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ON A NEW SOLUTION OF THE TRANSPORTATION PROBLEM

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ABSTRACT.The transportation problem plays a vital role in industry, commerce, logistics etc. To maximize profit, organizations are always looking for better ways to curtail cost and improve revenue. Basically, the standard solution of a transportation problem is a two-stage process. First is to find an initial basic feasible solution and secondly, perform optimality test to improve the solution. This process is tiring and time wasting. In this study, a new solution is introduced which overcomes the two-stage process and solves the transportation problem in a one stage process. In most instances the method was able to give an optimal solution.

Keywords and phrases: Transportation problem, initial basic feasible solution, optimality test,optimal solution.
2010 Mathematical Subject Classification:90B06

1. INTRODUCTION

The primary aim of a transportation model is to find the best way of transporting goods from several sources to several destinations with minimum cost and in the least amount of time. Its objective is to meet the needs at the various destinations while exhausting the resources at the various supply points.

The application of the transportation problem or $T.P$ can be clearly seen in logistics, industries, communication network, allotment etc.

In a real life circumstance, consider a company with three manufacturing plants S_1, S_2 and S_3 in a region. The plants have a limited number of goods they can produce. Suppose the company wishes to transport their goods from their three plants to their three warehouses W_1, W_2 and W_3 scattered in the region; the warehouses have a certain capacity they can take. Clearly, $s_1, s_2,$ and s_3 is

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the amount that can be produced in the three plants respectively, while w_1, w_2 and w_3 is the amount(of goods) required at the various warehouses. Indeed, C_{ij} is the cost of moving the goods from the i_{th} plant to the j_{th} warehouse while X_{ij} is the number of goods moved from the i_{th} plant to the j_{th} destination.

Priya Ms.S. et al(2016) published an article titled "Solving transportation problem using ICMM method". In this method, Priya suggested interchanging the odd number of columns (with supply and demand also) and finding the smallest cost in each column to obtain optimality [3] .

Hlayel et al (2012), solved the transportation problem using the best candidate method. [1]

In their paper, "A comparative Study of Initial Basic Feasible Solution Methods for a transportation problem", Soomro A.S et al.(2014) proposed a method "The Minimum Transportation Cost Method" to find the IBFS for the problem solved by Hakim , and found that it provides not only the minimum transportation cost but also an optimal solution.[6]

Gaurav et al.(2015) published their paper "Solving Time Minimizing Transportation Problem by Zero Point Method". In their work, they used the zero point method to solve time minimizing transportation problem and they compare the results obtained with the regular methods, which is solve by Tora Software to get feasible solution and they found that zero point method is best from other regular method.[8]

"An Algorithmic approach to solve transportation problems with the average total opportunity cost". Abul S.M. et al.(2017) used this method in solving a transportation problem and found that the initial basic feasible solution(IBFS) found by their method is better than other familiar methods discussed in their work. [2]

Ahmed M.M et al(2016) in their paper "A New Approach to Solve Transportation Problems" discussed a solution for solving the Initial Basic Feasible Solution of a transportation problem which gives better IBF solutions. [4]

2. MODEL OF A TRANSPORTATION PROBLEM.

The transportation model is defined:

$$\text{Minimize } Z = \sum_{i=1}^{m} X_{ij}C_{ij} \tag{1}$$

$$\sum_{j=1}^{n} X_{ij} \leq_{a_i}, i = 1, 2, 3...m(\text{Demand constraint}) \tag{2}$$

$$\sum_{i=1}^{m} X_{ij} \geq_{b_j}, j = 1, 2, 3...n(\text{Supply constraint}) \tag{3}$$

$$X_{ij} \geq 0, 1, 2, 3...n \tag{4}$$

This is a linear program with $m.n$ decision variables,$m + n$ functional constraints, and $m.n$ nonnegative constraints.

m is the number of resources.
n is the Number of destinations.
a_i is the Capacity of i_{th} source
b_j is the Demand of j_{th} destination.
c_{ij} is the The unit transportation cost between i_{th} source and j_{th} destination (in naira or as a distance in kilometers, miles, etc.).While, x_{ij} is the Size of material transported between i_{th} source and j_{th} destination (in tons, pounds, liters etc.)

A transportation problem is said to be unbalanced if and only if

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \tag{5}$$

There are two cases:
Case (1)

$$\sum_{i=1}^{m} a_i \geq \sum_{i=1}^{n} b_j \tag{6}$$

Case (2)

$$\sum_{i=1}^{m} a_i \leq \sum_{j=1}^{n} b_j \tag{7}$$

To balance the Transportation Problem,Introduce a dummy origin or source in the transportation table with a zero cost. The availability at this origin is:

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 0 \tag{8}$$

3. SOLUTION OF A TRANSPORTATION PROBLEM

0.1. **Tableau And Network Representation.**

The transportation problem is illustrated with the model of a linear program and it appears in a network and tableau form.

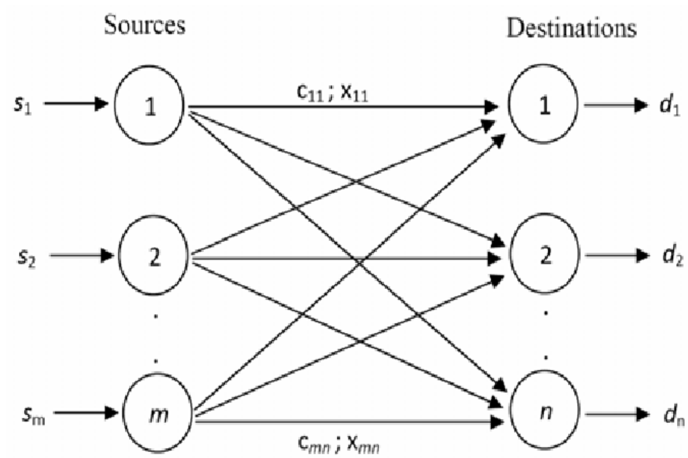

Fig. 1. The Transportation network.

Destination / Plants	D_1	D_2	D_3		D_{n-1}	D_n	Supply quantity
S_1	x_{11}	x_{12}	x_{13}	…	$x_{1,m-1}$	$x_{1,n}$	s_1
S_2	x_{21}	x_{22}	x_{23}	…	$x_{2,m-1}$	$x_{2,n}$	s_2
S_3	x_{31}	x_{32}	x_{33}	…	$x_{3,m-1}$	$x_{3,n}$	s_3
	.	.	.		.	.	.
	.	.	.		.	.	.
	.	.	.		.	.	.
S_{m-1}	$x_{m-1,1}$	$x_{m-1,2}$	$x_{m-1,3}$	…	$x_{m-1,n-1}$	$x_{m-1,n}$	s_{m-1}
S_m	$x_{n,1}$	$x_{n,2}$	$x_{n,3}$	…	$x_{m,n-1}$	$x_{m,n}$	s_m
Demand quantity	d_1	d_2	d_3	…	d_{n-1}	d_n	

Fig. 2. The Transportation Tableau.

Flowchart solution of the transportation Problem.

- The problem is formulated as a transportation model.
- Is the transportation model balanced?

- If yes, go to next step else,add dummy to the rows or column.
- Determine initial basic solutions.
- Go to next step if the solution is optimized else Go to fourth step.
- Using the optimal solution ,calculate the total transportation cost.

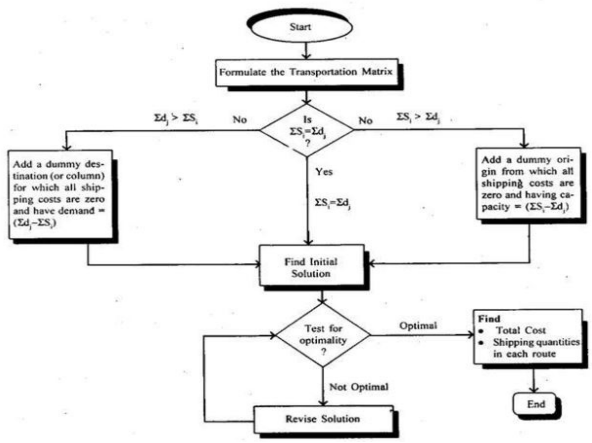

Fig. 3. Flowchart of transportation solution.

Solving The Transportation problem. There are three popular methods to finding an initial basic feasible solution and they include:

(1) Northwest Corner Rule
(2) Least Cost Method
(3) Vogel Approximation Method and
(4) The new method

0.2. **Northwest Corner Rule (NCR).** In this method, allocation of quantities being transported from source to some destination must start from the upper most left hand cell that is , the Northwest corner of the table.

The steps include:

(i) Make allocation in the northwest(upper left) corner of the Transportation Problem table. Compare the supply of plant 1 say S_1 with the demand at the warehouse or destination 1 say d_1. Then,

(i) If $d_1 < S_1$ i.e if the amount required at d_1 is less than the number of units available at S_1, set x_{11} equal to d_1, find the balance supply and demand and *proceed horizontally.*
(ii) If $d_1 = S_1$, set x_{11} equal to d_1, balance supply and demand and *proceed diagonally.* Remember to make a zero allocation to the least cost cell in S_1 / d_1.
(iii) If $d_1 > S_1$, set x_{11} equal to S_1, balance demand and supply and *proceed vertically.*

(ii) Continue with i to iii, step by step away from the upper left corner until you reach a value in the south-east corner.
(iii) Calculate the total transportation cost.

This method does not take into account the transportation cost and hence may not yield a good initial basic feasible solution.

0.3. **Least Cost Method (LCM).** Also called the matrix minimum method is a method of finding an initial basic feasible solution where allocation of resources begins from the least cost.

The Steps Include:

(1) Determine the cell having the least transportation cost (C_{ij}).
(2) Allocate as much as possible to this least cost.
(3) If there's a tie in least cost,select the cell having the greatest least cost.
(4) Delete the row or column which has been exhausted.
(5) Select the next least cost and allocate as much as possible.
(6) Continue in this manner till all row and column requirements are met.

0.4. **The Vogel Approximation Method (VAM).** This procedure is an iterative method of finding an initial basic feasible solution. It is an improved version of the least cost method.

The Steps Include:

(1) Find the difference between The least cost and next least cost of each row and column.(This difference is the row or column penalty).
(2) Select the row or column with the biggest penalty.
(3) In case of a tie in penalty, select the row or column with the greatest least cost.
(4) Make allocation as much as possible to the cell in that row /column.

(5) Delete the column or row that has been completely exhausted.
(6) Repeat steps 1 to 5 until all allocations are made.

0.5. **Test For Optimality.** In Optimality test, questions such as, can necessary adjustments be made in the initial basic feasible solution that can further minimize the transportation cost. In this regard,probe every unoccupied/unallocated cell and check if there will be a reduction in Transportation Problem cost if allocation is made in these unoccupied cell.
This is the second phase of solving a Transportation Problem.(The new method proposed in this paper seeks to solve the transportation problem in just one phase instead of the usual two stage solution.)
To begin, there are two methods of testing for optimality viz:

- The stepping stone method and
- The modified Distribution Method(MODI).

0.6. **The modified Distribution Method(MODI).** The MODI method is an improvement of the stepping stone method.In this method a closed path is traced for each unoccupied cell.Cell evaluation are found and the cell with the most negative evaluation becomes the basic cell.Cell evaluation of all the unoccupied cells are calculated simultaneously and only one closed path for the most negative cell is evaluated.

The steps are As Follows:

(1) Determine initial solutions.
(2) Introduce dual variables u_i and v_i corresponding to the demand and supply constraints.
(3) Compute your change of cost k_{ij} for all cells that are empty.
(4) To the empty cell which will produce highest net decrease in cost, allocate much to this cell.
(5) Do steps 2 upto 4 again till all k_{ij} is zero or is positive.

0.7. **The New Method.** The new method uses the concept of penalty for its iterations. It determines this penalty by finding the sum of the differences between the least cost and every other cost in the row or column and adding it to the number of rows or columns.

The Steps Include.

(1) Prepare a balanced transportation table
(2) For every *row*:

- Determine the difference between the least cost and every other cost in that *row*.
- Add these differences together with the number of *ROWS* present at the table.*(This becomes the penalty for that row)*.
- Write this penalty at the RHS of the row.

(3) For every *Column*:

- Determine the difference between the least cost and every other cost in that *column*.
- Add these differences together with the number of *COLUMNS* present at the table.*(This becomes the penalty for that column)*.
- Write this penalty at the bottom of the column.

(4) Select the row or column having the greatest penalty.
(5) Allocate as much as possible to the cell in that row or column having the least cost.
(6) In case of tie in the penalties, select the row or column having the *greatest* least cost.
(7) Repeat steps 2 to 6 until all allocations have been made.

4.NUMERICAL ILLUSTRATION

In this chapter, numerical illustrations have been collected and solved with the existing solutions. The new method was also applied in solving the numerical illustrations.

Illustration 1. Consider the tableau where X_{ij} is the number of wheat(in tons) transported from each grain house to each mill.$(i, j = 1, 2, 3)$. The total transportation cost for each route is the objective function. The solution is the number of tons of wheat to be transported from house to mill so as to minimize total cost of transportation.

Table 1. The wheat transportation Problem.

$\frac{To}{From}$	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

The Northwest Corner Rule. Here make the first allocation to the northwest corner and proceed to adjacent feasible cells. The allocation is feasible.

Hence starting solution consisting of 5 variables is:
$X_{1A} = 150\text{tons}, ; X_{2A} = 50\text{tons}, ; X_{2B} = 100\text{tons}, ; X_{2C} = 25\text{tons};$
$X_{3C} = 275\text{tons}$
Evaluation of the objective function gives the transportation cost:
$Z = 6(X_{1A}) + 8(X_{1B}) + 10(X_{1C}) + 7(X_{2A}) + 11(X_{2B}) + 11(X_{2C}) + 4(X_{3A}) + 5(X_{3B}) + 12(X_{3C})$
$= 6(150)+8(0)+10(0)+7(50)+11(100)+11(25)+4(0)+5(0)+2(275)$
$= \#5,925$
See the table below:

Table 2. Solution by Northwest corner rule.

$\frac{To}{From}$	A	B	C	Supply
1	$\boxed{150}$ 6	8	10	150
2	$\boxed{50}$ 7	$\boxed{100}$ 11	$\boxed{25}$ 11	175
3	4	5	$\boxed{275}$ 12	275
Demand	200	100	300	600

The Least Cost Method. Considering the test problem, using the least cost method make first allocation in cell 3B and cross out column A because it has been exhausted. Continue like this to the next least cost and get the final table:

Table 3. Solution by Least Cost.

$\frac{To}{From}$	A	B	C	Supply
1	6	$\boxed{25}$ 8	$\boxed{125}$ 10	150
2	7	11	$\boxed{175}$ 11	175
3	$\boxed{200}$ 4	$\boxed{75}$ 5	12	275
Demand	200	100	300	600

The total transportation cost gives #4,550. Hence the least cost gives a more optimized solution than the Northwest Corner rule.This advantage is because it considers cost.

The VAM Approach. First,consider the VAM penalty cost of each row and column.

Table 4. Solution by VAM.

$\frac{To}{From}$	A	B	C	Supply
1	6	8	$\boxed{150}$ 10	150
2	$\boxed{175}$ 7	11	11	175
3	$\boxed{25}$ 4	$\boxed{100}$ 5	$\boxed{150}$ 12	275
Demand	200	100	300	600

The total transportation cost using the VAM approach is #5,125. Hence, VAM and least cost method computes better solutions than the Northwest method.

The MODI Method. From the table ,the reduction in cost is now #4,525.

Table 5. The MODI solution.

	v_j	$v_A = 6$	$v_B = 7$	$v_C = 10$	
u_i	$\frac{To}{From}$	A	B	C	supply
u_i=0	1	$\boxed{25}$ 6	8	$\boxed{125}$ 10	150
u_2=1	2	7	11	$\boxed{175}$ 11	175
u_3	3	$\boxed{175}$ 4	$\boxed{100}$ 5	12	275
	Demand	200	100	300	600

From the table ,the reduction in cost is now #4,525.

The New Method. Prepare a balanced matrix from the test problem.
First Iteration(3 rows 3 columns)

Table 6. The wheat transportation Problem.

$\frac{To}{From}$	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

Row Penalty:

- 6 is the least cost in row 1. To get the penalty for row 1, subtract 6 from all the cells in row 1 and take the sum of the resulting values: $(8 - 6) + (10 - 6) = 6$.

- Add this sum to the number of rows, (6+3)=9 which becomes the penalty for row 1.
- Write this 9 at the RHS of row 1.
  Do same for the subsequent rows.
- For row 2 with least cost of 7, we have (11-7)+(11-7)+3 = 11 = penalty.
- For row 3 with least cost of 4, we have (5-4)+(12-4)+ 3 = 12 = penalty.

Column Penalty:

- 4 is the least cost in column A. To get the penalty for column A, subtract 4 from all the cells in column A and take the sum of the resulting values: $\{(6-4)+(7-4)=5\}$.
- Add this sum to the number of columns, (5+3)=8 which becomes the penalty for column A.
- Write this 8 at the bottom of column A.
  Do same for the subsequent columns.
- For column B with least cost of 5 we have $\{(8-5)+(11-5)\} = 12 = \textit{penalty}$.
- For column C with least cost of 10 we have $\{(11-10)+(12-10)\} = 6 = \textit{penalty}$.

Table 7. First Iteration.

$\frac{To}{From}$	A	B	C	supply	penalty
1	6	8	10	150	9
2	7	11	11	175	11
3	4	$\boxed{100}$ 5	12	275/175	12
Demand	200	100	300	600	-
penalty	8	12	6	-	-

12 is the greatest penalty having a tie at row 3 and column B. Allocate to cell 3B since it has the greatest of the least cost. Cross out column B.

Second Iteration(3 rows 2columns)

Table 8. Current table.

$\frac{To}{From}$	A	C	Supply
1	6	10	150
2	7	11	175
3	4	12	175
Demand	200	300	500

Row Penalty:

- 6 is the least cost in row 1. To get the penalty for row 1, subtract 6 from all the cells in row 1 and take the sum of the resulting values: $(10 - 6) = 4$.

- Add this sum to the number of rows, (4+3)=7 which becomes the penalty for row 1.

- Write this 7 at the RHS of row 1.
  Do same for the subsequent rows.

- For row 2 with least cost of 7, we have (11-7)+3 = 7 = penalty.

- For row 3 with least cost of 4, we have (12-4)+ 3 = 11 = penalty.

Column Penalty:

- 4 is the least cost in column A. To get the penalty for column A, subtract 4 from all the cells in column A and take the sum of the resulting values: $\{(6 - 4) + (7 - 4) = 5\}$.

- Add this sum to the number of columns, (5+2)=7 which becomes the penalty for column A.

- Write this 7 at the bottom of column A.
  Do same for the subsequent columns.

- For column C with least cost of 10 we have $\{(11 - 10) + (12 - 10)\} = 5 = penalty$.

Table 9. Second Iteration.

$\frac{To}{From}$	A	C	supply	penalty
1	6	10	150	5
2	7	11	175	7
3	$\boxed{175}$ 4	12	175/0	11
Demand	200/25	300	500	-
penalty	7	5	-	-

11 is the greatest penalty at row 3. Allocate to cell 3A since it has least cost of 4. Cross out row 3.

Third Iteration(2 rows 2 columns)

Table 10. Current table.

$\frac{To}{From}$	A	C	Supply
1	6	10	150
2	7	11	175
Demand	25	300	325

Since this is an m x m matrix, no need to add number of rows and columns to the sum of the differences.

Row Penalty:

- 6 is the least cost in row 1. To get the penalty for row 1, subtract 6 from all the cells in row 1 and take the sum of the resulting values: $(10 - 6) = 4$.
  which becomes the penalty for row 1.

- Write this 4 at the RHS of row 1.
  Do same for the subsequent rows.

- For row 2 with least cost of 7, we have (11-7) = 4 = penalty.

Column Penalty:

- 6 is the least cost in column A. To get the penalty for column A, subtract 6 from all the cells in column A and take the sum of the resulting values: $\{(7 - 6) = 1\}$.
  which becomes the penalty for column A.

- Write this 1 at the bottom of column A.
  Do same for the subsequent columns.

- For column C with least cost of 10 we have $\{(11-10)\} = 1 =$ penalty.

Table 11. Third Iteration.

$\frac{To}{From}$	A	C	supply	penalty
1	6	10	150	4
2	$\boxed{25}$ 7	11	175/150	4
Demand	25/0	300	325	-
penalty	1	1	-	-

Row 1 and 2 have 4 as greatest penalty(tie). Allocate as much as possible to row 2 cell 2A because it has 7 as least cost bigger than 6 which is the least cost of row 1. Cross out column A.

Fourth Iteration(2 rows 1 column)

Table 12. Current table.

$\frac{To}{From}$	C	Supply
1	10	150
2	11	150
Demand	300	300

Row Penalty:

- 10 is the least cost in row 1 and its the only cost. which becomes the penalty for row 1.
- Write this 10 at the RHS of row 1.
  Do same for the subsequent rows.
- For row 2 with least cost of 11, we have 11 as the penalty.

Column Penalty:

- 10 is the least cost in column C. To get the penalty for column C, subtract 10 from all the cells in column C and take the sum of the resulting values: $\{(11-10) = 1\}$.
  which becomes the penalty for column C.
- Write this 1 at the bottom of column C.
- No other column

Table 13. Fourth Iteration.

$\frac{To}{From}$	C	supply	penalty
1	10	150	10
2	$\boxed{150}$ 11	150/0	11
Demand	300/150	300	-
penalty	1	-	-

Allocate to row 2, cell 2C and cross out row 2.

Final Iteration(1 row 1 column)

Table 14. Current table.

$\frac{To}{From}$	C	Supply
1	10	150
Demand	150	150

Make allocation to 1C and that ends the iteration, hence the generalized allocation is:

Table 15. Final Iteration.

$\frac{To}{From}$	A	B	C	SUPPLY
1	6	8	$\boxed{150}$ 10	150
2	$\boxed{25}$ 7	11	$\boxed{150}$ 11	175
3	$\boxed{175}$ 4	$\boxed{100}$ 5	12	275
Demand	200	100	300	600

Therefore the total transportation cost is:
10(150)+7(25)+11(150)+4(175)+5(100) = #4,525
Which is the same as the amount computed from the MODI method for getting optimal solutions and better than other methods such as Northweat Corner Rule, Least cost method and Vogel Approximation Method (VAM).

Illustration 2. Consider the transportation of Mahindra manufacture's of products from their various manufacturing plants to their different warehouses. Obtain the least transportation cost. [3]

Table 16. Illustration 2.

$\frac{Whouse}{Plant}$	V	C	N	Supply
K	6	4	1	50
B	3	8	7	40
M	4	4	2	60
Demand	20	95	35	150

Using the Northwest corner rule, we obtain a transportation cost of #730.

Table 17. Solution by Northwest corner method.

$\frac{Whouse}{Plant}$	V	C	N	Supply
K	$\boxed{20}$ 6	$\boxed{30}$ 4	1	50
B	3	$\boxed{40}$ 8	7	40
M	4	$\boxed{25}$ 4	$\boxed{35}$ 2	60
Demand	20	95	35	150

By further optimality test we obtain the T.T.C of #555

Table 18. Solution by MODI method.

$\frac{Whouse}{Plant}$	V	C	N	Supply	v_i
K	6	$\boxed{15}$ 4	$\boxed{35}$ 1	50	0
B	$\boxed{20}$ 3	$\boxed{20}$ 8	7	40	4
M	4	$\boxed{60}$ 4	2	60	0
Demand	20	95	35	150	
u_j	-1	4	1		

Applying the new method yields a T.T.C of #555. .

Table 19. Solution by the new Method.

$\frac{Whouse}{Plant}$	V	C	N	Supply	penalties			
K	6	$\boxed{15}$ 4	$\boxed{35}$ 1	50/15	8	6	7	4
B	$\boxed{20}$ 3	$\boxed{20}$ 8	7	40/20	9	4	11	-
M	4	$\boxed{60}$ 4	2	60	4	5	7	4
Demand	20	95	35	150				
penalties	4	4	7					
	-	6	9					
	-	5	-					

Illustration 3. Consider the transportation problem from [7]

Table 20. Matrix representation(Unbalanced).

$\frac{To}{From}$	X	Y	Z	U	supply
A	60	120	75	180	8000
B	58	100	60	165	9200
C	62	110	65	170	6250
D	65	115	80	175	4900
E	70	135	85	195	6100
Demand	5000	2000	10000	6000	

However we notice that $\sum_{a_i} = 34450 \neq \sum_{b_j} = 23000$. Introduce a dummy column,call it V with zero allocations but with a total demand of 11450 which is the difference between 34450 and 23000.

Table 21. Matrix representation.

$\frac{To}{From}$	X	Y	Z	U	V	supply
A	60	120	75	180	0	8000
B	58	100	60	165	0	9200
C	62	110	65	170	0	6250
D	65	115	80	175	0	4900
E	70	135	85	195	0	6100
Demand	5000	2000	10000	6000	11450	

Initial basic feasible solutions: Solution by NorthWest Corner Rule

The application yields #3,086,000

Table 22. Solution by Northwest Corner.

$\frac{To}{From}$	X	Y	Z	U	V
A	$\boxed{5000}$ 60	$\boxed{2000}$ 120	$\boxed{1000}$ 75	180	0
B	58	100	60	$\boxed{9000}$ 165	$\boxed{200}$ 0
C	62	110	65	$\boxed{5800}$ 170	$\boxed{450}$ 0
D	65	115	80	175	$\boxed{4900}$ 0
E	70	135	85	195	$\boxed{6100}$ 0

Solution by VAM:

Table 23. Solution by VAM.

To / From	X	Y	Z	U	V
A	5000 60	120	3000 75	180	0
B	58	2000 100	1200 60	6000 165	0
C	62	110	5800 65	170	450 0
D	65	115	80	175	4900 0
E	70	135	85	195	6100 0

The total transportation yields #2,164,000

Solution by Least Cost Method:
The total T.P yields #2,404,500

Table 24. Solution by Least cost.

To / From	X	Y	Z	U	V
A	5000 60	120	750 75	180	2250 0
B	58	100	60	165	9200 0
C	62	110	6250 65	170	0
D	65	1900 115	3000 80	175	0
E	70	100 135	85	6000 195	0

Optimality Test: Modi Method:
After testing for optimality.

Table 25. MODI optimality.

To / From	X	Y	Z	U	V
A	5000 60	120	2250 75	180	450 0
B	58	2000 100	1200 60	6000 165	0
C	62	110	6250 65	170	0
D	65	115	80	175	4900 0
E	70	100 135	85	195	6100 0

Hence the total T.P yields #2,159,000

Solution using the new Method:
This solution yields

The total T.p yields #2,159,500. It is again clear that the new method yields a solution better than the other IBFS and same as the optimal solution.

Table 26. Solution by new Method.

$\frac{To}{From}$	X	Y	Z	U	V	supply	iterations						
A	$\boxed{5000}$ 60	120	$\boxed{2550}$ 75 75	180	0	8000 $\frac{7550}{2550}$	44 0	43 9	43 8	19 8	15 0	-	-
B	58	$\boxed{2000}$ 100	$\boxed{1200}$ 60	$\boxed{6000}$ 165	0	9200 $\frac{3200}{1200}$	388 8	387 7	386 6	154 4	145 5	147 7	145 5
C	62	110	$\boxed{6250}$ 65	170	0	6250	412 2	411 1	410 0	162 2	150 0	152 2	- -
D	65	115	80	175	$\boxed{4900}$ 0	4900	44 0	43 9	-	-	-	-	-
E	70	135	85	195	$\boxed{6100}$ 0	6100	490 0	-	-	-	-	-	-
Demand	5000	2000	10000 $\frac{7450}{1200}$	6000 6000	11450 $\frac{5350}{450}$								
ite	30	85	70	55	5								
ra	18	50	45	35	5								
tions	11	35	25	25	5								
	10	34	24	24	-								
	-	30	20	20	-								
	-	13	8	8	-								
	-	103	63	148	-								
	-	102	62	-	-								

4. CONCLUDING REMARKS

To determine the optimal solution of a transportation problem involves a two stage process. First the initial basic feasible solution(IBFS) is determined using one of Northwest corner rule, least cost method, vogel approximtion method etc. Secondly, After this is determined, an optimality test is performed on the IBFS using the MODI or any other method to determine a better solution. This two stage process is what the "new algorithm" was able to overcome.

Hence instead of determining an IBFS and then performing optimality test on them, a one algorithm is applied to generate the optimal solution. The new solution presented in this research work was able to determine the optimal solution of a transportation problem in a one stage process. It is faster, easy to implement and time saving.

Having presented three numerical illustrations, it is obvious that the new solution is able to determine an optimal solution in most instances. In the first illustration, the optimal solution by MODI method was #4,525 which was also the same derived using the new method. Other methods only got initial solutions.

Again, the MODI method generated an optimal solution of #555 in the second illustration, same as the solution gotten by the new method.

In the third illustration, the optimal solution is #2,159,000. This was confirmed by the MODI method and indeed by the new method. Clearly, the new method is able to determine an optimal solution in most instances.

NOMENCLATURE

TP: Transportation problem,
IBFS: Initial Basic Feasible Solution.
RHS: Right Hand Side.

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