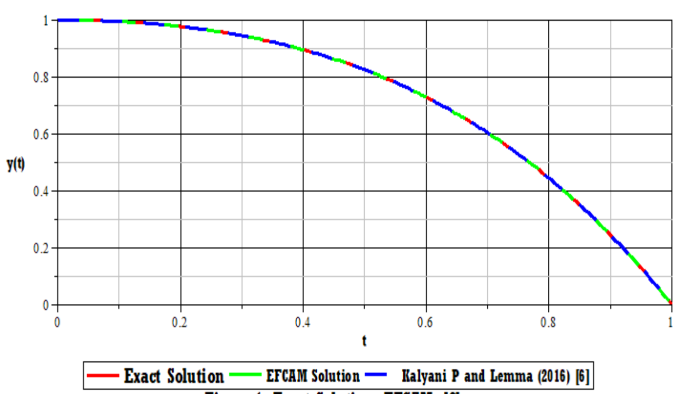

Figure 1. Exact Solution, EFCAM, [6]
Seventh order boundary value problem Example 1

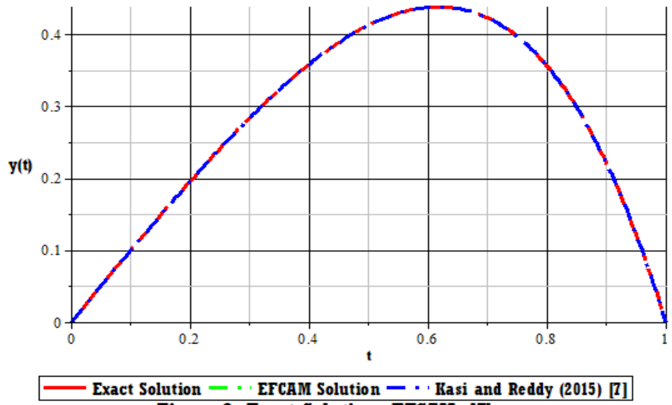

Figure 2. Exact Solution, EFCAM, [7]
Seventh order boundary value problem Example 2

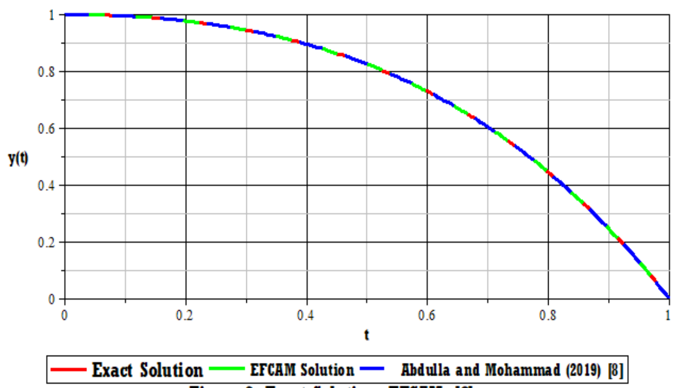

Figure 3. Exact Solution, EFCAM, [8]
Seventh order boundary value problem Example 3

4. CONCLUSION

In this paper, exponentially fitted collocation approximate technique has been introduced and applied to obtain numerical solutions of seventh order boundary value problems.To test the accuracy and efficiency of the proposed method, it has been tested on three examples of seventh order boundary value problems. It is found that the obtained results are giving little or no error comparing to exact solutions and existing numerical methods available in the literature. The advantage of this present method lies on utilization of MAPLE 18 software for all computational works which enhance the accuracy and efficiency.

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