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ADOMIAN DECOMPOSITION METHOD FOR DIRECT INTEGRATION OF BERNOULLI DIFFERENTIAL EQUATIONS

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ABSTRACT. We introduce the basic and less known methodology of Adomian Decomposition Method (ADM) that yields series solutions for differential equations. We then formulate the method to obtain analytic solutions, in a rapidly convergent series, to some class of higher order differential equations. ADM is a type of algorithm applicable to various ordinary or partial differential equations including Bernoulli Differential Equations (BDEs) as proved by the present paper. The results show excellent potentials of applying this method.

Keywords and phrases: Adomian Decomposition Method, Higher Order Differential Equations, Bernoulli Differential Equations (BDEs), Analytic and Approximate Solutions

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1. INTRODUCTION

Bernoulli differential equations (BDE) are nonlinear differential equations named after J. Bernoulli, a Swiss scientist. They are used in modern Physics for modeling the dynamics behind certain circuit elements known as Bernoulli memristors. These types of differential equations are special because they are nonlinear with exact solutions. The equation has a nonlinear term which is a function of the independent variable raise to a certain exponent, say n. When n is zero or one, the BDE is linear. But for $n \geq 2$, substitution is carried out to transform it to a linear form which can then be solved linearly [1] and [2]. In this paper we apply Adomian Decomposition Method (ADM) to solve BDE with $n \geq 2$.

In the 1980's, George Adomian introduced a new method to solve nonlinear functional equations [3]. This method has since been termed the Adomian Decomposition Method (ADM) and has been the subject of

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much investigation and modification [4], [5]. The Adomian Decomposition Method generates a solution in form of a series whose terms are determined by a recursive relation using these Adomian polynomials [6], [7] and [8]. The ADM involves separating appropriately the equation under investigation into linear and nonlinear portions, in such a way that the isolated linear part is easy to be inverted. On the one hand, the linear differential operator representing a linear portion of the equation is inverted (integrated) and the inverse operator is then applied to the equation considering any given conditions. On the other hand, the nonlinear portion is decomposed into a series of Adomian polynomials [9]. The method is satisfactory if the problem has a unique solution.
Consider differential equation of the form

$$y' + P(x)y = g(x)y^n \tag{1}$$

A case where $P(x) = c$, with c a constant, is commonly discussed in literature in the application of ADM, for example (see [10]). In this work, we considered a case for which P is a function of x. Also several articles involve the transformation of BDEs to obtain the analytic solution (see [4] and [11]). The ADM allow us to avoid auxiliary transformations of the dependent variable. In reviewing the basic methodology involved, a general nonlinear differential equation will be used for simplicity.

2. BASIC METHODOLOGY OF ADM

We consider differential equation of the form

$$y^{(m)} = f(x, y, y', \dots, y^{(m-1)}) \tag{2}$$

where I is an interval containing 0 (unless otherwise stated), m a natural number and f a continuous function that is smooth with respect to its last m variable(s).
We denote by L the differentiation (operator) and by L^{-1} its inverse (integration) under 0 initial condition.
Then we re-write equation (2) as

$$L^m y = f(x, y, y', y'', \dots, y^{(m-1)}) \quad x \in I\,, \tag{3}$$

Inverting (integrating) equation (3), we have

$$y \;=\; c_0 + c_1 x^2 + \cdots + c_{m-1}x^{m-1} \;+\; L^{-m}\left(f(x, y, y', y'', \dots, y^{(m-1)})\right) \tag{4}$$

Setting

$$y_0(x) = c_0 + c_1 x^2 + \cdots + c_{m-1}x^{m-1} \tag{5}$$

and

$$N(x, y) \;=\; L^{-m}\left(f(x, y, y', y'', \dots, y^{(m-1)}\right)$$

(4) takes the form

$$y = y_0 + N(x, y). \tag{6}$$

The ADM approach for solving (6) consists of finding a series solution

$$y = y_0 + y_1 + y_2 + y_3 + \ldots + y_n + \ldots$$

basing on the fact that the function $N(x, y)$ admits an adomian decomposition of the form

$$N(x,y) = A_0(x, y_0) + A_1(x, y_0, y_1) + A_2(x, y_0, y_1, y_2) + \ldots$$

$$+ A_n(x, y_0, y_1, y_2, \ldots, y_n) + \ldots$$

and by using the recursive formula :

$$y_{n+1} = A_n(x, y_0, y_1, y_2, \ldots, y_n)\,; \quad \text{for } n = 0, 1, 2, \ldots$$

The formula of A_n is :

$$A_n(x, y_0, y_1, y_2, \ldots, y_n) = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n} N\left(x, \sum_{k=0}^{+\infty}\lambda^k y_k\right)\,; \quad \text{for } n = 0, 1, 2, \ldots$$

For instance, when g is an infinitely many times differentiable function, then its ADM decomposition is

$$g(y) = \sum_{n=0}^{+\infty} A_n(y_0, y_1, y_2, \ldots, y_n)$$

with $y = \sum_{n=0}^{+\infty} y_n$ and

$$\begin{aligned}
A_0 &= g(y_0)\\
A_1 &= g'(y_0)y_1\\
A_2 &= g'(y_0)y_2 + g''(y_0)\tfrac{y_1^2}{2!}\\
A_3 &= g'(y_0)y_3 + g''(y_0)y_1y_2 + g^{(3)}(y_0)\tfrac{y_1^3}{3!}\\
A_4 &= g'(y_0)y_4 + g''(y_0)\left(\tfrac{y_2^2}{2!} + y_1y_3\right) + g^{(3)}(y_0)\tfrac{y_1^2y_2}{2!} + g^{(4)}(y_0)\tfrac{y_1^4}{4!}
\end{aligned} \tag{7}$$

A sufficient condition for the convergence of the ADM decomposition is provided in [12].

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For an instance, if

$$y^{(m)} = y^p \tag{8}$$

then

$$y = c_0 + c_1 x + \frac{c_2 x^2}{2!} + \cdots + \frac{c_{m-1} x^{m-1}}{(m-1)!} + L^{-m}(y^p) \tag{9}$$

where

$$y_0 = c_0 + c_1 x + \frac{c_2 x^2}{2!} + \cdots + \frac{c_{m-1} x^{m-1}}{(m-1)!}$$

ADM implies :

$$y_{n+1} = A_n = L^{-1}(B_n(y_0, y_1, y_2, ..., y_n))$$

where $B_n(y_0, y_1, y_2, ..., y_n)$ is the n^{th} component of the Adomian Decomposition of y^p, see formula (7). Therefore :

$$y_1 = L^{-1}(y_0^p).$$

$$y_2 = L^{-1}(p y_0^{p-1} y_1) = L^{-1}(p y_0^{p-1}(l^{-1}(y_0^p))).$$

$$\begin{aligned} y_3 &= L^{-1}\left[\tfrac{p(p-1)}{2!} y_0^{p-2} y_1^2 + p y_0^{p-1} y_2\right] \\ &= L^{-1}\left[\tfrac{p(p-1)}{2!} y_0^{p-2} (L^{-1}(y_0^p))^2 + p y_0^{p-1}(L^{-1}(p y_0^{p-1}(L^{-1}(y_0^p))))\right]. \end{aligned}$$

$$y_4 = L^{-1}\left[\frac{p(p-1)(p-2)}{6} y_0^{p-3} y_1^3 + p(p-1) y_0^{p-2} y_1 y_2 + p y_0^{p-1} y_3\right].$$

$\cdots$

3. APPLICATION OF ADM TO OBTAIN SOLUTIONS OF $y' = x^q y^p$

Let

$$y' = x^q y^p \,. \tag{10}$$

That is

$$L\, y = x^q y^p$$

which implies

$$y = c + L^{-1}\,(x^q y^p) \tag{11}$$

where the anti-differentiation L^{-1} is appropriately defined according to the sign of q. In fact we shall consider the cases in which L^{-1} is the integration under 0 initial condition, at 0 if $q \geq 0$ but at 1 if $q < 0$.
Here

$$y_0 = c$$

and ADM implies :

$$y_{n+1} = A_n = L^{-1}(x^q B_n(y_0, y_1, y_2, ..., y_n))$$

where $B_n(y_0, y_1, y_2, ..., y_n)$ is the n^{th} component of the Adomian Decomposition of y^p. Therefore :

$$
\begin{aligned}
&y_0 = c\\
&y_1 = L^{-1}\,(x^q y_0^p)\\
&y_2 = L^{-1}(x^q(py_o^{p-1}y_1)) = L^{-1}(x^q(py_o^{p-1}L^{-1}(x^q y_0^p)))\\
&y_3 = L^{-1}(x^q(\frac{p(p-1)}{2}y_0^{p-2}(y_1)^2 + py_0^{p-1}y_2)\\
&= L^{-1}(x^q(\frac{p(p-1)}{2}y_0^{p-2}(L^{-1}(x^q y_0^p))^2 + py_0^{p-1}L^{-1}(x^q(y_o^{p-1}L^{-1}(x^q y_0^p))))\\
&y_4 = L^{-1}(x^q(\frac{p(p-1)(p-2)}{6}y_0^{p-3}(y_1)^3\\
&\qquad + p(p-1)y^{p-2}(y_1)(y_2) + py_0^{p-1}y_3)\\
&= L^{-1}(x^q(\frac{p(p-1)(p-2)}{6}y_0^{p-3}(L^{-1}(x^q y_0^p))^3\\
&\qquad + p(p-1)y^{p-2}((L^{-1}(x^q y_0^p))(L^{-1}(x^p(py_0^{p-1}l^{-1}(x^q y_0^p)))))))\\
&\qquad + py_0^{p-1}L^{-1}(x^q(\frac{p(p-1)}{2}y_0^{p-2}(L^{-1}(x^q y_0^p))^2\\
&\qquad + py_0^{p-1}L^{-1}(x^q(y_o^{p-1}L^{-1}(x^q y_0^p))))).
\end{aligned}
\tag{12}
$$

$$\ldots \tag{13}$$

$$y = y_0 + y_1 + y_2 + y_3 + y_4 + \ldots$$

4. IMPLEMENTATION STRATEGIES

Example 1. We consider solution BDEs of the form (5) for

(i) $q = p = 1$, (ii) $q = 1$ and $p = -1$, (iii) $q = -1$ and $p = 1$, and for (iv) $q = p = -1$

(i) $y' = xy,\ \ y(0) = e = e^1$. Exact solution $y = e^{\frac{x^2}{2}+1},\ \ x \in \mathbb{R}$.
Besides, According to ADM, using equation (11)
$y_0 = c = e^1$, $\quad A_n = L^{-1}(xB_n)$ with $B_n = y_n$ and L^{-1} as the integration under 0 initial condition at 0. In this way
$y_{n+1} = L^{-1}(xy_n)$. Thus
$y_1 = L^{-1}(xy_0) = e^1(\frac{x^2}{2})$.
$y_2 = L^{-1}(x(L^{-1}(xy_0))) = e^1(\frac{x^4}{8})$.
$y_3 = L^{-1}(L^{-1}(x(x(L^{-1}(xy_0))))) = e^1(\frac{x^6}{48})$.
$y_4 = L^{-1}(x(L^{-1}(x(L^{-1}(x(L^{-1}(xy_0)))))) = e^1(\frac{x^8}{384})$.
$y_5 = e^1(\frac{1}{3840}x^{10})$.
...
So $\ y = e^1 + e^1(\frac{x^2}{2}) + e^1(\frac{x^4}{8}) + e^1(\frac{x^6}{48}) + e^1(\frac{x^8}{384}) + e^1(\frac{1}{3840}x^{10}) + \ldots$

Now we assume, by mathematical induction, that for some n,

$$y_n = e^1\left(\frac{x^{2n}}{2^n n!}\right).$$

Let us compute for y_{n+1}. We have
$y_{n+1} = e^1 L^{-1}((\frac{x^{2n}}{2^n n!})) = e^1(\frac{x^{2(n+1)}}{2^{n+1}(n+1)!})$.
Then we conclude that for every n, $y_n = e^1(\frac{x^{2n}}{2^n n!}) = e^1 \times \frac{1}{n!}\left(\frac{x^2}{2}\right)^n$
Therefore $y = e^1 \sum\limits_{n=0}^{\infty} \frac{x^{2n}}{2^n n!} = e^1(e^{\frac{x^2}{2}}) = e^{\frac{x^2}{2}+1}$.

(ii) $y' = xy^{-1}$, $y(0) = 1$. Exact solution $y = (1+x^2)^{\frac{1}{2}}$, $x \in \mathbb{R}$
Using equation (11), we have
$y_0 = c = 1$.
$y_1 = L^{-1}(xy_0^{-1}) = \frac{1}{2}x^2$ where L^{-1} is the integration under 0 initial condition at 0.
$y_2 = L^{-1}(x(-(L^{-1}(xy_0^{-1})))) = L^{-1}\left(\frac{x^2}{2}(-x)\right) = -\frac{1}{8}x^4$.
$y_3 = L^{-1}(x(-(L^{-1}(xy_0^{-1}))^2 - (L^{-1}x(-(L^{-1}(xy_0^{-1})))))) = L^{-1}(\frac{3x^5}{8}) = \frac{1}{16}x^6$.

$$\begin{aligned} y_4 &= L^{-1}(x(-(L^{-1}(xy_0^{-1}))^3 - 2((L^{-1}(xy_0^{-1}))(L^{-1}(x(L^{-1}(xy_0^{-1}))))) \\ &\quad - L^{-1}(x(-(L^{-1}(xy_0^{-1}))^2 + L^{-1}(x(-(L^{-1}(xy_0^{-1}))))) \\ &= \frac{5}{128}x^8. \end{aligned}$$

$y_5 = \frac{7x^{10}}{256}$.
...
Thus $y = 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6 + \frac{5}{128}x^8 + \frac{7}{256}x^{10} + \dots$.
Using an appropriate mathematical induction principle as in example 1(i) above, we have
$y_n = \begin{pmatrix} 1/2 \\ n \end{pmatrix} x^{2n}$
and so

$$y = \sum_{n=0}^{\infty} \begin{pmatrix} 1/2 \\ n \end{pmatrix} x^{2n} = \sqrt{1+x^2}$$

(iii) $y' = x^{-1}y$, $y(1) = 1$.
Exact solution $y = x$, $x \geq 1$.
Using equation (11), we have
$y_0 = c = 1$.
$y_1 = L^{-1}(x^{-1}y_0) = \ln x$. Note that here L^{-1} is the integration under 0 initial condition at 1.
$y_2 = L^{-1}(x^{-1}(L^{-1}(x^{-1}y_0))) = \frac{1}{2}(\ln x)^2$.
$y_3 = L^{-1}(x^{-1}(L^{-1}(x^{-1}(L^{-1}(x^{-1}y_0))) = \frac{1}{6}(\ln x)^3$.

$y_4 = L^{-1}(x^{-1}(L^{-1}(x^{-1}(L^{-1}(x^{-1}(L^{-1}(x^{-1}y_0))))))) = \frac{1}{24}(\ln x)^4.$
$y_5 = \frac{(\ln x)^5}{120}.$
...

Using an appropriate mathematical induction principle as in example 1(i) above we have
$y_n = \frac{(\ln x)^n}{n!}.$

Thus $y = 1 + \ln(x) + \frac{(\ln x)^2}{2} + \frac{(\ln x)^3}{6} + \frac{(\ln x)^4}{24} + \cdots + \frac{(\ln x)^n}{n!} + \ldots$
That is $y = \sum_{n=0}^{\infty} \frac{\left(\ln x\right)^n}{n!} = e^{(\ln x)}$
and so $y = x$.

(iv) $y' = x^{-1}y^{-1}, \quad y(1) = 1.$
Exact solution $y = (1 + 2\ln x)^{\frac{1}{2}}, \; x \geq 1.$
Following (11), to solve
$y = c + L^{-1}(x^{-1}y^{-1})$ with $y_0 = c = 1$, where L^{-1} holds for the operator that assigns to a continuous function defined at 1, its anti-derivative that vanishes at 1, we have :
$y_1 = L^{-1}(x^{-1}y_0^{-1}) = \ln x.$
$y_2 = L^{-1}(x^{-1}(L^{-1}(x^{-1}y_0^{-1}))) = L^{-1}\left(\frac{1}{x}(-\ln x)\right) = -\frac{1}{2}(\ln x)^2.$

$$\begin{aligned} y_3 &= L^{-1}(x^{-1}(-(L^{-1}(x^{-1}y_0^{-1}))^2 - (L^{-1}x^{-1} - (L^{-1}(x^{-1}y_0^{-1}))))) \\ &= L^{-1}\left(\tfrac{1}{x}(\tfrac{3}{2}(\ln x)^2)\right) = \tfrac{1}{2}(\ln x)^3. \end{aligned}$$

$$\begin{aligned} y_4 &= L^{-1}(x(-(L^{-1}(x^{-1}y_0^{-1}))^3 - 2((L^{-1}(x^{-1}y_0^{-1}))(L^{-1}(x(L^{-1}(x^{-1}y_0^{-1}))))) \\ &\quad - L^{-1}(x(-(L^{-1}(x^{-1}y_0^{-1}))^2 + L^{-1}(x(-(L^{-1}(x^{-1}y_0^{-1}))))) \\ &= -\tfrac{5}{8}(\ln x)^4. \end{aligned}$$

$y_5 = \frac{7}{8}(\ln x)^5.$
...
Using an appropriate mathematical induction principle as in example 1(i) above we have
$y_n = \begin{pmatrix} 1/2 \\ n \end{pmatrix} (\ln x)^n.$

Thus $y = 1 + \ln(x) - \frac{(\ln x)^2}{2} + \frac{(\ln x)^3}{2} - \frac{5(\ln x)^4}{4} + \frac{7(\ln x)^5}{8} + \ldots$
$y = \sum_{k=0}^{\infty} \begin{pmatrix} 1/2 \\ k \end{pmatrix} (\ln x)^k$;
That is $y = (1 + \ln(x))^{\frac{1}{2}}.$

Example 2. $y' + \frac{y}{x} = y^2, \quad y(1) = 1.$
Exact solution $y = \frac{1}{x}(1 - \ln x)^{-1}$ for $1 \leq x < e$.

Firstly we shall transform this equation in order to facilitate the Adomian decomposition. For $x \neq 0$, we have

$$y' + \frac{y}{x} = y^2 \iff xy' + y = xy^2$$
$$\iff \frac{d}{dx}\Big(xy\Big) = xy^2 .$$

Thus $y' + \frac{y}{x} = y^2 \iff xy = c + L^{-1}\left(xy^2\right);$ where L^{-1} is the integration under 0 initial condition at 1. In this way, our problem is equivalent to

$y = \frac{c}{x} + \frac{1}{x}L^{-1}\left(xy^2\right)$. By ADM

$y_0 = \frac{c}{x}$.

$y_{n+1} = A_n = \frac{1}{x}L^{-1}(xB_n);$ where $B_n = B_n(y_0, y_1, y_2, ..., y_n)$ is the n^{th} component of the Adomian Decomposition of y^2. Therefore:

$y_1 = \frac{1}{x}L^{-1}(x(y_0^2)) = \frac{1}{x}L^{-1}(\frac{c^2}{x}) = c^2 \frac{\ln x}{x}$.

$$\begin{aligned} y_2 &= \tfrac{1}{x}L^{-1}(x(2y_0^1 y_1^1)) \\ &= \tfrac{1}{x}L^{-1}(x(2(\tfrac{c}{x})^1(c^2 \tfrac{\ln x}{x}))) \\ y_2 &= c^3 \tfrac{(\ln x)^2}{x}. \end{aligned}$$

$$\begin{aligned} y_3 &= \tfrac{1}{x}L^{-1}(x(2y_0 y_2 + y_1^2)) \\ &= \tfrac{1}{x}L^{-1}(x(2(\tfrac{c}{x})^1(c^3 \tfrac{\ln^2 x}{x}))) \\ y_3 &= c^4 \tfrac{\ln^3 x}{x}. \end{aligned}$$

$$\begin{aligned} y_4 &= \tfrac{1}{x}L^{-1}(x(2y_0 y_3 + 2y_1 y_2)) \\ &= \tfrac{1}{x}L^{-1}\left(4c^5 \tfrac{\ln^3 x}{x}\right) \\ y_4 &= c^5 \tfrac{\ln^4 x}{x}. \end{aligned}$$

...

Using an appropriate mathematical induction principle as in example 1(i) above we have

$y_n = c^n \frac{(\ln x)^n}{x}$.

It follows that

$$\begin{aligned} y &= \frac{1}{c\,x} + \frac{\ln x}{c^2\,x} + \frac{(\ln x)^3}{c^4\,x} + \frac{(\ln x)^4}{c^5\,x} + \frac{(\ln x)^5}{c^6\,x} + \frac{(\ln x)^6}{c^7\,x} + \cdots + c^n \frac{(\ln x)^n}{x} + \ldots \\ &= \frac{c}{x} \sum_{n=0}^{\infty} (\ln x)^n . \end{aligned}$$

For $1 \leq x < e$, we have

$$y = \frac{c}{x(1-\ln x)}$$

and since $y(1) = 1$, there holds

$$y = \frac{1}{x(1-\ln x)}, \qquad 1 \leq x < e\,.$$

Example 3. $y' - \frac{y}{x} = xy^2, \quad y(1) = \frac{3}{2}$.
Exact solution $y = x\left(1 - \frac{x^3}{3}\right)^{-1}, \quad 0 < x < \sqrt[3]{3}$.
$y' = xy^2 + \frac{y}{x}$.
Like in the previous example, we have for all $x \neq 0$:

$$\begin{aligned} y' - \tfrac{y}{x} = xy^2 &\iff \tfrac{y'}{x} - \tfrac{y}{x^2} = y^2 \\ &\iff \tfrac{d}{dx}\left(\tfrac{y}{x}\right) = y^2 \\ &\iff \tfrac{y}{x} = c + L^{-1}(y^2) \\ &\iff y = cx + xL^{-1}(y^2)\,. \end{aligned}$$

where L^{-1} is the integration under 0 initial condition at 1. In this way, our problem is equivalent to
$y = cx + xL^{-1}(y^2)$.
$y_0 = c\,x$.

$$\begin{aligned} y_1 = xL^{-1}(y_0^2) &= xL^{-1}(c^2x^2) \\ &= c\int_1^x \left(c^2t^2\right)\,dt \\ &= cx\,\tfrac{c(x^3-1)}{3}\,. \end{aligned}$$

$$\begin{aligned} y_2 = xL^{-1}(2y_0y_1) &= x\int_1^x \left(2(ct)^2\ \tfrac{c(t^3-1)}{3}\right)\,dt \\ &= cx\int_1^x 2ct^2\ \tfrac{c(t^3-1)}{3}\,dt \\ &= cx\left[\left(\tfrac{c(t^3-1)}{3}\right)^2\right]_1^x \\ &= cx\left(\tfrac{c(x^3-1)}{3}\right)^2\,. \end{aligned}$$

$$
\begin{aligned}
y_3 = xL^{-1}(2y_0y_2 + y_1^2) &= x\int_1^x \left[2(ct)^2\left(\tfrac{c(t^3-1)}{3}\right)^2 + \left(ct\,\tfrac{c(t^3-1)}{3}\right)^2\right] dt \\
&= cx\int_1^x \left[3ct^2\left(\tfrac{c(t^3-1)}{3}\right)^2\right] dt \\
&= cx\left(\tfrac{c(x^3-1)}{3}\right)^3 .
\end{aligned}
$$

$$
\begin{aligned}
y_4 = xL^{-1}(2y_0y_3 + 2y_1y_2) &= x\int_1^x \Bigg[2(ct)^2\left(\tfrac{c(t^3-1)}{3}\right)^3 \\
&+ \quad 2\left(ct\,\tfrac{c(t^3-1)}{3}\right)ct\left(\tfrac{c(t^3-1)}{3}\right)^2\Bigg] dt \\
&= cx\int_1^x \left[4ct^2\left(\tfrac{c(t^3-1)}{3}\right)^3\right] dt \\
&= cx\left(\tfrac{c(x^3-1)}{3}\right)^4 .
\end{aligned}
$$

Using an appropriate mathematical induction principle as in example 1(i) above we have for every nonnegative integer n :

$$
y_n = cx\left(\frac{c(x^3-1)}{3}\right)^n .
$$

Thus in a neighbourhood of 1 we have

$$
y = \sum_{n=0}^{\infty} cx\left(\frac{c(x^3-1)}{3}\right)^n .
$$

Thus if $x > 0$ and $|c(x^3-1)| < 3$, we have

$$
y = cx\frac{1}{1-\frac{c(x^3-1)}{3}}.
$$

According to the initial condition $y(1) = \frac{3}{2}$, we have $c = \frac{3}{2}$ and so

$$
y = \frac{x}{1-\frac{x^3}{3}} \qquad \text{for} \quad 0 < x < \sqrt[3]{3}.
$$

4. CONCLUDING REMARKS

In this work we have presented the basic methodology of the ADM. The method was formulated for solutions of variable index BDEs in term of the given condition. We have successfully applied Adomian Decomposition Method to Bernoulli differential equations. We have given the general Adomian polynomial for the nonlinear term in the Bernoulli differential equations and applied it to pratical problems. Some implementation strategies for positive and negative index of the nonlinear term are considered. The possibilities of obtaining analytic solution of the used examples shows the efficiency of the method. In fact the result obtained by Adomian Decomposition Method from each Bernoulli differential equation (BDE) were exactly the same as those of the analytical solutions. Adomian Decomposition Method gives the exact solutions directly without the usual transformation processes that take place when using the analytical method to find the solutions to the BDEs.

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