

*Special Issue: The International Conference on Non-linear Analysis
(organized to mark the 70th birthday anniversary of Prof. Charles
Ejike Chidume)*

CELEBRATING THE 70TH BIRTHDAY OF PROFESSOR CHARLES E. CHIDUME

VASILE BERINDE

ABSTRACT. This is an abbreviated version of the plenary lecture given by the author to the International Conference on Nonlinear Analysis. In Honour of Prof. C. E. Chidume, University of Nigeria, Nsukka, 12-15 March 2018. Most of the aspects included in that lecture, especially pictures and graphics, are not more included here for obvious reasons and, instead of them, some other information has been selected in the effort to shape a sincere exercise of admiration to an outstanding mathematician, Professor Charles E. Chidume, African University of Science and Technology (AUST), Abuja, Nigeria.

1. INTRODUCTION

Professor Charles Ejike Chidume celebrated his 70th birthday in 2018. An important event, "The International Conference on Non-linear Analysis. In Honour of Prof. Charles E. Chidume", which was preceded by an extremely impressive valedictory lecture, has been organised in his home country and at his Alma Mater, University of Nigeria, Nsukka, Nigeria, in the period 13-15 March 2018. I have been amongst the very few guest speakers at this event and have given a lecture on his research and mentoring activity.

This paper is an adapted written version of that plenary lecture [3]. Despite the fact that most of the aspects included in the lecture, especially the oral part and the accompanying illustrations, cannot be included here for obvious reasons, I shall try to give an extended abstract of it, accompanied by some other information that has been selected in my effort to shape a sincere exercise of admiration to an outstanding mathematician and friend, Professor Charles E. Chidume.

Received by the editors November 14, 2019; ; Accepted: November 14, 2019
www.nigerianmathematicalsociety.org; Journal available online at <https://ojs.ictp.it/jnms/>

It is important to state that Professor C. E. Chidume is still pursuing a very active research career at AUST (African University of Science and Technology), Abuja, with an enthusiastic research group that I visited and to which I delivered some invited lectures in March 2018, during my visit in Nigeria.

He is highly regarded and well-respected both internationally and nationally in the mathematics community for his research accomplishments, exemplary mentorship, and equally, for his pioneering work, teaching activity and important contributions in the area of iterative approximation of fixed points.

2. HOW I MET CHARLES CHIDUME

I met Charles Chidume for the first time virtually, that is, by reading his papers from the time of my very first mathematical research work devoted to iterative approximation of fixed points. This happened in my postdoc period, when I realised how important for Fixed Point Theory is not only to obtain new existence theorems or new existence and uniqueness theorems and use them in various concrete applications - what I basically did in my PhD thesis - but also to construct appropriate iterative schemes in order to approximate those fixed points. Such an approach inevitably directs to the important problem of providing convergence theorems for such iterative schemes in more general contexts.

While my first research results were concerned exclusively with the fixed point problem for self mappings defined on a (generalized) metric space and satisfying a certain generous metric contractive condition - that was always guaranteeing the convergence of the method of successive approximations or Picard iteration - later on I discovered how important is to consider weaker contraction conditions, like the nonexpansiveness type conditions, when the fixed point problem becomes more interesting and at the same time more complicated.

At this stage, I planned to write a monograph whose first accomplished part was its title: **Iterative Approximation of Fixed Points**. This was a very important step, indeed, since it essentially designed the contents of the book. Then, normally, I started to collect bibliography for this ambitious project - I collected in a rather short period of time more than 1000 significant papers, all in printed form - and just at that moment I realized how huge and variate research work has been done on this topic.

This was the moment when I came across with some of Chidume's papers: all of them to which I have had access have been included in the short list for the planned book and were included in the list of References of the first edition [1] of the monograph *Iterative Approximation of Fixed Points*, printed at Efemeride Publishing House, in 2002. Its second revised and enlarged edition [2] has been published in 2007 by Springer. It is very probably that the very favourable review to that book in Mathematical Reviews, see [12], written by a very respected specialist in the field, Prof. Billy E. Rhoades from Indiana University (Bloomington), attracted Charles Chidume's attention about myself since, at the beginning of 2004, I received an official letter from Prof. Chidume, inviting me to visit ICTP Trieste, to which he was affiliated for a long period of time.

Due to my overwhelming teaching and administration duties - equally then and nowadays ! - I was able to visit the International Centre for Theoretical Physics (ICTP) Trieste and Prof. Chidume only in November 2004. Despite the fact it was a very short visit - no more than three weeks - it was extremely important and fruitful experience for me. First of all, this visit offered to me the opportunity to meet in person a mathematician who previously impressed me by his consistent and well written papers - some of them will be mentioned later - but the visit was also memorable by the extraordinary atmosphere that I found inside and around ICTP.



The author and C. E. Chidume, ICTP, Trieste, Italy, November 2004



Professor C. E. Chidume as a plenary speaker at ICAM6, Baia Mare, September 18-21, 2008

A second visit of mine to ICTP Trieste was possible - for the same reasons - only after 4 years, in June 2008, when Charles Chidume was just finishing his very important monograph [4], that was published in 2009 by Springer.

This second visit of mine has been followed by a visit of C. Chidume to my home university, North University of Baia Mare, where he was a main speaker to The Sixth International Conference on Applied Mathematics (ICAM6), Baia Mare, September 18-21, 2008. C. Chidume accepted my invitation to come to Baia Mare mainly because he knew that there he will have the opportunity to meet in person a Romanian mathematician he admired much: Professor Ștefan Mărușter (1937-2017), the one who introduced the class of demicontractive mappings, which is an extremely important class of nonexpansive type mappings in the iterative approximation of fixed points.

Coming back to the moment of the Conference dedicated to him, it was for me a privilege to present, after my lecture, the certificate awarded by Romanian Mathematical Society to Professor Chidume, for his merits in promoting the results of some Romanian mathematicians: Ș. Mărușter, C. Zălinescu, Ioana Ciorănescu, D. Pascali, V. Barbu, G. Isac etc.



The Honour certificate awarded by Romanian Mathematical Society to Professor C. E. Chidume

This award was essentially due to the fact that Professor C. E. Chidume valorised some contributions of Ștefan Mărușter and has the merit that, in the framework of his research group, see [11], it was remarked that the demicontractive operators, introduced in 1977 by Hicks and Kubicek [8] under this name, are equivalent to the class of operators possessing "property (A)", introduced in 1977 by Mărușter [10], for solving the fixed point problem $x = Tx$ in Hilbert spaces and, in the case of the nonlinear equation $U(x) = 0$ in \mathbb{R}^n , in 1973, see [9].

In fact, Chidume and Mărușter have also co-authored a very important survey paper [6], where the paternity issue of demicontractive mappings has been explicitly and definitively solved, Mărușter's priority being clearly stated and proved.

3. MAIN SCIENTIFIC CONTRIBUTIONS OF PROFESSOR C. E. CHIDUME

It is difficult to select amongst the very diversified mathematical contributions of Professor C. Chidume the few ones that would be

the most significant, because all he did in his career was situated at the highest possible level:

- (1) with respect to his scientific and professional career
- (2) with respect to the topics of his research work
- (3) since he has published mainly in high level mathematical journals
- (4) since he has a unique topic of research where he is a master
- (5) since he was extremely rigorous with himself and with his students
- (6) since he has the merit of relaunched the interest for Iterative Approximation of Fixed Points
- (7) since he has a great visibility and impact amongst researchers working in this area
- (8) since he has had a significant number of students and followers
- (9) since he has had continuous and consistent contact with native country
- (10) since he has an important contribution to raising the mathematical level of his country, Nigeria, and of the African continent as well.

We exemplify only item no. 3, by indicating the journals where he published a significant number of papers, according to Web of Science:

- Journal of Mathematical Analysis and Applications (25 papers)
- Nonlinear Analysis Theory Methods Applications (22 papers)
- Proceedings of the American Mathematical Society (14 papers)
- Fixed Point Theory and Applications (9 papers)
- Applied Mathematics and Computation (5 papers)
- Fixed Point Theory (4 papers)
- Carpathian Journal of Mathematics (3)
- Indian Journal of Pure Applied Mathematics (3)
- Numerical Functional Analysis and Optimization (3)
- Journal of Fixed Point Theory and Applications (2)
- Journal of Nonlinear and Convex Analysis (2)
- Journal of Nonlinear and Variational Analysis (2)
- Optimization (2)
- Proceedings of the Edinburgh Mathematical Society (2)
- ...

If I would be asked to choose exactly one contribution from the very many important results obtained by C. E. Chidume during his long, productive and really impressive research career, I would be in a serious difficulty, because I am highly appreciating all of his contributions, most of them being presented in his dense monograph [4].

However, if I would be forced to have a preference anyway, it is very probably that I would consider the following iterative method

$$x_{n+1} = (1 - \lambda_n)x_n + \lambda_n T x_n - \lambda_n \theta_n (x_n - x_1), n \geq 1, \quad (1)$$

which is a very unusual perturbation of the classical normal Mann iteration

$$x_{n+1} = (1 - \lambda_n)x_n + \lambda_n T x_n, n \geq 0. \quad (2)$$

The algorithm (1) has been considered by Chidume and Zegeye in [5] in order to approximate the fixed points of Lipschitzian pseudo-contractive mappings $T : K \longrightarrow K$, with $Fix(T) \neq \emptyset$, where K is a nonempty, closed and convex subset of a real Banach space E .

It is well known that, for many important classes of nonexpansive type mappings (nonexpansive, quasi-nonexpansive, strictly pseudocontractive in the sense of Browder, demicontractive, pseudocontractive etc.), in case that Mann iteration (2) converges to a fixed point of T , the convergence is generally only *weak*, while strong convergence is usually obtained by requiring some additional conditions (like demicompactness, for example).

One of the most impressive merits of the iterative algorithm (1) is that, despite the fact that the expression in the right hand side of (1) is not more a convex combination - like in the case of Mann iteration (2) - it converges *strongly* to a fixed point of T and, even being simpler computationally, it successfully replaces Ishikawa iteration and the hybrid Halpern-Ishikawa iteration, under the following setting, see for example Theorem 6.11 in [4]:

- (1) E is a real Banach space with a uniformly Gâteaux differentiable norm;
- (2) $K \subset E$ is nonempty, closed and convex;
- (3) T is Lipschitz pseudo-contractive with $Fix(T) \neq \emptyset$;
- (4) $\{\lambda_n\}$ and $\{\theta_n\}$ are real sequences in $(0, 1)$ satisfying some technical conditions.

The same interesting algorithm (1) has been used in a more recent paper [7] to approximate zeros of inclusion problems for bounded accretive operators in uniformly smooth Banach spaces.

4. QUANTITATIVE AND QUALITATIVE IMPACT AND INTERNATIONAL VISIBILITY

Professor Charles Chidume has an outstanding record of scholarship. He has published a fundamental monograph in the field of iterative approximation of fixed points [4], devoted to the important role of geometric properties of Banach spaces in the study of nonlinear fixed point iterations, 4 textbooks, and more than 165 refereed papers, according to MathScinet, with 1401 citations inside MathScinet; 155 refereed papers, according to zbMATH; 140 papers indexed in SCOPUS, with 2558 citations inside SCOPUS and 112 papers indexed in Web of Science, which collected so far 1778 independent citations inside WoS.

By using one of the reports offered by *zbMATH* database, we present the distribution of Professor Chidume' main publications, according to **Mathematics Subject Classification (MSC 2010)**, which gives a better view on his mathematics contributions. Note that the topic of a certain reviewed paper may fall into two, three or many different MSC 2010 subjects.

- Operator theory (47-XX): **148** reviewed papers;
- Numerical analysis (65-XX): **19** reviewed papers;
- Calculus of variations and optimal control; optimization (49-XX): **9** reviewed papers;
- General topology (54-XX): **8** reviewed papers;
- Functional analysis (46-XX): **7** reviewed papers;
- Operations research, mathematical programming (90-XX): **4** reviewed papers;
- Integral equations (45-XX): **3** reviewed papers;
- Computer science (68-XX): **2** reviewed papers;
- Information and communication, circuits (94-XX): **2** reviewed papers;
- Real functions (26-XX): **1** reviewed paper;
- Integral transforms, operational calculus (44-XX): **1** reviewed paper;

Now, by using the corresponding reports extracted from *Math-Scinet* database, we have the following distribution of Professor Chidume' publications (figures are not essentially different of the ones from *zbMATH* but they illustrate clearer the fact that he had a very specific and constant area of research).

- Operator theory: **145** reviewed papers;
- Numerical analysis: **8** reviewed papers;

- Calculus of variations and optimal control; optimization: **6** reviewed papers;
- Integral equations: **4** reviewed papers;
- Integral transforms, operational calculus: **1** reviewed paper;

We generally evaluate the international impact and visibility of a scholar by considering the figures offered by various scientometric and bibliographic databases. In this respect, we give a list of his most cited papers, according to Web of Science (from SCOPUS and MathScinet we get quite similar figures).

In Web of Science we found 112 papers indexed, which collected together 1778 independent citations. The first ten most cited papers by C. E. Chidume and collaborators are given in the following list.

1. Chidume, C. E. and Mutangadura, S. A., *An example on the Mann iteration method for Lipschitz pseudocontractions*, Proc. Amer. Math. Soc. **129** (2001), No: 8, 2359–2363. (**165** independent citations);
2. Chidume, C. E., *Iterative approximation of fixed points of Lipschitzian strictly pseudocontractive mappings*, Proc. Amer. Math. Soc. **99** (1987), no. 2, 283–288. (**116** independent citations);
3. Chidume, C. E., Ofoedu, E. U. and Zegeye, H., *Strong and weak convergence theorems for asymptotically nonexpansive mappings*, J. Math. Anal. Appl. **280** (2003), no. 2, 364–374. (**107** independent citations);
4. Chidume, C. E., *Approximation of fixed points of strongly pseudocontractive mappings*, Proc. Amer. Math. Soc. **120** (1994), no. 2, 545–551. (**100** independent citations);
5. Chidume, C. E. and Chidume, C. O., *Iterative approximation of fixed points of nonexpansive mappings*, J. Math. Anal. Appl. **318** (2006), no. 1, 288–295. (**77** independent citations);
6. Chidume, C. E. and Shahzad, N., *Strong convergence of an implicit iteration process for a finite family of nonexpansive mappings*, Nonlinear Anal. **62** (2005), no. 6, 1149–1156. (**71** independent citations);
7. Chidume, C. E., *An iterative process for nonlinear Lipschitzian strongly accretive mappings in L_p spaces*, J. Math. Anal. Appl. **151** (1990), no. 2, 453–461. (**70** independent citations);
8. Chidume, C. E. and Moore, Chika, *Fixed point iteration for pseudocontractive maps*, Proc. Amer. Math. Soc. **127** (1999), no. 4, 1163–1170. (**64** independent citations);

9. Chidume, C. E., *Iterative solution of nonlinear equations with strongly accretive operators*, J. Math. Anal. Appl. **192** (1995), no. 2, 502–518. (**63** independent citations);
10. Chidume, C. E. and Zegeye, H., *Approximate fixed point sequences and convergence theorems for Lipschitz pseudocontractive maps*, Proc. Amer. Math. Soc. **132** (2004), no. 3, 831–840. (**54** independent citations);

It would be also interesting to have a look on Google Scholar, to see that C. E. Chidume's papers collected so far 5104 citations. The first four most cited papers by C. E. Chidume, according to Google Scholar, are given in the following list.

1. Chidume, C., *Geometric properties of Banach spaces and non-linear iterations*, Lecture Notes in Mathematics, 1965. Springer-Verlag London, Ltd., London, 2009. (**390** citations);
2. Chidume, C. E. and Mutangadura, S. A., *An example on the Mann iteration method for Lipschitz pseudocontractions*, Proc. Amer. Math. Soc. **129** (2001), No: 8, 2359–2363. (**279** citations);
3. Chidume, C. E., *Iterative approximation of fixed points of Lipschitzian strictly pseudocontractive mappings*, Proc. Amer. Math. Soc. **99** (1987), no. 2, 283–288. (**245** citations);
4. Chidume, C. E., Ofoedu, E. U. and Zegeye, H., *Strong and weak convergence theorems for asymptotically nonexpansive mappings*, J. Math. Anal. Appl. **280** (2003), no. 2, 364–374. (**203** citations);

In my opinion, to have a complete image of the scenery, we have to also consider an important invisible side of the visibility of a scientist, i.e., the number and quality of his students and followers, expressed by their own scientific contributions. According to a recent CV, Professor Chidume supervised (or is still supervising) more than 20 PhD students. What is really impressive is the fact that, 6 of his former PhD students are already Full Professors (it would be not surprising if this list has been enlarged in the meantime):

- (1) Professor A.N. Eke, University of Nigeria, Nsukka, Nigeria.
- (2) Professor M.O. Osilike, University of Nigeria, Nsukka, Nigeria.
- (3) Professor Chika Moore, Nnamdi Azikiwe University, Awka, Nigeria.
- (4) Professor Habtu Zegeye, Formally of Bahir Dar University, Ethiopia, now of the University of Botswana, Southern Africa.

- (5) Professor Eric U. Ofoedu, Nnamdi Azikiwe University, Awka, Nigeria.
- (6) Professor Bashir Ali, Bayero University, Kano, Nigeria.

Therefore, apart of the remarkable impact and visibility of Professor Chidume himself, described above, we have to add a weighted quota of the scientific contributions of his students and followers for papers that were not co-authored with their supervisor.

I mention here only the data for four of them, extracted from MathScinet: Habtu Zegeye (122 publications, 746 citations); M.O. Osilike (66 publications, 615 citations), Bashir Ali (44 publications, 77 citations) and Eric U. Ofoedu (32 publications, 158 citations).

5. CONCLUSIONS

I kindly invite the readers and all active researchers working in the field of Nonlinear Analysis to join me in congratulating Professor Charles E. Chidume for his 70th anniversary and for his impressive academic activity by wishing him a long, happy, healthy, and productive life, alongside of his beautiful family, by adding many more contributions to the field of iterative approximation of fixed points as a scientist, as a mentor, and also as a research group leader.

REFERENCES

- [1] Berinde, V., *Iterative Approximation of Fixed Points*, Efemeride, Baia Mare, 2002.
- [2] Berinde, V., *Iterative Approximation of Fixed Points*, Second edition. Lecture Notes in Mathematics, 1912. Springer, Berlin, 2007.
- [3] Berinde, V., *In honour and celebration of the 70th birthday of Professor Charles E. Chidume*, International Conference on Nonlinear Analysis. In Honour of Prof. C. E. Chidume, University of Nigeria, Nsukka, 12-15 March 2018.
- [4] Chidume, C. E., *Geometric Properties of Banach Spaces and Nonlinear Iterations*, Lecture Notes in Mathematics, 1965. Springer-Verlag London, Ltd., London, 2009.
- [5] Chidume, C. E. and Zegeye, H., *Approximate fixed point sequences and convergence theorems for Lipschitz pseudocontractive maps*, Proc. Amer. Math. Soc. **132** (2004), no. 3, 831–840.
- [6] Chidume, C. E. and Mărușter, Șt., *Iterative methods for the computation of fixed points of demicontractive mappings*, J. Comput. Appl. Math. **234** (2010), no. 3, 861–882.
- [7] Chidume, C. E., *Strong convergence theorems for bounded accretive operators in uniformly smooth Banach spaces*. Nonlinear analysis and optimization, 31–41, Contemp. Math., 659, Amer. Math. Soc., Providence, RI, 2016.
- [8] Hicks, T. L. and Kubicek, J. D., *On the Mann iteration process in a Hilbert space* J. Math. Anal. Appl. **59** (1977), no. 3, 498–504.
- [9] Mărușter, Șt., *Sur le calcul des zéros d'un opérateur discontinu par itération*, Canad. Math. Bull. **16** (1973), 541–544.
- [10] Mărușter, Șt., *The solution by iteration of nonlinear equations in Hilbert spaces*, Proc. Amer. Math. Soc. **63** (1977), no. 1, 69–73.

- [11] Moore, Chika, *Iterative approximation fixed points of demicontractive maps*, The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, Scientific Report IC/98/214, 1998, 12 pages
- [12] Rhoades, B. E., Review Zbl 1036.47037 to [Berinde, Vasile, *Iterative Approximation of Fixed Points*, Efemeride, Baia Mare, 2002].

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE, TECHNICAL UNIVERSITY OF CLUJ-NAPOCA, NORTH UNIVERSITY CENTER AT BAI A MARE, VICTORIEI 76, 430122 BAI A MARE, ROMANIA

E-mail address: `vberinde@cunbm.utcluj.ro`