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ON IDEAL AMENABILITY OF TRIANGULAR BANACH ALGEBRAS

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ABSTRACT. We provide conditions under which the triangular Banach algebra T is I_T-weakly amenable and ideally amenable for a closed two-sided ideal I_T of T. For Banach algebras A and B, we show that in the case where A and B are commutative and are both ideally amenable, $A\hat{\otimes}B$ is ideally amenable. Thus providing a partial answer to the question raised by M.E Gorgi and T. Yazdanpanah.

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1. INTRODUCTION

In [4], Gorgi and Yazdanpanah introduced two notions of amenability for a Banach algebra A. These are the notions of I-weak amenability and ideal amenability for a Banach algebra A, where I is a closed two-sided ideal in A. They related these notions to weak amenability and amenability of Banach algebras, and showed that ideal amenability is different from amenability and weak amenability. These authors finally posed the following question: If A and B are ideally amenable Banach algebras, then is $A\hat{\otimes}B$ ideally amenable? Partial answer to this question was given by the first author in [6].

In this paper, we shall also provide a partial answer to the above question for commutative case.

In [3], Forest and Marcoux investigated the Arens regularity and n-weak amenability of a triangular Banach algebra T in relation to that of the algebras A and B and their action on the module M. In particular, they showed that T is Arens regular if and only if both A and B are Arens regular and A and B act regularly on M, and that T is weakly amenable if and only if both A and B are weakly amenable, where A and B are unital in this case. The triangular

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Banach algebras are algebras of the form

$$T = \begin{bmatrix} A & M \\ 0 & B \end{bmatrix},$$

where A and B are themselves Banach algebras and M is a Banach A, B-module.

In this paper, we shall also extend the results of Forest and Marcoux in [3] by providing conditions under which T is I_T-weakly amenable and ideally amenable for closed two-sided ideal I_T of T.

2. PRELIMINARY

First, we recall some standard notions; for further details, see [1] and [7].

Let A be an algebra and let X be an A-bimodule. A *derivation* from A to X is a linear map $D : A \to X$ such that

$$D(ab) = Da \cdot b + a \cdot Db \quad (a, b \in A)\,.$$

For example, for $x \in X$, the map $\delta_x : A \to X$ defined by $\delta_x(a) = a \cdot x - x \cdot a \quad (a \in A)$ is a derivation; derivations of this form are called the *inner derivations*.

Let A be a Banach algebra, and let X be an A-bimodule. Then X is a Banach A-bimodule if X is a Banach space and if there is a constant $k > 0$ such that

$$\|a \cdot x\| \leq k\|a\|\|x\|, \quad \|x \cdot a\| \leq k\|a\|\|x\| \quad (a \in A,\, x \in X)\,.$$

By renorming X, we can suppose that $k = 1$. For example, A itself is Banach A-bimodule and X', the dual space of a Banach A-bimodule X is a Banach A-bimodule with respect to the module operations given by

$$\langle x,\, a \cdot \lambda\rangle = \langle x \cdot a,\, \lambda\rangle, \quad \langle x,\, \lambda \cdot a\rangle = \langle a \cdot x,\, \lambda\rangle \quad (x \in X),$$

for $a \in A$ and $\lambda \in X'$; we say that X' is the *dual module* of X. In particular every closed two-sided ideal I of A is Banach A-bimodule and I' the dual space of I is a dual A-bimodule. Successively, the duals $X^{(n)}$ are Banach A-bimodules; in particular $A^{(n)}$ is a Banach A-bimodule for each $n \in \mathbb{N}$. We take $X^{(0)} = X$.

Let A be a Banach algebra and let X be a Banach A-bimodule. Then we denote by $\mathcal{Z}^1(A, X)$ the space of all continuous derivations from A into X and $\mathcal{N}^1(A, X)$ the space of all inner derivations from A into X. The first cohomology group of A with coefficients in X is the quotient space

$$\mathcal{H}^1(A, X) = \mathcal{Z}^1(A, X)/\mathcal{N}^1(A, X)\,.$$

The Banach algebra A is *amenable* if $\mathcal{H}^1(A, X') = \{0\}$ for each Banach A-bimodule X and *weakly amenable* if $\mathcal{H}^1(A, A') = \{0\}$. Further, as in [2], A is *n-weakly amenable* for $n \in \mathbb{N}$, if $\mathcal{H}^1(A, A^{(n)}) = \{0\}$, and A is *permanently weakly amenable*, if it is n-weakly amenable for each $n \in \mathbb{N}$. For instance, each C^*-algebra is permanently weakly amenable [2, Theorem 2.1]. Each group algebra is n-weakly amenable whenever n is odd. Also, Mewomo in [5], showed that the semigroup algebra $\ell^1(S)$ is $(2k+1)$-weakly amenable for $k \in \mathbb{Z}^+$ and Rees matrix semigruop S. Recently, the authors in [4] defined A as *I-weakly amenable* if $\mathcal{H}^1(A, I') = \{0\}$ for a closed two-sided ideal I of A and ideally amenable if it is I-weakly amenable for every closed two-sided ideal I of A. Clearly, an amenable Banach algebra is ideally amenable and an ideally amenable Banach algebra is weakly amenable.

Let A and B be Banach algebras and suppose X is a Banach A, B-module, that is, X is a Banach space, a left A-module and a right B-module and the actions of A and B are continuous in that

$$\|a.x.b\| \leq \|a\|_A \|x\|_X \|b\|_B.$$

With the case $A = B$, X becomes a Banach A-bimodule. With the right action of A on the dual space X' of X and a left action of B on X' given as

$$\langle x, \lambda \cdot a\rangle = \langle a \cdot x, \lambda\rangle, \quad \langle x, b \cdot \lambda\rangle = \langle x \cdot b, \lambda\rangle \quad (x \in X),$$

for all $a \in A, b \in B, \lambda \in X'$, then X' becomes a Banach B, A-module.

Let A and B be Banach algebras and let M be a Banach A, B-module. We define the corresponding triangular algebra

$$T = \begin{bmatrix} A & M \\ 0 & B \end{bmatrix},$$

with the sum and product being given by the usual 2×2 matrix operations. The norm on T is

$$\| \begin{bmatrix} a & m \\ 0 & b \end{bmatrix} \| = \|a\|_A + \|m\|_M + \|b\|_B.$$

3. IDEAL AMENABILITY OF T

In this section, we first give some elementary properties of T which will assist us in our main result in the section. Suppose the Banach algebra A is unital (i.e. A has an identity element e_A). In this work, we do not require that $\|e_A\|_A = 1$.

We recall that a Banach A, B-module M is unital if A has a unit e_A and B has a unit e_B such that

$$e_A.m = m.e_B = m \; for \; all \; m \in M.$$

Proposition 1: *Let A and B be Banach algebras and let*

$$T = \begin{bmatrix} A & M \\ 0 & B \end{bmatrix}$$

be the corresponding triangular Banach algebra. Then

(i) T is unital if and only if A, B and M are unital.
(ii) T is commutative if and only if A and B are commutative and $AM = MB = \{0\}$.

Proof : (i) Suppose e_A and e_B are the unit elements in A and B respectively. We show that $\begin{bmatrix} e_A & 0 \\ 0 & e_B \end{bmatrix}$ is a unit element of T.
Let $\begin{bmatrix} a & m \\ 0 & b \end{bmatrix} \in T$, then

$$\begin{bmatrix} a & m \\ 0 & b \end{bmatrix} \begin{bmatrix} e_A & 0 \\ 0 & e_B \end{bmatrix} = \begin{bmatrix} ae_A & me_B \\ 0 & be_B \end{bmatrix} = \begin{bmatrix} a & m \\ 0 & b \end{bmatrix}.$$

Similarly,

$$\begin{bmatrix} e_A & 0 \\ 0 & e_B \end{bmatrix} \begin{bmatrix} a & m \\ 0 & b \end{bmatrix} = \begin{bmatrix} e_Aa & e_Am \\ 0 & e_Bb \end{bmatrix} = \begin{bmatrix} a & m \\ 0 & b \end{bmatrix}.$$

Hence T is unital.
(ii) Suppose A and B are commutative and $AM = MB = \{0\}$. Then

$$\begin{bmatrix} a_1 & m_1 \\ 0 & b_1 \end{bmatrix} \begin{bmatrix} a_2 & m_2 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_1a_2 & a_1m_2 + m_1b_2 \\ 0 & b_1b_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_2a_1 & 0 \\ 0 & b_2b_1 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 & m_2 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} a_1 & m_1 \\ 0 & b_1 \end{bmatrix}.$$

Thus T is commutative.
The converse follows easily. □

Corollary 2 : *Suppose A is a commutative Banach algebra. Then $T = \begin{bmatrix} A & A' \\ 0 & A \end{bmatrix}$ is not commutative.*

Proof : Since certainly A' does not annihilate A. Then T is not commutative by Proposition 1. □

Theorem 1 : *Let A and B be commutative unital weakly amenable Banach algebra and let $T = \begin{bmatrix} A & M \\ 0 & B \end{bmatrix}$ be the corresponding triangular Banach algebra. Suppose $AM = MB = \{0\}$. Then*
(i) T is permanently weakly amenable.
(ii) T is ideally amenable.

Proof : (i) Since A and B are weakly amenable and unital, then T is weakly amenable by [3, Corollary 3.5]. Since T is commutative by Proposition 1 and weakly amenable, then T is permanently weakly amenable by [2].

(ii) Since T is commutative and weakly amenable, then T is ideally amenable by [4, Theorem 1.3]. □

Corollary 2 : *Let A and B be commutative unital weakly amenable Banach algebras. Then the triangular Banach algebra $T = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ is ideally amenable.*

Proof : Follow from Theorem 1 with $M = \{0\}$. □

Example 1 : *Let G be a locally compact group. We write $M(G)$ for the space of all (finite) complex regular Borel measures on G. $(M(G), *)$ with the convolution product*

$$\langle f, \mu\ \nu\rangle = \int_G \left(\int_G f(gh) d\mu(g) \right) d\nu(h) \quad (f \in C_0(G), \mu, \nu \in M(G))$$

is a unital Banach algebra called the measure algebra of G, which is commutative if and only if G is abelian. For the case in which the group G is discrete and abelian, $M(G) = l^1(G)$ is a unital commutative Banach algebra. It is known that $l^1(G)$ is weakly amenable for all groups G. Thus, in the case where G is discrete and abelian, we have that $T = \begin{bmatrix} M(G) & 0 \\ 0 & M(G) \end{bmatrix}$ is ideally amenable.

Let A and B be Banach algebras, and let I_A, I_B be closed two-sided ideals of A and B respectively. By Lemma 1 below, $I_T = \begin{bmatrix} I_A & M \\ 0 & I_B \end{bmatrix}$ is closed two-sided ideal of T and so $I'_T = \begin{bmatrix} I'_A & M' \\ 0 & I'_B \end{bmatrix}$ is a dual T-bimodule. We next consider the problem of I_T weak amenability and ideal amenability of T.

Since as a Banach space, T is isomorphic to the l^1-direct sum of A, M and B and I'_A, I'_B and I'_T are dual modules, it is clear that $I'_T \cong I'_A \oplus_1 M' \oplus_1 I'_B$.

It was shown in [3] that the action of T upon T' is given by

$$w \circ \tau = \begin{bmatrix} x\alpha + y\mu & z\mu \\ 0 & z\beta \end{bmatrix} \tag{3.1}$$

and

$$\tau \circ w = \begin{bmatrix} \alpha x & \mu x \\ 0 & \mu y + \beta z \end{bmatrix} \tag{3.2}$$

for $w = \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \in T$ and $\tau = \begin{bmatrix} \alpha & \mu \\ 0 & \beta \end{bmatrix} \in T'$.

As in [3], we take the action of T on I_T' as those given in (3.1) and (3.2).

We next consider the problem of I_T-weak amenability and ideal amenability of T. The next lemma is useful in establishing this problem.

Lemma 1 : *Let A and B be Banach algebras. Suppose I_A and I_B are closed two-sided ideals of A and B respectively. Then $I_T = \begin{bmatrix} I_A & M \\ 0 & I_B \end{bmatrix}$ is a closed ideal of T.*

Proof : Clearly I_T is a closed subspace of T.
Let $\begin{bmatrix} a & m \\ 0 & b \end{bmatrix} \in T$ and $\begin{bmatrix} i_a & m_1 \\ 0 & i_b \end{bmatrix} \in I_T$. Then

$$\begin{bmatrix} a & m \\ 0 & b \end{bmatrix} \begin{bmatrix} i_a & m_1 \\ 0 & i_b \end{bmatrix} = \begin{bmatrix} ai_a & am_1 + mi_b \\ 0 & bi_b \end{bmatrix} \in I_T$$

since I_A and I_B are ideals in A and B respectively. Thus $AI_T \subset I_T$. Similarly $I_TA \subset I_T$, and so I_T is a closed ideal of T. $\square$

Lemma 2 : *Let $\delta_A : A \to I_A'$ and $\delta_B : B \to I_B'$ be continuous derivations. Then $D_{\delta_A} : T \to I_T'$, $D_{\delta_B} : T \to I_T'$, and $D : T \to I_T'$, defined by*

$$D_{\delta_A}\left(\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} \delta_A(a) & 0 \\ 0 & 0 \end{bmatrix},$$

$$D_{\delta_B}\left(\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & \delta_B(b) \end{bmatrix},$$

and

$$D\left(\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} \delta_A(a) & 0 \\ 0 & \delta_B(b) \end{bmatrix},$$

are continuous derivations. Furthermore, δ_A is inner if and only if D_{δ_A} is inner and δ_B is inner if and only if D_{δ_B} is inner.

Proof : Let $\begin{bmatrix} a_1 & m_1 \\ 0 & b_1 \end{bmatrix}, \begin{bmatrix} a_2 & m_2 \\ 0 & b_2 \end{bmatrix} \in T$, then

$$D_{\delta_A}\left(\begin{bmatrix} a_1 & m_1 \\ 0 & b_1 \end{bmatrix}\right)\left(\begin{bmatrix} a_2 & m_2 \\ 0 & b_2 \end{bmatrix}\right) = D_{\delta_A}\left(\begin{bmatrix} a_1a_2 & a_1m_2 + m_1b_2 \\ 0 & b_1b_2 \end{bmatrix}\right) = \begin{bmatrix} \delta_A(a_1a_2) & 0 \\ 0 & 0 \end{bmatrix}.$$

Moreover,

$$\begin{bmatrix} a_1 & m_1 \\ 0 & b_1 \end{bmatrix} D_{\delta_A}\left(\begin{bmatrix} a_2 & m_2 \\ 0 & b_2 \end{bmatrix}\right) + D_{\delta_A}\left(\begin{bmatrix} a_1 & m_1 \\ 0 & b_1 \end{bmatrix}\right)\begin{bmatrix} a_2 & m_2 \\ 0 & b_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & m_1 \\ 0 & b_1 \end{bmatrix}\begin{bmatrix} \delta_A(a_2) & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \delta_A(a_1) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a_2 & m_2 \\ 0 & b_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1\delta_A(a_2) & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \delta_A(a_1)a_2 & 0 \\ 0 & 0 \end{bmatrix}$$

(using (3.1) and (3.2) of the action of T on I'_T)

$$= \begin{bmatrix} a_1\delta_A(a_2) + \delta_A(a_1)a_2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \delta_A(a_1a_2) & 0 \\ 0 & 0 \end{bmatrix}$$

since $\delta_A : A \to I'_A$ is a derivation.
Thus D_{δ_A} is a derivation.
The proof that D_{δ_A} and D are derivation are similar.

Suppose δ_A is inner. Then there exists $\alpha \in I'_A$ such that $\delta_A(a) = a.\alpha - \alpha.a$. Consider $\tau = \begin{bmatrix} \alpha & 0 \\ 0 & 0 \end{bmatrix} \in I'_T$. Then

$$D_\tau\left(\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \alpha & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} a\alpha & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \alpha a & 0 \\ 0 & 0 \end{bmatrix}$$

(using (3.1) and (3.2))

$$= \begin{bmatrix} a\alpha - \alpha a & 0 \\ 0 & 0 \end{bmatrix}$$

$$= D_{\delta_A}\left(\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\right),$$

and so D_{δ_A} is inner.
Conversely, suppose D_{δ_A} is inner. Then there exists $\tau = \begin{bmatrix} \alpha & \mu \\ 0 & \beta \end{bmatrix} \in I'_T$ such that

$$D_{\delta_A}\left(\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\begin{bmatrix} \alpha & \mu \\ 0 & \beta \end{bmatrix} - \begin{bmatrix} \alpha & \mu \\ 0 & \beta \end{bmatrix}\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} a\alpha + m\mu & b\mu \\ 0 & b\beta \end{bmatrix} - \begin{bmatrix} \alpha a & \mu a \\ 0 & \mu m + \beta b \end{bmatrix}$$

(using (3.1) and (3.2))

$$= \begin{bmatrix} a\alpha + m\mu - \alpha a & b\mu - \mu a \\ 0 & b\beta - \mu m - \beta b \end{bmatrix}.$$

But $D_{\delta_A}\left(\begin{bmatrix} a & m \\ 0 & b \end{bmatrix}\right) = \begin{bmatrix} \delta_A(a) & 0 \\ 0 & 0 \end{bmatrix}$.

It follows that $\delta_A(a) = a\alpha + m\mu - \alpha a$ for all $a \in A, m \in M$. We may assume without loss of generality that $m = 0$. Then for any $a \in A$, we have

$$\delta_A(a) = a\alpha - \alpha a,$$

and so δ_A is inner.

The proof that δ_B is inner is similar. □

Forest and Marcoux in [3] gave the following theorem:

Theorem 2 : *Let A and B be unital Banach algebras and M be a unital Banach A, B-module. Let $T = \begin{bmatrix} A & M \\ 0 & B \end{bmatrix}$ be the corresponding triangular Banach algebra. Then*

$$\mathcal{H}^1(T, T') \cong \mathcal{H}^1(A, A') \oplus \mathcal{H}^1(B, B').$$

It clearly follow from the Theorem 2 and the above lemmas that

$$\mathcal{H}^1(T, I'_T) \cong \mathcal{H}^1(A, I'_A) \oplus \mathcal{H}^1(B, I'_B),$$

for every closed two-sided ideals I_A of A and I_B of B. Thus, we have the next result.

Theorem 3 : *Let A and B be unital Banach algebras and let M be a unital Banach A, B-module. Let $T = \begin{bmatrix} A & M \\ 0 & B \end{bmatrix}$ be the corresponding triangular Banach algebra. Then*

(i) T is I_T-weakly amenable if and only A is I_A-weakly amenable and B is I_B weakly amenable.

(ii) T is ideally amenable if and only if both A and B are ideally amenable.

Proof : (i) Suppose T is I_T-weakly amenable, then $\mathcal{H}^1(T, I'_T) = \{0\}$ and so $\mathcal{H}^1(A, I'_A) = \mathcal{H}^1(B, I'_B) = \{0\}$. Thus A is I_A-weakly amenable and B is I_B weakly amenable.

Conversely, suppose A is I_A-weakly amenable and B is I_B weakly amenable. Then $\mathcal{H}^1(A, I'_A) = \mathcal{H}^1(B, I'_B) = \{0\}$, and so $\mathcal{H}^1(T, I'_T) = \{0\}$. Thus T is I_T-weakly amenable.

(ii) This follows from (i) and the fact that for every closed two-sided ideal I_A of A and I_B of B, $I_T = \begin{bmatrix} I_A & M \\ 0 & I_B \end{bmatrix}$ is a closed two-sided ideal of T. □

4. IDEAL AMENABILITY OF $A\hat{\otimes}B$

In this section, we try to provide a partial answer to the problem of ideally amenability of $A\hat{\otimes}B$ raised by Gorgi and Yazdanpanah in [4].

Let A and B be Banach algebras. The projective tensor product $(A\hat{\otimes}B, \|.\|_\pi)$ of A and B is defined on [1, p. 165]; $(A\hat{\otimes}B, \|.\|_\pi)$ is a Banach A-bimodule and a Banach algebra. We denote by $A^\#$ the unitization of A. Clearly A is a closed two-sided ideal of $A^\#$.

As in [4, Theorem 1.9], we have the following theorem.

Theorem 5 : *Let A be a Banach algebra and let I be a closed two-sided ideal in A with a bounded approximate identity. If A is ideally amenable, then I is ideally amenable*

The following lemma is very useful in establishing our main result in this section.

Lemma 3 : *Let A and B be Banach algebras.*
(i) If A and B are commutative, Then $A\hat{\otimes}B$ is commutative.
(ii) $A\hat{\otimes}B$ has a bounded approximate identity if and only if both A and B have bounded approximate identity.
(iii) For every closed two-sided ideal I_A of A and I_B of B, $I_A\hat{\otimes}I_B$ is closed two-sided ideal of $A\hat{\otimes}B$.

Proof : (i) This is elementary since for $(a \otimes b), (c \otimes d) \in A\hat{\otimes}B$

$$(a \otimes b).(c \otimes d) = ac \otimes bd = ca \otimes db = (c \otimes d).(a \otimes b)$$

if and only if A and B are commutative.

(ii) This follows from [1, Proposition 2.9.21].

(iii) This is easy. □

Theorem 7 : *Let A and B be commutative ideally amenable Banach algebras. Then $A\hat{\otimes}B$ is ideally amenable.*

Proof : By Lemma 3 (i) $A\hat{\otimes}B$ is commutative. A and B are ideally amenable implies A and B are weakly amenable, and so $A\hat{\otimes}B$ is weakly amenable by [1, Proposition 2.8.71]. Since $A\hat{\otimes}B$ is commutative and weakly amenable, then it is ideally amenable by [4, Theorem 1.3]. □

Remark 1 : Whenever A and B are commuative and ideally amenable, $A\hat{\otimes}B$ is ideally amenable. For the case in which A and

B are non commutative, suppose A and B have bounded approximate identity and are both ideally amenable. Then $I = A\hat{\otimes}B$ has a bounded approximate identity and it is closed two-sided ideal of $A^{\#}\hat{\otimes}B^{\#}$ and by Theorem 5, $I = A\hat{\otimes}B$ is ideally amenable if $A^{\#}\hat{\otimes}B^{\#}$ is ideally amenable.

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