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ON THE POISEUILLE FLOW OF A THIRD GRADE FLUID IN AN HORIZONTAL CHANNEL

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ABSTRACT. The work presents an analysis of the Poiseuille flow of a third Grade fluid in a horizontal channel. The dimensionless variable were used to dimensionalize the governing equations of the flow using suitable variables. A semi-analytical method in form of perturbation method was use to obtain a solution of velocity distribution and temperature profile. The graphical results were used to study the non-Newtonian, Electrokinetic separation distance based on plate height, specific internal energy parameter of the model.

Keywords and phrases: Third grade fluid, Poiseuille flow, Perturbation method, Velocity and temperature profiles
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1. INTRODUCTION

Poiseuille flow in a steady axisymmetric flow in an infinitely long circular pipe of radius. The scientists and engineers are very much interested in the geometry of flows of such types of fluids are compared to Newtonian fluids, the analysis of the behavior of the motion of such fluids is much more complicated and not easy to handle because of non-linear relationship between stress and rate of slain. In recent years, many non-Newtonian models have been proposed. Among these models, model of fluids of differential type [1] have received considerable attention. Fluid of third grade is a subclass of fluids of differential type, which has been studied successfully in various types of flow situation [2],[3],[4] and is known to capture the non-Newtonian affects such as shear thinning, shear thickening as well as normal stress.

Mebine [5] studied the effect of thermal radiation on Magneto-Hydrodynamics (MHD) Couette flow with heat transfer between

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two parallel plates. Singh [6] considered the problem magneto hydrodynamics (MHD) of free convection flow of an optically thin fluid bounded by two horizontal porous parallel plate Chaudhary and Jain [7] worked on an exact solution to the unsteady free-convection boundary layer flow past an impulsively started vertical surface with Newtonian heating using Laplace transform method to obtain solutions in a closed form. Akgul *et. al* [8] considered the electro-osmotic flow of a third grade fluid between micro-parallel plates. Approximately analytical solutions are obtained by perturbation techniques. Constant viscosity and temperature dependent viscosity cases are treated separately and they also obtained the numerical solutions, also the influence of some parameters on the velocity and temperature profiles are shown. Sharma and Khan [9] have investigated the Magnetohydrodynamics flow of a viscous fluid through a porous medium induced by torsionally oscillating disk and presented approximate solutions of the flow characteristics. Ahmad and Asghar [10] gave an exact solution for Magnetohydrodynamics boundary layer flow of a second grade fluid over a permeable stretching surface with arbitrary velocity and appropriate wall transpiration. Ali *et. al* [11] analyzed the problem of unsteady electrically conducting second grade fluid passing through a porous space and established an exact solution for the transient flow due to oscillating wall boundary using Laplace transform method. Abdulhameed *et. al* [12] studied the unsteady Magneto hydrodynamic flow of incompressible viscous fluid over flat plates with impulsive and oscillating motions, and with wall transpiration through a porous medium. They obtained results by applying an extension of the variable separation technique combined with similarity arguments. Among important studies on third grade fluidd model: Aziz and Aziz [13] examined the analytical solution for unsteady Magnetohydrodynamic flow of a third grade fluid past a porous plate within a porous medium due to an arbitrary wall with suction/injection velocity. They obtained results by applying a Lie symmetric and numerical methods. Aziz *et. al* [14] have presented Group invariant solutions for steady Magnetodynamic flow of a third grade fluid in porous medium due to the arbitrary velocity of non-porous plate. Other important studies related to the second-grade and third-grade fluids are [15]-[19]. Akinbobola and Okoya [20] studied the flow of second grade fluid over a stretch sheet with variable thermal conductivity and viscosity in the presence of heat source/sink. In the aforementioned above studies, it is observed that no analysis on the combined effects electro-osmotic and Br in the presence of heat source/sink, viscous dissipation, work done by deformation and thermal radiation in a second grade fluid flow has been carried out. In this paper, the Poiseuilles flow of a third grade fluid in a horizontal channel is considered.

2. PROBLEM FORMULATION

The fluid is assumed to be an incompressible laminar flow. The equation of motion is given as continuity equation, momentum equation, and the energy equation as follows:

$$\nabla . v^* = 0, \tag{1}$$

$$\rho \frac{dv^*}{dt} = \nabla \tau + \rho b, \tag{2}$$

where $b = \rho_\epsilon E$ and

$$\rho \frac{d\varsigma}{dt} = \tau . grad(v^*) - \nabla q + \sigma E_x^2, \tag{3}$$

where v^* is the velocity vector, ρ is the fluid density, τ is the stress tensor, b is the body force consisting of the electrical field E only with gravity not included, ρ_ϵ is the net electric charge density, ς is the specific internal energy, q is the heat flux vector, σ is the permittivity of electric field and $E_x^2 \sigma$ term represents Joule heating. The electric field is in the y-direction only.

3. METHODOLOGY

The Poisson Boltzmann equation is related to the potential distribution within the electric double layer which can be expressed in the y-direction as follows;

$$\frac{d^2 \Psi^*}{dy^{*2}} = -\frac{\rho e}{\in}, \tag{4}$$

where Ψ^* is the electrical potential, $\in$ is the dielectric constant or permitivity of the fluid and the ρe is the net electric charge density. If we assume that the equilibrium Boltzmann equation is a uniform dielectric constant, the numbers of type-i ions are of the form:

$$n_i = n_{i0} \exp \left(\frac{-ze\Psi^*}{k_b \theta} \right), \tag{5}$$

where n_{i0}, z, e, k_b and θ are the bulk ionic concentration, valence of type-i ions elementary charge, Boltzmann's constant and absolute temperature respectively. The net electric charge density can be expressed assuming a symmetric electrolyte as follows:

$$\rho_e = -2zen_0 \sinh(\Psi). \tag{6}$$

Substituting 5 and 6 in to 4 gives:

$$\frac{d^2\Psi^*}{dy^{*2}} = \frac{2zen_0}{\in} \sinh\left(\frac{ze\Psi^*}{k_b\theta^*}\right). \tag{7}$$

Applying Debye-Huckel linear approximation $\sinh\left(\frac{ze\Psi^*}{k_b\theta^*}\right) = \left(\frac{ze\Psi^*}{k_b\theta^*}\right)$. Then 7 becomes

$$\frac{d^2\Psi^*}{dy^{*2}} \cong \frac{2n_0z^2e^2\Psi^*}{\in k_b\theta^*}. \tag{8}$$

When the electrical potential is small compared to the thermal energy of the ions. Equation 8 can be written as

$$\frac{d^2\Psi^*}{dy^{*2}} = k^2\Psi^*, \tag{9}$$

where $k = ze\sqrt{\frac{2n_0}{\in k_b\theta^*}}$ is the Debye-Huckel parameter and $\frac{1}{k}$ is the Debye length.

The boundary conditions are given as

$$\frac{d\Psi}{dy}(0) = 0, \qquad \Psi'(h) = \varsigma^*. \tag{10}$$

Introducing the following non-dimensional parameters

$$y = \frac{y^*}{h}, \qquad \Psi = \frac{ze\Psi}{k_b\theta}, \qquad K = kh, \qquad \varsigma = \frac{ze\varsigma^*}{k_b\theta^*}. \tag{11}$$

On solving 9 subject to the boundary conditions 10 gives

$$\Psi = \frac{\varsigma\cosh(Ky)}{\cosh K}. \tag{12}$$

The viscosity is said to be a constant and using the velocity components, the momentum equation becomes:

$$\mu\frac{d^2u^*}{dy^*} + 2\beta\frac{d}{dy^*}\left(\frac{du^*}{dy^*}\right)^3 + E_x\rho_e = \frac{\partial p^*}{\partial x^*}, \tag{13}$$

where μ is the kinetic viscosity, β is the material constant, E_x is the electrical field, ρ_e is the net electric charge density,

$$\frac{d}{dy^*}\left[(2\alpha_1+\alpha_2)\left(\frac{du^*}{dy^*}\right)^2\right] = \frac{\partial p^*}{\partial y^*}, \tag{14}$$

along the y axis and $\frac{\partial p^*}{\partial z^*} = 0$ along the z axis.

If a modified pressure is defined as

$$\overline{p^*} = p^* - \left[(2\alpha_1 + \alpha_2) \left(\frac{du^*}{dy^*} \right)^2 \right]. \tag{15}$$

Equation 14 reduces to

$$\overline{p^*} = p^*(x). \tag{16}$$

Equation 13 gives

$$\mu \frac{d^2 u^*}{dy^*} + 2\beta \frac{d}{dy^*} \left(\frac{du^*}{dy^*} \right)^3 + E_x \rho_e = \frac{d\overline{p^*}}{dx^*}. \tag{17}$$

The pressure gradient is assumed to be constant and equation 16 is

$$\mu \frac{d^2 u^*}{dy^*} + 2\beta \frac{d}{dy^*} \left(\frac{du^*}{dy^*} \right)^3 + E_x \rho_e = c_0. \tag{18}$$

Introducing the non dimensionless parameters

$$u^* = uU, \qquad y^* = yh, \tag{19}$$

$$\frac{d^2 u}{dy^2} + 6\Lambda_1 \left(\frac{du}{dy} \right)^2 \frac{d^2 u}{dy^2} - \Lambda_2 \frac{d^2 \Psi}{dy^2} = \Lambda_3, \tag{20}$$

where

$$\Lambda_1 = \frac{\beta U^2}{\mu h^2}, \qquad \Lambda_2 = \frac{\epsilon k_b \theta^* E_x}{ze\mu U}, \qquad \Lambda_3 = \frac{h^2 c_0}{\mu U}, \tag{21}$$

U is the reference veolcity, Λ_1 is the non-Newtonian behaviour, λ_2 is the electro-kinetic effects, γ_1 is the Brinkman number, γ_2 is the Joule heating, θ_m^* and θ_s^* are mean and surface temperatures and c_0 the constant pressure gradient. Substituting 12 into 19 to obtain

$$\frac{d^2 u}{dy^2} + 6\Lambda_1 \left(\frac{du}{dy} \right)^2 \frac{d^2 u}{dy^2} - \Lambda_2 \frac{\varsigma K^2 \cosh(Ky)}{\cosh(K)} = \Lambda_3. \tag{22}$$

The energy equation is given by

$$\mu \left(\frac{du^*}{dy^*} \right)^2 + 2\beta \left(\frac{du^*}{dy^*} \right)^4 + K_{th} \left(\frac{d^2 \theta^*}{dy^{*2}} \right) + E_x^2 \sigma = c_0, \tag{23}$$

$$\gamma_1 = \frac{\mu U^2}{K_{th}(\theta_m^* - \theta_s^*)}, \qquad \gamma_2 = \frac{E_x^2 \sigma h^2}{K_{th}(\theta_m^* - \theta_s^*)}, \qquad \theta = \frac{\theta^* - \theta_s^*}{\theta_m^* - \theta_s^*}, \tag{24}$$

where $K_t h$ and σ are thermal conductivity of the fluid and the permittivity of electricity respectively, also E_x^σ term represents Joule heating, ρ is density, θ_m^* is the temperature along the x axis, θ_s^* is outer temperature and g is acceleration due to gravity.

Therefore, the non-dimensional form of 23 gives:

$$\frac{d^2\theta}{dy^2} + \gamma\left(\frac{du}{dy}\right)^2 + 2\Lambda_1\gamma_1\left(\frac{du}{dy}\right)^4 + \gamma_2 = 0. \tag{25}$$

To solve equation 22 and 25, a solution in term of perturbation method is assumed in the form

$$u = u_0 + \epsilon u_1 + \epsilon^2 u_2,\ \theta = \theta_0 + \epsilon\theta_1 + \epsilon^2\theta_2,\ \Lambda_1 = \epsilon\lambda_1,\ \Lambda_2 = \epsilon\lambda_2, \tag{26}$$

where ϵ is the perturbation parameter in a small quantity.
Substituting 26 into 20 gives

$$\frac{d^2u_0}{dy^2} + \epsilon\frac{d^2u_1}{dy^2} + \epsilon^2\frac{d^2u_2}{dy^2} + 6\epsilon\lambda_1\left(\frac{du_0}{dy} + \epsilon\frac{du_1}{dy} + \epsilon^2\frac{du_2}{dy}\right)$$

$$+\left(\frac{d^2u_0}{dy^2} + \epsilon\frac{d^2u_1}{dy^2} + \epsilon^2\frac{d^2u_2}{dy^2}\right) - \frac{\epsilon\lambda_2\varsigma\cosh(Ky)K^2}{\cosh(K)} = \Lambda_3. \tag{27}$$

Further simplification of 27 and arrange in order of ϵ, we have

$$6\lambda_1\left(\tfrac{du_2}{dy}\right)^2\left(\tfrac{d^2u_2}{dy^2}\right)\epsilon^7 +$$
$$\left(12\lambda_1\left(\tfrac{du_1}{dy}\right)\left(\tfrac{du_2}{dy}\right)\left(\tfrac{d^2u_2}{dy^2} + 6\lambda_1\left(\tfrac{du_2}{dy}\right)^2\left(\tfrac{d^2u_1}{dy^2}\right)\right)\right)\epsilon^6 +$$

$$\left(6\lambda_1\left(2\left(\tfrac{d^2u_0}{dy^2}\right)\left(\tfrac{du_2}{dy}\right)\right) + \left(\tfrac{du_1}{dy}\right)^2 + 12\lambda_1\left(\tfrac{du_1}{dy}\right)\left(\tfrac{du_2}{dy}\right)\left(\tfrac{d^2u_1}{dy^2}\right)\right)\epsilon^5$$
$$\left(+6\lambda_1\left(\tfrac{du_2}{dy}\right)^2\left(\tfrac{d^2u_2}{dy^2}\right)\right)\epsilon^5 + \left(12\lambda_1\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)\left(\tfrac{d^2u_2}{dy^2}\right)\right)$$

$$+\left(\left(6\lambda_1\left(2\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_2}{dy}\right) + \left(\tfrac{du_1}{dy}\right)^2\right)\left(\tfrac{d^2u_1}{dy^2}\right)\right.\right.$$

$$\left.+12\lambda_1\left(\tfrac{du_1}{dy}\right)\left(\tfrac{du_2}{dy}\right)\left(\tfrac{d^2u_0}{dy^2}\right)\right)\epsilon^4 + \left(6\lambda_1\left(\tfrac{du_0}{dy}\right)^2\left(\tfrac{d^2u_2}{dy^2}\right)\right)\epsilon^3$$
$$\left(+12\lambda_1\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)\left(\tfrac{d^2u_1}{dy^2}\right)\right.$$

$$+6\lambda_1\left(2\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_2}{dy}\right) + \left(\tfrac{du_1}{dy}\right)^2\right)\left(\left(\tfrac{d^2u_0}{dy^2}\right)\right)\epsilon^3$$

$$+\left(6\lambda_1\left(\tfrac{du_0}{dy}\right)^2\left(\tfrac{d^2u_1}{dy^2}\right) + 12\lambda_1\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)\left(\tfrac{d^2u_0}{dy^2}\right) + \left(\tfrac{d^2u_2}{dy^2}\right)\right)\epsilon^2$$

$$+\left(\left(\frac{d^2u_1}{dy^2}\right)-\frac{\lambda_2\varsigma\cosh(Ky)K^2}{\cosh(K)}+6\lambda_1\left(\frac{du_0}{dy}\right)^2\left(\frac{d^2u_0}{dy^2}\right)\right)\epsilon \tag{28}$$
$$+\left(\frac{d^2u_0}{dy^2}\right)=\Lambda_3.$$

Using the same approach 24 for equation 25 arrange in order of ϵ

$$2\lambda_1\gamma_1\left(\tfrac{du_2}{dy}\right)^4\epsilon^9+8\lambda_1\gamma_1\left(\tfrac{du_1}{dy}\right)\left(\tfrac{du_2}{dy}\right)^3\epsilon^8$$
$$+2\lambda_1\gamma_1\left(2\left(2\left(\tfrac{du_0}{dy}\right)\right)\left(\tfrac{du_2}{dy}\right)+\left(\tfrac{du_1}{dy}\right)^2\right)\left(\tfrac{du_2}{dy}\right)+4\left(\tfrac{du_1}{dy}\right)^2\left(\tfrac{du_2}{dy}\right)^2\right)\epsilon^7$$
$$+2\lambda_1\gamma_1\left(4\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)\left(\tfrac{du_2}{dy}\right)^2+4\left(2\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_2}{dy}\right)+\left(\tfrac{du_1}{dy}\right)^2\right)\right)\epsilon^6$$
$$\left(\left(\tfrac{du_1}{dy}\right)\left(\tfrac{du_2}{dy}\right)\right)\epsilon^6$$
$$+2\lambda_1\gamma_1\left(2\left(\tfrac{du_0}{dy}\right)^2\left(\tfrac{du_2}{dy}\right)^2+8\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)^2\left(\tfrac{du_2}{dy}\right)+\right)\epsilon^5$$
$$\left(\left(2\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_2}{dy}\right)+\left(\tfrac{du_1}{dy}\right)^2\right)^2\right)\epsilon^5 \tag{29}$$
$$+2\lambda_1\gamma_1\left(2\left(\tfrac{du_0}{dy}\right)^2\left(\tfrac{du_2}{dy}\right)^2\left(\tfrac{du_1}{dy}\right)\left(\tfrac{du_2}{dy}\right)+4\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)\left(2\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_2}{dy}\right)\right)\right)\epsilon^4$$
$$+2\lambda_1\gamma_1\left(2\left(\tfrac{du_0}{dy}\right)^2\left(2\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_2}{dy}\right)^3+\left(\tfrac{du_1}{dy}\right)^2\right)+4\left(\tfrac{du_0}{dy}\right)^2\left(\tfrac{du_1}{dy}\right)^2\right)\epsilon^3$$
$$+\left(\tfrac{d^2\theta_2}{dy^2}+\gamma\left(2\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_2}{dy}\right)+\left(\tfrac{du_1}{dy}\right)^2\right)+8\lambda_1\gamma_1\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)\right)\epsilon^2$$
$$+\left(\tfrac{d^2\theta_1}{dy^2}+2\gamma\left(\tfrac{du_0}{dy}\right)\left(\tfrac{du_1}{dy}\right)+2\lambda_1\gamma_1\left(\tfrac{du_0}{dy}\right)^4\right)\epsilon$$
$$+\tfrac{d^2\theta_0}{dy^2}+\gamma_1\left(\tfrac{du_0}{dy}\right)^2+\gamma_2=0.$$

On Solving (28) and (29) resulted into

$$u=\tfrac{\Lambda_3y^2}{2}-\tfrac{\Lambda_3}{2}y+\tfrac{1}{2}\tfrac{1}{\cosh(K)}\left(-\lambda_1\Lambda_3^3\cosh(K)y^4+2\lambda_1\Lambda_3^3\cosh(K)y^3-\right.$$
$$\left.\tfrac{3}{2}2\lambda_1\Lambda_3^3\cosh(K)y^2+2\lambda_2\varsigma\cosh(Ky)\right)-\tfrac{1}{4}\left(4\lambda_2\varsigma\cosh(K)-\lambda_1\Lambda_3^3\right.$$
$$\left.\cosh(K)+4\lambda_2\varsigma\right)y-\lambda_2\varsigma-\tfrac{1}{8}\tfrac{1}{\cosh(K)K^2}$$
$$\left(\lambda_1\Lambda_3^2\left\{-12\lambda_2\varsigma K^2-96\lambda_2\varsigma-16\lambda_1\Lambda_3^3\cosh(K)y^6K^2\right.\right.$$
$$+48\lambda_1\Lambda_3^3\cosh(K)y^5K^2-60\lambda_1\Lambda_3^3\cosh(K)y^4K^2-98y\lambda_2\varsigma\sinh(Ky)$$
$$+96\lambda_2\varsigma\cosh(Ky)+40\lambda_1\Lambda_3^3\cosh(K)-48y\lambda_2\cosh(Ky)K^2$$
$$+48\lambda_2\varsigma K\sinh(Ky)+16y^3\lambda_2\varsigma K^2-16\lambda_2\varsigma\cosh(K)y^3K^2$$

$$-15\lambda_1\Lambda_3^3\cosh(K)y^2K^2 + 12\lambda_2\varsigma\cosh(Ky)K^2 - 24y^2\lambda_2\varsigma K^2(63)$$
$$+24\lambda_2\varsigma\cosh(Ky)y^2K^2\})$$
$$+(\lambda_1\Lambda_3^2(-20\lambda_2\varsigma K^2e^K - 96\lambda_2\varsigma e^K + 48\lambda_2\varsigma + 48\lambda_2\varsigma e^{2K} \tag{30}$$
$$-3\lambda_1\Lambda_3^3\cosh(K)K^2e^K + 6\lambda_2\varsigma K^2e^{2K} + 6\lambda_2\varsigma K^2 - 24\lambda_2\varsigma K^2e^{2K}$$
$$+8\lambda_2\varsigma K^2\cosh(K)K^2e^K)e^{-K}y).$$

$$\theta = -\tfrac{\gamma_1\Lambda_3^2(2y-1)^4}{192} - \tfrac{\gamma_2y^2}{2} + \tfrac{\gamma_2y}{2} + \tfrac{\gamma_2\Lambda_3^2}{192} + \tfrac{1}{240}\tfrac{1}{K^2\cosh(K)}(\gamma_1\Lambda_3($$
$$+960\lambda_2\varsigma + 16\lambda_1\Lambda_3^3\cosh(K)y^6K^2 - 48\lambda_1\Lambda_3^3\cosh(K)y^5K^2$$
$$+60\lambda_1\Lambda_3^3\cosh(K)y^4K^2 - 40\lambda_1\Lambda_3^3\cosh(K)y^3K^2$$
$$-480y\lambda_2\varsigma K\sinh(Ky) + 960\lambda_2\varsigma\cosh(Ky) - 80y^3\lambda_2\varsigma K^2$$
$$+80\lambda_2\varsigma\cosh(K)y^3K^2 + 15\lambda_1\Lambda_3^3\cosh(K)y^2K^2$$
$$+240\lambda_2\varsigma\sinh(Ky) + 960\lambda_2\varsigma\cosh(Ky) - 80y^3\lambda_2\varsigma K^2$$
$$+80\lambda_2\varsigma\cosh(K)y^3K^2 + 15\lambda_1\Lambda_3^3\cosh(K)y^2K^2$$
$$+240\lambda_2\varsigma\sinh(Ky) + 120y^2\lambda_2\varsigma K^2 - 120\lambda_2\varsigma\cosh(K)y^2K^2))$$
$$+\tfrac{1}{240}\tfrac{1}{K^2\cosh(K)}(\gamma_1\Lambda_3(+960\lambda_2\varsigma e^K + 120\lambda_2\varsigma Ke^{2K}$$
$$-3\lambda_1\Lambda_3^3\cosh(K)K^2e^K - 120\lambda_2\varsigma K - 40\lambda_2\varsigma K^2e^K - 480\lambda_2\varsigma e^{2K}$$
$$+\frac{1}{3360}\frac{1}{\cosh(K)^2K^4}(\gamma_1(-3360\lambda_2\varsigma K^3\lambda_1\Lambda_3^3\cosh(K)\sinh(Ky)$$
$$-241920\lambda_2\varsigma\lambda_1\Lambda_3^3\cosh(K)\sinh(Ky)K + 40\lambda_2\varsigma\cosh(K)K^2e^K$$
$$-480\lambda_2\varsigma)e^{-K}y) - 40320\lambda_2\varsigma K^2\lambda_1\Lambda_3^3\cosh(K)\cosh(Ky)$$
$$-6720\lambda_2^2\varsigma^2K^3\sinh(Ky) - 840\lambda_2^2\varsigma^2K^4\cosh(Ky)^2$$
$$+161280\lambda_2\varsigma K^2\lambda_1\Lambda_3^3\cosh(K)y\cosh(Ky)$$

$$+26880\lambda_2\varsigma\lambda_1\Lambda_3^3\cosh(K)K^3y^3\sinh(Ky)$$
$$+20160\lambda_2\varsigma\lambda_1\Lambda_3^3\cosh(K)y\sinh(Ky)$$
$$-40320\lambda_2\varsigma K^3\lambda_1\Lambda_3^3\cosh(K)y^2\sinh(Ky)$$
$$-161280\lambda_2\varsigma\lambda_1\Lambda_3^3\cosh(K)K^2y^2\cosh(Ky)$$
$$+6720\lambda_2^2\varsigma^2\cosh(K)K^3\sinh(Ky)$$
$$-645120\lambda_2\varsigma\lambda_1\Lambda_3^3\cosh(K)\cosh(Ky)$$
$$-2800\Lambda_3^3\lambda_1\cosh(K)^2K^4\lambda_2\varsigma y^3-3360\Lambda_3^3\lambda_1\cosh(K)K^4y^5\lambda_2\varsigma$$
$$+3360\Lambda_3^3\lambda_1\cosh(K)^2K^4\lambda_2\varsigma y^4+4480\Lambda_3^3\lambda_1\cosh(K)^2K^4\lambda_2\varsigma y^3$$
$$+840\Lambda_3^3\lambda_1\cosh(K)^2K^4\lambda_2\varsigma y^2-3360\Lambda_3^3\lambda_1\cosh(K)K^4y^2\lambda_2\varsigma$$
$$+3360\Lambda_3^3\lambda_1\cosh(K)^2K^4y\lambda_2\varsigma-5040\Lambda_3^6\lambda_1^2\cosh(K)^2K^4y^6$$
$$-6720K^4\lambda_2^2\varsigma^2\cosh(K)y-1680\lambda_2^2\varsigma^2\cosh(K)^2K^4y^2$$
$$+3360K^4\lambda_2^2\varsigma^2\cosh(K)y^2+5040\Lambda_3^6\lambda_1^2\cosh(K)^2K^4y^5$$
$$+2880\Lambda_3^6\lambda_1^2\cosh(K)^2K^4y^7-3150\Lambda_3^6\lambda_1^2\cosh(K)^2K^4y^4$$
$$+1260\Lambda_3^6\lambda_1^2\cosh(K)^2K^4y^3-315\Lambda_3^6\lambda_1^2\cosh(K)^2K^4y^2$$
$$-720\Lambda_3^6\lambda_1^2\cosh(K)^2K^4y^8+40320K^2\lambda_2\varsigma\lambda_1\Lambda_3^3\cosh(K)$$
$$+80640\lambda_1\Lambda_3^3\cosh(K)y\lambda_2\varsigma K^2-840\Lambda_3^3\lambda_1\cosh(K)K^4e^K\lambda_2\varsigma y^3$$
$$-840\Lambda_3^3\lambda_1\cosh(K)K^{-4}e^{-K}\lambda_2\varsigma y^3+13440\lambda_1\Lambda_3^3\cosh(K)y^3\lambda_2\varsigma K^2$$
$$-20160\lambda_1\Lambda_3^3\cosh(K)y^2\lambda_2\varsigma K^2+645120\lambda_2\varsigma\lambda_1\Lambda_3^3\cosh(K)$$
$$+840K^4\lambda_2^2\varsigma^2+6720K^4\lambda_2^2\varsigma^2y+840K^6\lambda_2^2\varsigma^2y^2-1680K^4\lambda_2^2\varsigma^2y^2$$
$$+1260\Lambda_3^3\lambda_1\cosh(K)K^4e^{-K}\lambda_2\varsigma y^2-5040\Lambda_3^3\lambda_1\cosh(K)K^3e^K\lambda_2\varsigma y^2$$
$$+10080\lambda_1\Lambda_3^3\lambda_2\varsigma e^KK^2\cosh(K)y^2+3360\Lambda_3^3\lambda_1\cosh(K)K^3e^K\lambda_2\varsigma y^3$$

$$
\begin{aligned}
&-3360\Lambda_3^3\lambda_1\cosh(K)K^3e^{-K}\lambda_2\varsigma y^3+5040\Lambda_3^3\lambda_1\cosh(K)K^3e^{-K}\lambda_2\varsigma y^2\\
&-6720\lambda\Lambda_3^3\cosh(K)e^{K}\lambda_2\varsigma K^2y^3+1260\Lambda_3^3\lambda_1\cosh(K)K^4e^{K}\lambda_2\varsigma y^2\\
&+1260\Lambda_3^3\lambda_1\cosh(K)K^4e^{-K}\lambda_2\varsigma y^2-5040\Lambda_3^3\lambda_1\cosh(K)K^3e^{K}\lambda_2\varsigma y^2\\
&+10080\lambda_1\Lambda_3^3\lambda_2\varsigma e^{K}K\cosh(K)y^2+10080\lambda_1\Lambda_3^3\lambda_2\varsigma e^{K}K^2\cosh(K)y^2\\
&+3360\Lambda_3^3\lambda_1\cosh(K)K^3e^{K}\lambda_2\varsigma y^3-3360\Lambda_3^3\lambda_1\cosh(K)K^3e^{-K}\lambda_2\varsigma y^3\\
&+5040\Lambda_3^3\lambda_1\cosh(K)K^3e^{-K}\lambda_2\varsigma y^2-6720\lambda_1\Lambda_3^3\cosh(K)e^{K}\lambda_2\varsigma K^2y^3\\
&+10080\lambda_1\Lambda_3^3\cosh(K)e^{-K}\lambda_2\varsigma K^2y^2))-\tfrac{1}{3360}\tfrac{1}{\cosh(K)^2K^4}(\gamma_1(\\
&-45\lambda_1^2\Lambda_3^6\cosh(K)^4e^{2K}\lambda_2^2\varsigma^2\cosh(K)K^3e^{3k}\lambda_2^2\varsigma^2\cosh(K)K^3\\
&-3360\lambda_2^2\varsigma^2\cosh(K)K^4e^{2K}-1680\lambda_2^2\varsigma^2\cosh(K)K^4e^{2K}\\
&+2464\lambda_1\Lambda_3^3\cosh(K)\lambda_2\varsigma^2K^4e^{2K}+56\lambda_1\Lambda_3^3\cosh(K)^2\lambda_2\varsigma^2K^4e^{2K}\\
&+11424\Lambda_3^3\lambda_1\cosh(K)K^2e^{2K}\lambda_2\varsigma+420\Lambda_3^3\lambda_1\cosh(K)K^4e^{3K}\lambda_2\varsigma\\
&+120960\Lambda_3^3\lambda_1\cosh(K)Ke^{3K}\lambda_2\varsigma\Lambda_3^3\lambda_1\cosh(K)K^2e^{3K}\lambda_2\varsigma\\
&-16800-120960\Lambda_3^3\lambda_2\varsigma e^{K}K\cosh(K)-16800\lambda_1\Lambda_3^3\lambda_2\varsigma e^{K}K^2\cosh(K)\\
&+5460K^4\lambda_2^2\varsigma^2e^{2K}-210K^4\lambda_2^2\varsigma^2e^{4K}-3360\lambda_2^2\varsigma^2K^3e^{2K}\\
&+420\Lambda_3^3\lambda_1\cosh(K)K^4e^{K}\lambda_2\varsigma-3360\lambda_2^2\varsigma^2\cosh(K)K^3e^{K}-210K^4\lambda_2^2\varsigma^2\\
&+645120\lambda_2\varsigma e^{2k}\cosh(K)K^3e^{-K}\lambda_1\Lambda_3^3\cosh(K)\\
&-322560\Lambda_3^3\lambda_1\cosh(K)Ke^{3K}\lambda_2\varsigma-322560\lambda_1\Lambda_3^3\lambda_1\varsigma e^{K}\\
&+3360\lambda_2^2\varsigma^2K^3e^{K})e^{-2k}y)).
\end{aligned}
\tag{31}
$$

4. RESULTS AND DISCUSSION

Figures (1, 2, 5 and 6) showed the dimensionless velocity profile for different non-Newtonian, electro-kinetic, electro kinetic separation distance based on the height of plate and the specific internal

energy parameters. It was discovered that the maximum velocity was attained as the non-Newtonian increases as electro kinetic increases at the point 0.2 it started decreasing as electro kinetic increases. 5 showed velocity profile for different elctro Kinetic separation distance based on the plate height K. At the upper layer, the velocity reduces while at the lower layer, the reverse is the case. This is due to the fact that electro kinetic separation distance affects the movement of the fluid at both layers. It can be seen also that the fluid velocity profile is parabolic in nature with the maximum magnitude along the channel center line and minimum at the walls of the plate. 6 discussed the effect of different specific internal energy. As the specific internal energy increases, the velocity reduces because of the internal heat produced within the fluid flow and hence reduces the motion of the fluid. It can be seen that magnitude along the channel centre line is maximum and minimum at the walls.
Figure (3, 4, 7 shown the temperature distribution for different Joule heating, Brinkman number and specific internal energy it was observed that as these parameters increase the temperature distribution is also increases. This is due to the heat within the flow region.

5. CONCLUSION

This paper presents the Poiseuille flow of a third grade fluid in a horizontal channel. The influence of electro-kinetic, Brinkman number and the internal specific energy on the flow fluid is significant as the electro kinetic parameters retarded the flow while Brinkman number and internal specific energy enhances the temperature field due to the thickness in boundary layer as the parameter increases. The effect of non-Newtonian parameter, the electro kinetic parameter and Joule heating parameter on the velocity and temperature profile are depicted.

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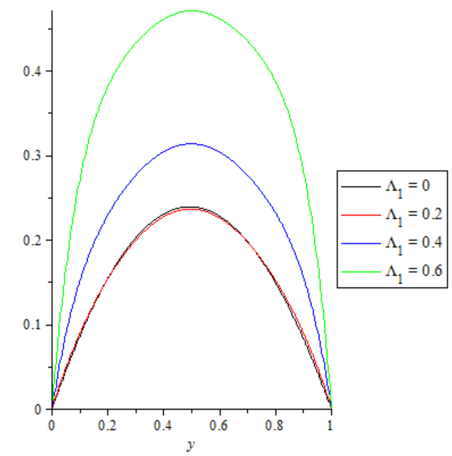

FIGURE 1. Velocity profile for different Non-Newtonian parameter Λ_1, $\varsigma = 1$, $K = 5$, $\Lambda_2 = 0.2$, $\Lambda_3 = -2$

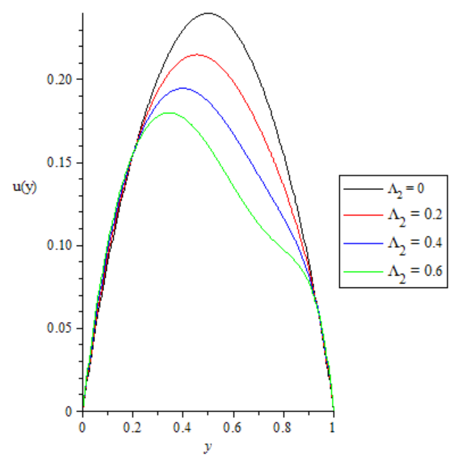

FIGURE 2. Velocity profile for different Electro-Kinetic parameter Λ_2, ($\varsigma = 1$, $K = 5$, $\Lambda_1 = 0.2$, $\Lambda_3 = -2$)

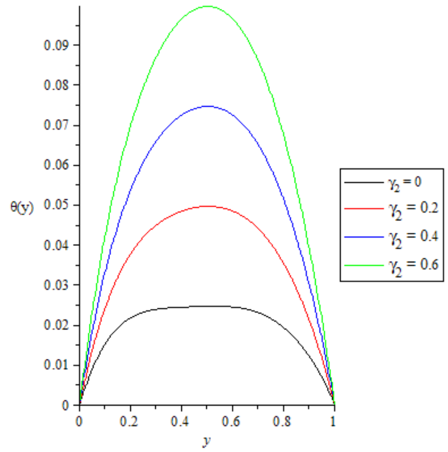

FIGURE 3. Temperature profile for different Joule heating parameter γ_2, ($\varsigma = 1$, $K = 5$, $\Lambda_1 = 0.1$, $\Lambda_2 = 0.2$, $\Lambda_3 = -2$, $\gamma_1 = 2$)

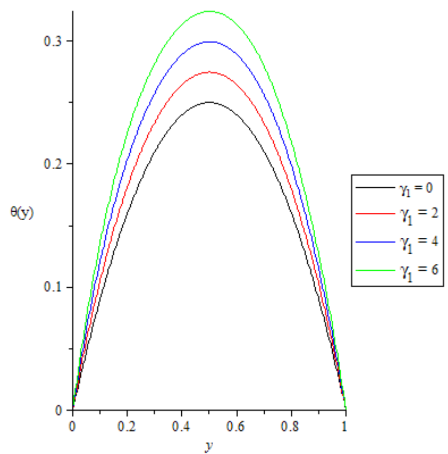

FIGURE 4. Temperature distribution for Brinkman number γ_1, ($\varsigma = 1$, $K = 5$, $\Lambda_1 = 0.2$, $\Lambda_2 = 0.2$, $\Lambda_3 = -2$, γ_2)

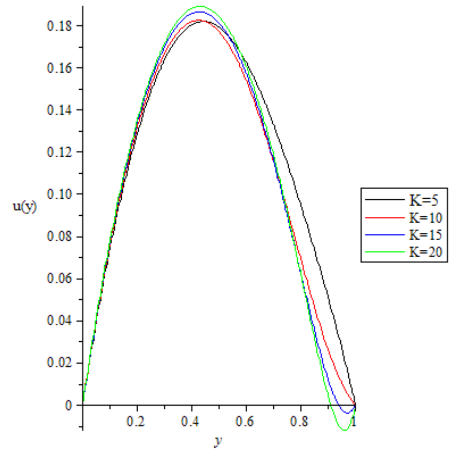

FIGURE 5. Velocity profile for different Electro-Kinetic separation K, ($\varsigma = 1$, $\Lambda_1 = 0.1$, $\Lambda_2 = 0.2$, $\Lambda_3 = -2$)

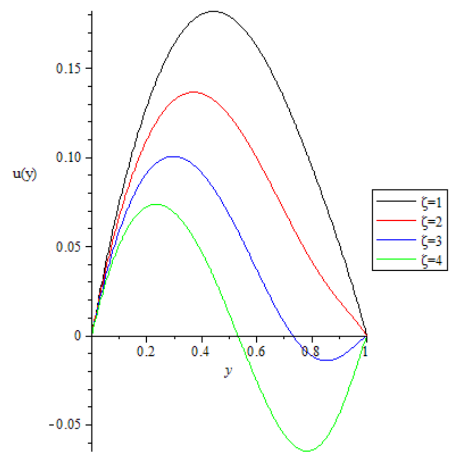

FIGURE 6. Velocity profile for different values of specific internal energy ς, ($K = 5$, $\Lambda_1 = 0.1$, $\Lambda_2 = 0.2$, $\Lambda_3 = -2$, $\gamma_2 = 2$)

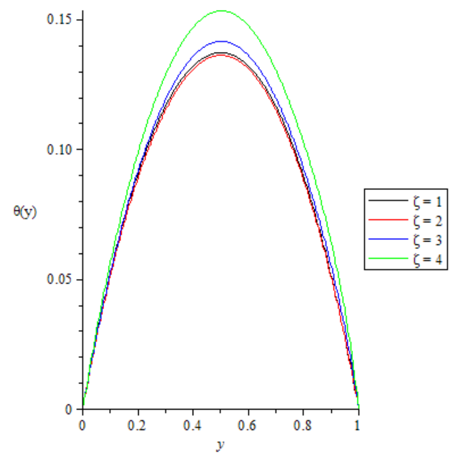

FIGURE 7. Dimensionless Temperature distribution for different values of specific internal energy ς, ($K = 5$, $\Lambda_1 = 0.1$, $\Lambda_2 = 0.2$, $\Lambda_3 = -2$, $\gamma_2 = 2$)