Journal of the Nigerian Mathematical Society
Vol. 35, pp. 532-545, 2016

COMPLETE SYNCHRONIZATION OF 5D HYPERCHAOTIC SYSTEM

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ABSTRACT. In this paper, we present the synchronization of a 5D hyperchaotic system,In this case, the dimension of the phase space that embeds the chaotic system is five, which will require the minimum number of coupled first order autonomous differential equations to be five which is a more complex system when compared with the 4-D system. More complex attractors and randomness displayed by the system make the embedded synchronized information difficult to be intruded and we believe that this will create more construction variations for error space vectors because of the higher number of variables present. We demonstrate the realization of complete synchronization of this 5-Dimensional hyperchaotic system. Using the active backstepping technique, the usual master-slave synchronization scheme for low order chaotic systems is extended to study the synchronization of higher order systems. Our numerical results confirm the effectiveness of the proposed analytical technique. This proposal was achieved, and we believe that the result will be useful in ensuring better security when applied in communication and encryption of information.

Keywords: Complete, Synchronization, 5D Hyperchaotic system

1. INTRODUCTION

Researchers have found that Nonlinear deterministic dynamical systems exhibits sensitive dependence on initial conditions. To this end, different methods have been employed to describe their existence in the fields of sciences, medicine and engineering Strogatz (2000) [1]. Various attributes of nonlinear dynamical systems such as chaos, bifurcation, multistability, pattern formation and synchronization have been found very useful in many disciplines.

As noted by Vincent et al (2015), synchronization of chaotic and hyperchaotic systems has been referred to as a major breakthrough [2] and one of the most important attributes of nonlinear dynamical systems. This is because of its potential applications in modelling

Received by the editors July 14, 2015; Accepted: August 01, 2016
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brain activities, chemical reactions and more importantly in information processing and secure communication. Increasing interest in the study of synchronization of chaotic systems has led to the discovery of various types of synchronization which include complete synchronization [3], phase synchronization [4], lag synchronization [5], generalized synchronization [6], [7], measure synchronization [8],[9] and [10], projective synchronization [11], [12], and [13], anticipated synchronization [14],[15], reduced-order synchronization [16] and function projective [17].

Several methods of achieving synchronization between two or more non-linear systems have been proposed and well developed.These include, the adaptive control [18], active control and robust synchronization [19], impulsive control [20], adaptive fuzzy feedback [21], sliding mode control [22] and backstepping technique [23]. Backstepping technique has been shown to exhibit outstanding performance in the synchronization of identical and non-identical chaotic systems, stabilization and tracking [24] and controlling of hyperchaotic systems [25], and useful in either the strict feedback or the non-strict feedback systems [26].

The temporal complexity and apparent randomness of chaotic systems is the most important characteristics of chaos [27]. So, the primary motivation of synchronization is that, one can hide certain electronic information to be transmitted in chaotic signal and retrieve by the technique of chaotic synchronization.

However, Meng et al [27] opined that absolute security of information and communication based on low-dimensional chaotic [2] and hyperchaotic [28] systems, cannot be fully guaranteed. This is because it can be reconstructed easily and separated from the secure information. Hence, concerted efforts were made to generate higher dimensional systems. In this direction, 4-Dimensional systems have been well studied, most of which are known to exhibit instability in two directions implying that the possibility for the existence of two positive Lyapunov exponents is ascertained. However, higher dimensional systems, with dimension greater than four are less investigated.

In this paper, therefore, we present the synchronization of a 5D hyperchaotic system coined by Yang and Chen (2013) [29]. In this case, the dimension of the phase space that embeds the chaotic system is five, which will require the minimum number of coupled first order autonomous differential equations to be five. The system was coined to have three Lyapunov exponent (number of terms in the coupled equations giving rise to instability). All the three having non-linear functions. By the description of hyperchaos, as given by Gao et al [30], this is a more complex system when compared with the 4-D system described by Yuxia et al [31].

2. SYSTEM DESCRIPTION

We consider the following new 5D hyperchaotic system reported by Yang and Chen (2013) [29].

$$\begin{aligned} \dot{x} &= a(y-x)+p, \\ \dot{y} &= cx-xz+w, \\ \dot{z} &= -bz+xy, \\ \dot{p} &= -hp-xz, \\ \dot{w} &= -k_1x-k_2y, \end{aligned} \tag{1}$$

where a, b, $h \neq 0$ a,b and c are the system parameters and h, k_1, k_2 are three control parameters, determining the chaotic and hyperchaotic behaviors of the system. The $5D$ hyperchaotic system (3) has five Lyapunov exponents, three of which are positive Lyapunov exponents for a given set of system parameters. The attractors of the system at the states xy, xz, xp, xw, yz, yp, yw and zp for $a = 10$, $b = 8/3$, $c = 28$, $h = 2.25$, $k_1 = -0.12$, $k_2 = 11.3$ are as shown in figure 1.

3. DEFINITION AND FORMULATION

Let us consider the following master-slave n dimensional chaotic systems, where the master systems is given by

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, x_3, ..., x_n), \\ \dot{x}_2 &= f_2(x_1, x_2, x_3, ..., x_n), \\ \dot{x}_n &= f_n(x_1, x_2, x_3, ..., x_n), \end{aligned} \tag{2}$$

and the controlled slave system is given by

$$\begin{aligned} \dot{y}_1 &= g_1(x_1, x_2, x_3, ..., x_m) + u_1, \\ \dot{y}_2 &= g_2(x_1, x_2, x_3, ..., x_m) + u_2, \\ \dot{y}_m &= g_m(x_1, x_2, x_3, ..., x_m) + u_m, \end{aligned} \tag{3}$$

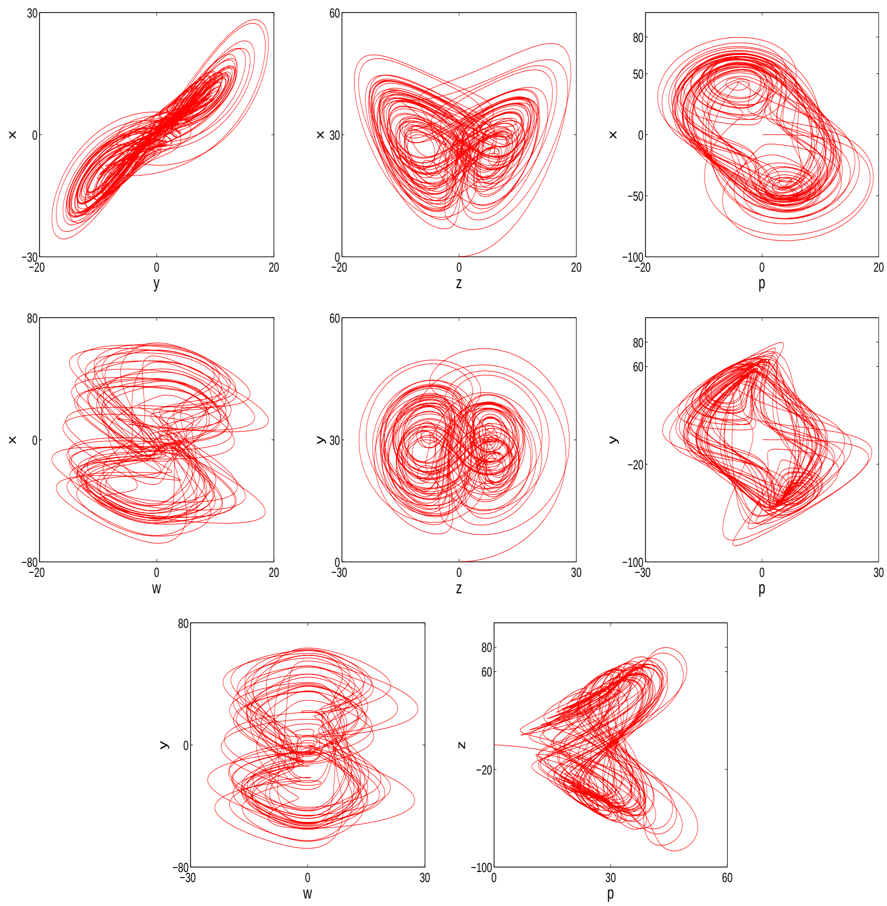

FIGURE 1. The phase portraits of the $5D$ system in different planes $x-y$, $x-z$, $x-p$, $x-w$, $y-z$, $y-p$, $y-w$ and $z-p$ showing possible planes in which synchronization could take place for $a=10$, $b=8/3$, $c=28$, $h=2.25$, $k_1=-0.12$, $k_2=11.3$ are shown respectively

where x_i, y_i $(i=1,2,...n)\ \varepsilon\ R^n$ are state space variables of the systems, f_n and $g_m : R^n \to R^n$ are continuous nonlinear functions and u_i $(i=1,2,...n) : R^m \to R^m$ is a nonlinear control function.

Definition 1: *If there exists two constants matrices* A,B ε R^n *and* A, $B \neq 0$, *such that* $\lim_{\infty} \|By_i - Ax_i\| = 0$ *where* $\|.\|$ *is the matrix norm and* A,B *are scaling matrices, then systems* (2) *and* (3) *are said to be in complete synchronization.*
Comment 1: *The error states in relation to the definition* 1 *are*

strictly chosen to satisfy the definition e_{ij} $(i = j = 1, 2, 3...n)$; *where* i, j *are the indices of the error and* n *refers to the number of dimensions of the chaotic system.* In this work, we consicered the case $(i = j = 5)$ based on our introduction.

4. DESIGN OF CONTROLLERS FOR 5-D HYPERCHAOTIC SYSTEM

Let us redefine the variables of system (1) as follows, $x = x_1$, $y = x_2$, $z = x_3$, $p = x_4$ *and* $w = x_5$ for the master system and $x = y_1$, $y = y_2$, $z = y_3$, $p = y_4$ *and* $w = y_5$ for the slave system. Thus, for the five dimensional system [29], let the master system be

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4, \\ \dot{x}_2 &= cx_1 - x_1x_3 + x_5, \\ \dot{x}_3 &= -bx_3 + x_1x_2, \\ \dot{x}_4 &= -hx_4 - x_1x_3, \\ \dot{x}_5 &= -k_1x_1 - k_2x_2, \end{aligned} \tag{4}$$

and

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1,) + y_4 + u_1 \\ \dot{y}_2 &= cy_1 - y_1y_3 + y_5 + u_2, \\ \dot{y}_3 &= -by_3 + y_1y_2 + u_3, \\ \dot{y}_4 &= -hy_4 - y_1y_3 + u_4, \\ \dot{y}_5 &= -k_1y_1 - k_2y_2 + u_5, \end{aligned} \tag{5}$$

the slave system, where u_1, u_2, u_3, u_4 and u_5 are the set of nonlinear controllers. The error dynamics are chosen as: e_{11}=y_1-x_1, e_{22}=y_2-x_2, e_{33}=y_3-x_3, e_{44}=y_4-x_4 and e_{55}=y_5-x_5. Using these notations and differentiating the error dynamics, we have

$$\begin{aligned} \dot{e}_{11} &= a(e_{22} - e_{11}) + e_{44} + u_1, \\ \dot{e}_{22} &= ce_{11} - e_{11}e_{33} - x_1e_{33} - x_3e_{11} + e_{55} + u_2, \\ \dot{e}_{33} &= -be_{33} + e_{11}e_{22} + x_{11}e_{22} + x_2e_{11} + u_3, \\ \dot{e}_{44} &= -he_{44} - e_{11}e_{33} - x_1e_{33} - x_3e_{11} + u_4, \\ \dot{e}_{55} &= -k_1e_{11} - k_2e_{22} + u_5. \end{aligned} \tag{6}$$

With error dynamics (6), if appropriate u_1, u_2, u_3, u_4 and u_5 are chosen, such that the system is stable and unchanged, then the asymptotic stabilization would be realized leading to globally stable synchronization of the system. If $z_1 = e_{11}$, its time derivative is

$\dot{z}_1 = \dot{e}_{11}$ and we can write the first part of (6) as

$$\dot{z}_1 = a\left(e_{22} - e_{11}\right) + e_{44} + u_1. \tag{7}$$

. We can stabilize (7) using the Lyapunov function

$$v_1 = \frac{1}{2}z_1^2. \tag{8}$$

By substituting for $\dot{z}_1$ in the derivative of (8) and choosing $e_{22} = \alpha_1\left(z_1\right)$ as a virtual controller, we have $u_1 = -e_{44}$, so that

$$\dot{v}_1 = -az_1^2 \leq 0. \tag{9}$$

Since the error between e_{22} and $\alpha_1\left(z_1\right)$ is estimative as $z_2 = e_{22}$-$\alpha_1\left(z_1\right)$ and $\alpha_1\left(z_1\right) = 0$, we can write the (z_1, z_2) subsystem as

$$\begin{aligned} \dot{z}_1 &= -a(z_2 - z_1), \\ \dot{z}_2 &= cz_1 - z_1e_{33} - x_1e_{33} - x_3z_1 + e_{55} + u_2. \end{aligned} \tag{10}$$

We stabilize the second part of equation (10) by describing the second Lyapunov function given as

$$v_2 = v_1 + \frac{1}{2}z_2^2, \tag{11}$$

By substituting for $\dot{z}_2$ in the derivative of (11) and choosing $e_{33} = \alpha_2\left(z_2\right)$ as a virtual controller choosing $u_2 = -z_2 - cz_1 + z_1x_3 - e_{55}$, we have

$$\dot{v}_2 = -az_1^2 - z_2^2 \leq 0. \tag{12}$$

Thus, the (z_1, z_2) subsystem is negative definite and assymptotically stable. Since the error between e_{33} and $\alpha_2\left(z_2\right)$ is estimative as $z_3 = e_{33}$-$\alpha_2\left(z_2\right)$ and $\alpha_2\left(z_2\right) = 0$, let

$$z_3 = e_{33}. \tag{13}$$

and we can write the (z_1, z_2, z_3) subsystem as

$$\begin{aligned} \dot{z}_1 &= -a(z_2 - z_1), \\ \dot{z}_2 &= -z_2 - z_1z_3, \\ \dot{z}_3 &= -bz_3 + e_{11}z_2 + x_1z_2 + x_2e_{11} + u_3, \end{aligned} \tag{14}$$

and stabilize (14) by defining the third Lyapunov function given as

$$v_3 = v_2 + \frac{1}{2}z_3^2. \tag{15}$$

By substituting for $\dot{z}_3$ in the derivative of (15) choosing $e_{11} = \alpha_3(z_3) = 0$ as a virtual controller and choosing $u_3 = -x_1 z_2 - x_2 z_1$,we have

$$\dot{v}_3 = -az_1^2 - z_2^2 - bz_3^2 \leq 0. \tag{16}$$

Thus, the (z_1, z_2, z_3) subsystem is assymptotically stable.Let $e_{44} = z_4$ and its derivative $\dot{e}_{44} = \dot{z}_4$ we can write the (z_1, z_2, z_3, z_4) subsystem as

$$\begin{aligned} \dot{z}_1 &= a(z_2 - z_1), \\ \dot{z}_2 &= -z_2 - z_1 z_3, \\ \dot{z}_3 &= -bz_3 + z_1 z_2, \\ \dot{z}_4 &= -hz_4 - z_1 z_3 - x_1 z_3 - x_3 z_1 + u_4.. \end{aligned} \tag{17}$$

We can stabilize (17) by defining the fourth Lyapunov function given as

$$v_4 = v_3 + \frac{1}{2} z_4^2. \tag{18}$$

By substituting for $(\dot{v}_3)$ and $(\dot{z}_4)$ in the derivative of (18) and choosing $u_4 = z_1 z_3 + x_1 z_3 + x_3 z_1$, we have

$$\dot{v}_4 = -az_1^2 - z_2^2 - bz_3^2 - hz_4^2 \leq 0. \tag{19}$$

Thus, the (z_1, z_2, z_3, z_4) subsystem is assymptotically stable. Let $z_5 = e_{55}$, we can write the $(z_1, z_2, z_3, z_4, z_5)$ system as

$$\begin{aligned} \dot{z}_1 &= a(z_2 - z_1), \\ \dot{z}_2 &= -z_2 - z_1 z_3, \\ \dot{z}_3 &= -bz_3 + z_1 z_2, \\ \dot{z}_4 &= -hz_4, \\ \dot{z}_5 &= -k_1 z_1 - k_2 z_2 + u_5,, \end{aligned} \tag{20}$$

and stabilize (20) by defining the fifth Lyapunov function given as

$$v_5 = v_4 + \frac{1}{2} z_5^2. \tag{21}$$

By susbstituting for $\dot{v}_4$ and $\dot{z}_5$ in the derivative of (21) and choosing $u_5 = -z_5 + k_1 z_1 + k_2 z_2$, we have

$$\dot{v}_5 = -az_1^2 - z_2^2 - bz_3^2 - hz_4^2 - z_5^2 \leq 0. \tag{22}$$

Thus, the whole system is assymptotically stable and we can write

$$\begin{aligned}
\dot{z}_1 &= a(z_2 - z_1), \\
\dot{z}_2 &= -z_2 - z_1 z_3, \\
\dot{z}_3 &= -bz_3 + z_1 z_2, \\
\dot{z}_4 &= -hz_4, \\
\dot{z}_5 &= -z_5.
\end{aligned} \tag{23}$$

5. NUMERICAL SIMULATIONS

Here we present our numerical simulation in order to verify the effectiveness of the controllers. We used the fourth-order Runge - Kutta algorithm. The system parameters are chosen as $a = 10$, $b = 8/3$, $c = 28$, $h = 2.25$, $k_1 = -0.12$, $k_2 = 11.3$ when the initial conditions were $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $p_1 = 0.1$, $w_1 = 0.1$, $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $p_2 = 0.7$ and $w_2 = 0.5$.We comment that these initial conditions gave rise to the observed attractors and time series before the activation of the synchronization control at $t \geq 20$. The step size was maintained at $H = 0.005$. The controllers u_i $(i = 1, 2, ...5)$ were activated at $t \geq 20$. The result for the synchronized states e_1, e_2, e_3, e_4 *and* e_5 for the system is shown in Figures (2), (3), (4), (5) and (6) respectively. The globally synchronized state is shown in Figure (7). In all the states, synchronization took place when each of the controllers was activated at $t \geq 20$ which signifies that complete synchronization of the *5D hyperchaotic system* has been achieved.

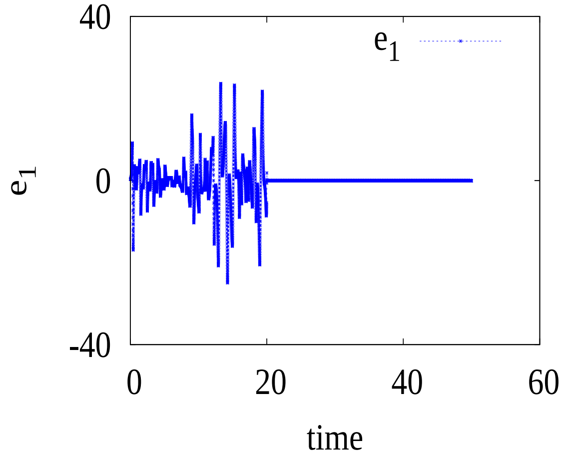

FIGURE 2. Synchronized state of e_1 when $a = 10$, $b = 8/3$, $c = 28$, $d = 2.25$, $p = -0.12$, $k = 11.3$ when the initial conditions were $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $q_1 = 0.1$, $w_1 = 0.1$, $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $q_2 = 0.7$ and $w_2 = 0.5$, $H = 0.005$ at $t \geq 20$

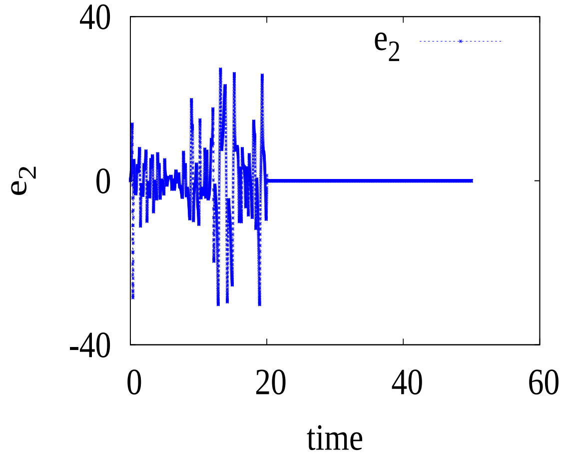

FIGURE 3. Synchronized state of e_2 when $a = 10$, $b = 8/3$, $c = 28$, $d = 2.25$, $p = -0.12$, $k = 11.3$ when the initial conditions were $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $q_1 = 0.1$, $w_1 = 0.1$, $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $q_2 = 0.7$ and $w_2 = 0.5$, $H = 0.005$ at $t \geq 20$

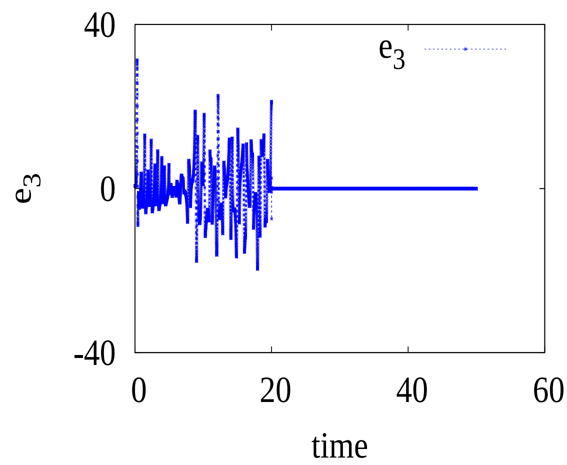

FIGURE 4. Synchronized state of e_3 when $a = 10$, $b = 8/3$, $c = 28$, $d = 2.25$, $p = -0.12$, $k = 11.3$ when the initial conditions were $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $q_1 = 0.1$, $w_1 = 0.1$, $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $q_2 = 0.7$ and $w_2 = 0.5$, $H = 0.005$ at $t \geq 20$

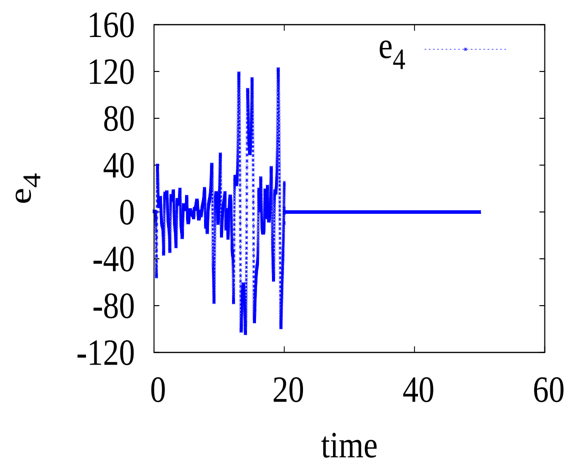

FIGURE 5. Synchronized state of e_4 when $a = 10$, $b = 8/3$, $c = 28$, $d = 2.25$, $p = -0.12$, $k = 11.3$ when the initial conditions were $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $q_1 = 0.1$, $w_1 = 0.1$, $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $q_2 = 0.7$ and $w_2 = 0.5$, $H = 0.005$ at $t \geq 20$

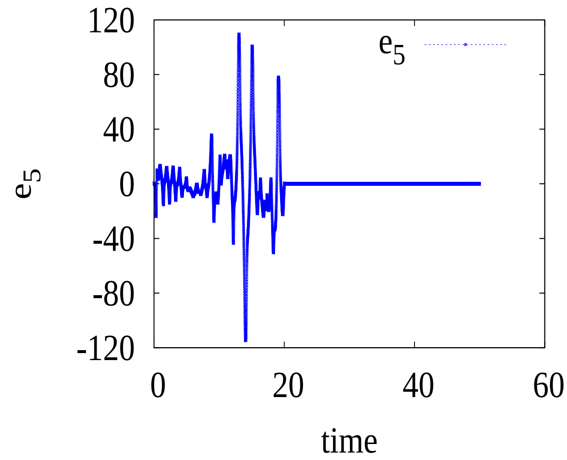

FIGURE 6. Synchronized state of e_5 when $a = 10$, $b = 8/3$, $c = 28$, $d = 2.25$, $p = -0.12$, $k = 11.3$ when the initial conditions were $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $q_1 = 0.1$, $w_1 = 0.1$, $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $q_2 = 0.7$ and $w_2 = 0.5$, $H = 0.005$ at $t \geq 20$

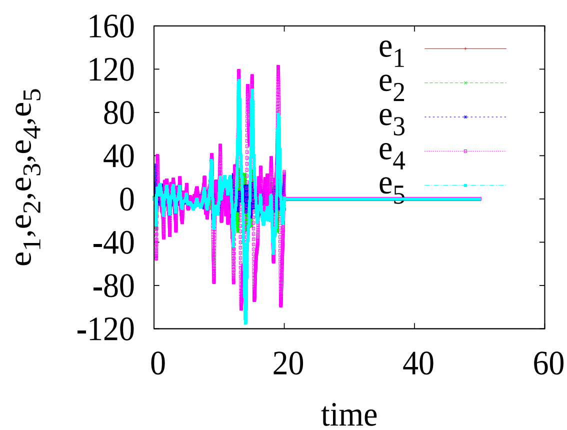

FIGURE 7. The globally synchronized state of e_1, e_2, e_3, e_4 and e_5 when $a = 10$, $b = 8/3$, $c = 28$, $d = 2.25$, $p = -0.12$, $k = 11.3$ when the initial conditions were $x_1 = 0.1$, $y_1 = 0.1$, $z_1 = 0.1$, $q_1 = 0.1$, $w_1 = 0.1$, $x_2 = 0.5$, $y_2 = 0.01$, $z_2 = 0.8$, $q_2 = 0.7$ and $w_2 = 0.5$, $H = 0.005$ at $t \geq 20$

6. CONCLUSION

In this paper, we have demonstrated, analysed and validated the realization of complete synchronization of 5-dimensional hyperchaotic system using the active backstepping technique. We extended the usual master-slave synchronization scheme for low order chaotic systems to study the synchronization of this higher order systems. Each of the 5-dynamical states was successfully synchronized and by implication, electronic information can be hidden in any or all of this 5d Hyperchaotic system and such information can be transferred, communicated and retrieved by applying the control inputs for each or all the dynamical states. Our numerical results confirm the effectiveness of the analytical technique and we belief that they are observable in laboratory experiments.

ACKNOWLEDGEMENTS

S.O. Ogundipe acknowledges research support from the Federal Government of Nigeria through the Tertiary Education Trust Fund (TETFUND). U.E. Vincent acknowledges research support from the Royal Society of London through the Newton International Fellowship Alumni Funding.

REFERENCES

(1) S H Strogatz: Nonlinear dynamics and chaos: with applications to physics, biology, chemistry and engineering, in: Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry and Engineering. *Persues Books Publishing: LLC,USA*(2000)
(2) U E Vincent, A Saseyi, P V E McClintock: Multi-switching combination synchronization of chaotic systems. *Nonlinear Dynamics.* 80(1),845-854(2015)
(3) M Zhan, X Wang, X Gong, G W Wei, C H Lai: Complete synchronization and generalized synchronization of one-way coupled time-delay systems. *Physical Review E.*68(36208),1-5(2003)
(4) M G Rosenblum, A S Pikovsky, J Kurths: Phase synchronization of chaotic oscillators.*Physical Review Letters.* 76(11),1804-1807(1996)
(5) M G Rosenblum, A S Pikovsky, J Kurths: From phase to lag synchronization in coupled chaotic oscillators. *Physical Review Letters.* 78 (22)4193-4196(1997)
(6) N F Rulkov, M M Sushchik, S Tsimring, H D I Abarbanel: Generalized synchronization of chaos in directionally coupled chaotic systems. *Physical Review E*51(2),980-994 (1995)
(7) P K Roy, C Hens, I Grosu, S K Dana: Engineering generalized synchronization in chaotic oscillators. *Chaos*21(01310),61-67 (2011)
(8) X Wang, Z Ying, G Hu: Controlling hamiltonian systems by using measure synchronization. *Physics Letters A*298(5-6),383-387(2002)
(9) U E Vincent, A N Njah, O Akinlade: Measure synchronization in a coupled hamiltonian system associated with nonlinear schrdinger equation. *Modern Physics Letters B*19(15),737-742(2005)

(10) X Wang, M Zhan, C H. Lai, H Gang: Measure synchronization in coupled 4 hamiltonian systems. *Physical Review Letters E.* 67(6) (06621),51-55(2003)

(11) M F Hu, Z Y Xu: Adaptive projective synchronization of unified chaotic systems and its application to secure communication. *Chinese Physics.* 16(11),3231-3237(2007)

(12) Z G Li, D L Xu: Stability criterion for projective synchronization in three-dimensional chaotic systems. *Physics Letter A.*282(3),175-179(2001)

(13) R Mainieri, J Rehacek: Projective synchronization in three-dimensional chaotic systems. *Physical Review Letter A.* 82(15)3042-3045(1999)

(14) M Kostur, T P Hnggi, J L Meteos: Anticipated synchronization in coupled inertia ratchets with time-delayed feedback: a numerical study. *Physical Review E.*72(3),(036210)1-15(2005)

(15) H U Voss: Anticipated synchronization: *Physical Review E*61(5A),5115-5119 (2000)

(16) R Femat, G Sols-Perales: Synchronization of chaotic systems with different order. *Physical Review E.*65(3),(036226)1-5(2002)

(17) S O Kareem, K S Ojo, A N Njah: Function projective synchronization of identical and non-identical modified finance and Shimizu-Morioka systems. *Pramana Journal of Physics.*79(1),71-79(2012)

(18) W Shu-Guo, Z Song: Adaptive function projective synchronization of uncertain complex dynamical networks with disturbance. *Chinese Physics B.*22(7)(070503),1-5(2013)

(19) C C Yang: Robust synchronization and anti-synchronization of identical ϕ oscillators via adaptive sliding mode control. *Journal of Sound and Vibration.*331(3),501-509(2012)

(20) J Lu, D W Ho, J Cao, J Kurths: Single impulsive controller for globally exponential synchronization of dynamical networks. *Nonlinear Analysis:Real World Applications.*14,581-593(2013)

(21) Y Li, S Tong, T Li: Adaptive fuzzy output feedback control for a single-link flexible robot manipulator driven dc motor via backstepping. *Nonlinear Analysis: Real World Applications.*14,483-494(2013)

(22) T Yashar, J Wang: Chaos control and synchronization of a hyperchaotic Zhou system by integral sliding mode control. *Entropy.*16539-6552(2014)

(23) K S Ojo, A N Njah, S T Ogunjo: Comparison of backstepping and modified active control in projective synchronization of chaos in an extended BonhofferVan der Pol oscillator. *Pramana Journal of Physics.*80(5),825-835(2013)

(24) M Kristic, I Kanellakopoulus, P O Kototovic: Nonlinear and adaptive control design. *Nonlinear and Adaptive Control Design.*1-30,(1995) *(John Wiley:New York,NY, USA)*

(25) H Zhang, X K Ma, M Li, J L Zou: Controlling and tracking hyperchaotic Rössler system via active backstepping design. *Chaos, Solitons and Fractals.*26(2),353-361(2005)

(26) A R Sahab, Z M Taleem: Generalized backstepping method based control Chen chaotic system using adaptive neuro-fuzzy inference, International *Journal of Information and Electronic Engineering.*2(5),667-671(2012)

(27) Z Meng, X Wang, X Gong, G Wei, C Lai: Complete synchronization and generalized synchronization of one-way coupled time-delay systems. *Physical Review E.*68,(036208)1-5(2003)

(28) A A Ajayi, K S Ojo, U E Vincent, A N Njah: Multiswitching synchronization of a driven hyperchaotic circuit using active backstepping. *Journal of Nonlinear Dynamics.*2014(918586)1-10(2014)

(29) Q Yang, C Chen: A 5D hyperchaotic system with three Lyapunov exponent coined. *International Journal of Bifurcation and Chaos.*23(6)(13501)091-924(2013)

(30) T Gao, G Chen, Z Chen, S Cang: The generation and circuit implementation of a new hyper chaos based upon Lorenz system. *Physics Letter A*.361,78-86(2007)

(31) L Yuxia, K S Wallace, Tang, G Chen: Hyperchaos evolved from the generalized Lorenz equation. *International Journal of Circuit Theory and Applications*.33(4),235-251 (2005)

(32) R Luo, Y Wang, S Deng: Combination synchronization of three classic chaotic systems using active backstepping design. *Chaos*. 21(4),(043114)(2011)

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