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SPLIT EQUALITY FIXED POINT PROBLEM FOR α -DEMICONTRACTIVE MAPPINGS IN HILBERT SPACES

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ABSTRACT.In this paper, we study the split equality fixed point problem for α- demicontractive mappings in the framework of infinite dimensional real Hilbert spaces. The study is a continuation of a recent study of a new iterative algorithm by Zhaoli Ma *et al* [11] in which they proved weak and strong convergence theorems of the new algorithm for strictly-pseudocontractive mappings. As an extension of their work, we extend the class of mappings to α -demicontractive mappings which is more general than the class of strictly-pseudocontractive mappings with nonempty fixed point set. The results presented in this paper extend and complement many related papers in literature.

Keywords and phrases: Split equality problem, α- demicontractive mappings; bounded linear operator; fixed point set, weak convergence, strong convergence.
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1. INTRODUCTION

The split equality fixed point problem was introduced by Moudafi and Al-Shemas [6]. Their results serve as a generalisation of the split feasibility problem introduced by Censor and Elfving [3] which appears as inverse problems in phase retrieval, medical image reconstruction, intensity modulated radiation therapy, computer tomograph and so on (see Byrne [1], Censor *et al* [2], Censor *et al* [3]). In the words of Moudafi and Al-Shemas [6], the Split equality

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fixed point problem is concerned with finding

$$x \in C = F(U) \textit{ and } y \in Q = F(T) \textit{ such that } Ax = By. \tag{1.1}$$

Where $A : H_1 \to H_3$ and $B : H_2 \to H_3$ are two bounded linear operators, H_1, H_2 and H_3 are real Hilbert spaces. While, $U : H_1 \to H_1$ and $T : H_2 \to H_2$ are firmly quasi-nonexpansive mappings. In a bid to estimate the solution of problem (1.1), Moudafi and Al-Shemas [6] formulated the under given iteration and achieved weak convergence theorem.

$$\begin{cases} x_{n+1} = U(x_n - \gamma_n A^*(Ax_n - By_n)); \\ y_{n+1} = T(y_n + \gamma_n B^*(Ax_n - By_n)) \end{cases} \tag{1.2}$$

Where A^* and B^* are the respective adjoints of A and B with λ_A and λ_B being the spectral radii of A^*A and B^*B respectively; and $\gamma_n \in \left(\epsilon, \dfrac{2}{[(1-t)^2 + \beta^2](\lambda_A + \lambda_B)} - \epsilon\right)$.

Inspired by the above innovation, Zhaoli *et al* [11] constructed the iterative algorithm below in order to solve (1.1) for strictly psudocontractive mappings in Hilbert spaces.

$$\begin{cases} \forall\ x_1 \in H_1,\ \forall\ y_1 \in H_2 \\ x_{n+1} = t_n x_n + (1-t_n)T_1(x_n - \gamma_n A^*(Ax_n - By_n)) \\ y_{n+1} = t_n y_n + (1-t_n)T_2(y_n + \gamma_n B^*(Ax_n - By_n)) \end{cases} \tag{1.3}$$

for all $n \in \mathbb{N}$ where λ_A and λ_B stand for the spectral radii of A^*A and B^*B respectively, $\{t_n\}$ is a sequence in $(0,1)$ with $\dfrac{k+1}{2} < t \le t_n \le \beta < 1$ (for some $t, \beta \in (0,1)$) and $\{\gamma_n\}$ is a positive real sequence such that $\gamma_n \in \left(\epsilon, \dfrac{2(1-\beta)}{[(1-t)^2 + \beta^2](\lambda_A + \lambda_B)} - \epsilon\right)$ for ϵ small enough.

Later, Chidume *et al* [4] developed a Krasnolseskii-type iterative scheme below so as to solve (1.1) for demicontractive mappings U and T.

$$\begin{cases} \forall\ x_1 \in H_1,\ \forall\ y_1 \in H_2 \\ x_{n+1} = (1-\alpha)(x_n - \gamma_n A^*(Ax_n - By_n)) \\ +\alpha U(x_n - \gamma_n A^*(Ax_n - By_n)) \\ y_{n+1} = (1-\alpha)(y_n + \gamma_n A^*(Ax_n - By_n)) \\ +\alpha T(y_n + \gamma_n A^*(Ax_n - By_n)) \end{cases} \tag{1.4}$$

In both cases above, the authors extended the original work of Moudafi and Al-Shemas [6] by extending the class of mappings and by gaining strong convergence theorems under mild conditions. At this juncture, it becomes natural to investigate whether strong convergence theorems for the split equality fixed point problem can still be obtained when the class of mappings is extended to α -demicontractive mappings which is more general than the aforementioned classes of mappings.

Motivated by the work of Moudafi and Al-Shemas [6], Zhaoli *et al* [11], Chidume *et al* [4] and Măruşter and Măruşter [5], this present work aims at proving strong convergence of (1.3) for the class of α-demicontractive mappings . As demonstrated by Osilike and Onah [9] as well as Măruşter and Măruşter [5], the class of α-demicontractive mappings properly contains the class of demicontractive mappings.

Hence, our theorems extend and generalise the results of Moudafi and Al-Shemas [6], Zhaoli *et al* [11] and Chidume *et al* [4]to mention but a few.

2. PRELIMINARY

Here, we recall some relevant definitions and lemmas which will be needed in the proof of our main result. In what shall follow, we denote strong and weak convergence by "$\rightarrow$" and "$\rightharpoonup$" respectively, the fixed point set of a mapping T by $F(T)$ and the solution set of (1.1) by Γ.

Defiinition 2.1 [Demiclosedness principle] Let H be a real Hilbert space and $T : H \rightarrow H$ be a mapping, then $(I - T)$ is said to be demiclosed at zero if for any sequence, $\{x_n\} \subset H$ with $x_n \rightharpoonup x^*$ and $(I - T)x_n \rightarrow 0$, we have $x^* = Tx^*$.

Defiinition 2.2 A single valued mapping $T : C \rightarrow C$ is said to be *demicompact*, if for any bounded sequence $\{x_n\} \subset C$ with $\|(I - T)x_n\| \rightarrow 0$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\{x_{n_k}\}$ converges strongly to a point $p \in C$.

Defiinition 2.3 Let H be a real Hilbert space with $C \subset H$ being nonempty then a mapping $T : C \rightarrow C$ is said to be k- strictly pseudo-contractive if there exists a constant $k \in (0, 1)$ such that

$$\|Tx-Ty\|^2 \ \leq \ \|x-y\|^2+k\|(I-T)x-(I-T)y\|^2 \ \forall \ x, y \in C \quad (2.1)$$

Defiinition 2.4 Let H be a real Hilbert space with $C \subset H$ being nonempty then a mapping $T : C \rightarrow C$ is said to be demi-contractive if $F(T) \neq \emptyset$ and there exists a constant $k \in (0, 1)$ such that

$$\|Tx - p\|^2 \ \leq \ \|x - p\|^2 + k\|x - Tx\|^2 \ \forall \ x \in C, \ \forall \ p \in F(T) \quad (2.2)$$

Defiinition 2.5 Let H be a real Hilbert space with $C \subset H$ being nonempty then a mapping $T : C \to C$ is said to be α-demi-contractive if $F(T) \neq \emptyset$ and there exist $\alpha \geq 1$ and a constant $\lambda > 0$ such that

$$\langle x - Tx, x - \alpha p\rangle \geq \lambda \|x - Tx\|^2 \ \forall (x, p) \in C \times F(T) \tag{2.3}$$

Clearly, (2.3) is equivalent to

$$\|Tx - \alpha p\|^2 \ \leq \ \|x - \alpha p\|^2 + k\|x - Tx\|^2 \ \forall (x, p) \in C \times F(T) \tag{2.4}$$

where $k = 1 - 2\lambda \in [0, 1)$.

Remark 2.6 [5] Every k- strictly pseudo-contractive mapping with nonempty fixed point set is demi-contractive. Again, every demi-contractive is α-demi-contractive with α =1 but the converse is false as shown by Osilike and Onah [9] as well as Măruşter and Măruşter [5].

Lemma 2.7 ([Opial Lemma [8]) Let H be a real Hilbert space and let $\{\mu_n\}$ be a sequence in H such that there exists a nonempty set $W \subset H$ satisfying the following:
(i) for every $\mu^* \in W$, $\{\|\mu_n - \mu^*\|\}$ converges and (ii) any weak cluster point of the sequence $\{\mu_n\}$ belongs to W, then, there exists $w^* \in W$ such that $\{\mu_n\}$ weakly converges to w^*.

Lemma 2.8 [7,10] Let $\{a_n\}$ be a sequence of nonnegative real numbers satisfying the following relations $a_{n+1} \leq a_n + \sigma_n, n \geq 0$, such that $\sum_{n=1}^{\infty} \sigma_n < \infty$. Then, $\{a_n\}$ is convergent. If in addition that $\{a_n\}$ has a subsequence, $\{a_{n_k}\}$ that converges to 0, then $\{a_n\}$ converges to 0 as $n \to \infty$.

3. THE HEART OF THE MATTER

Theorem 3.1: Suppose H_1, H_2 and H_3 are real Hilbert spaces with $A : H_1 \to H_3$ and $B : H_2 \to H_3$ being two bounded linear operators, While, $T_1 : H_1 \to H_1$ as well as $T_2 : H_2 \to H_2$ are α-demicontractive mappings having constants k_1 and k_2 respectively for the same $\alpha \geq 1$. Suppose in addition that $I - T_1$ and $I - T_2$ are demiclosed at 0 with T_1 as well as T_2 being uniformly continuous and $\Gamma = \{(p, q) \in F(T_1) \times F(T_2) : Ap = Bq\} \neq \emptyset$. Then, for arbitrary $x_1 \in H_1$ and $y_1 \in H_2$, the iterative scheme defined for all $n \in \mathbb{N}$ by

$$\begin{cases} x_{n+1} = t_n x_n + (1 - t_n)T_1(x_n - \gamma_n A^*(Ax_n - By_n)) \\ \\ y_{n+1} = t_n y_n + (1 - t_n)T_2(y_n + \gamma_n B^*(Ax_n - By_n)) \end{cases} \tag{3.1}$$

converges weakly to a solution of (1.1).Where λ_A and λ_B stand for the spectral radii of A^*A and B^*B respectively, $\{t_n\}$ is a sequence in $(0,1)$ with $\frac{k+1}{2} < t \leq t_n \leq \beta < 1$ (for some $t, \beta \in (0,1)$) and $\{\gamma_n\}$ is a positive real sequence such that
$\gamma_n \in \left(\epsilon, \frac{2(1-\beta)}{[(1-t)^2+\beta^2](\lambda_A+\lambda_B)} - \epsilon\right)$ for ϵ small enough and $k = max\{k_1, k_2\}$.

Proof. From the assumption, $\Gamma \neq \emptyset$, let $(p,q) \in \Gamma$ be arbitrary and for purpose of simplicity, let $w_n = x_n - \gamma_n A^*(Ax_n - By_n)$ and $z_n = y_n + \gamma_n B^*(Ax_n - By_n)$. Then,

$$
\begin{aligned}
\|x_{n+1} - \alpha p\|^2 &= \|[t_n x_n + (1-t_n)T_1 w_n - \alpha p\|^2 \\
&= \|t_n(x_n - \alpha p) + (1-t_n)(T_1 w_n - \alpha p)\|^2 \\
&= t_n^2\|x_n - \alpha p\|^2 + (1-t_n)^2\|T_1 w_n - \alpha p\|^2 \\
&\quad +2t_n(1-t_n)\langle x_n - \alpha p, T_1 w_n - \alpha p\rangle \\
&\leq t_n^2\|x_n - \alpha p\|^2 + (1-t_n)^2\|w_n - \alpha p\|^2 \\
&\quad +k(1-t_n)^2\|w_n - T_1 w_n\|^2 \\
&\quad +2t_n(1-t_n)\langle x_n - \alpha p, T_1 w_n - \alpha p\rangle \\
&\quad = t_n^2\|x_n - \alpha p\|^2 + k(1-t_n)^2\|(I-T_1)w_n\|^2 \\
&\quad +(1-t_n)^2\|x_n - \gamma_n A^*(Ax_n - By_n) - \alpha p\|^2 \\
&\quad +2t_n(1-t_n)\langle x_n - \alpha p, T_1 w_n - w_n + w_n - \alpha p\rangle \\
&= t_n^2\|x_n - \alpha p\|^2 + (1-t_n)^2\|x_n - \alpha p\|^2 \\
&\quad +(1-t_n)^2\|\gamma_n A^*(Ax_n - By_n)\|^2 \\
&\quad -2(1-t_n)^2\langle x_n - \alpha p, \gamma_n A^*(Ax_n - By_n)\rangle \\
&\quad +k(1-t_n)^2\|(I-T_1)w_n\|^2 \\
&\quad +2t_n(1-t_n)\langle x_n - \alpha p, (T_1 - I)w_n\rangle \\
&\quad +2t_n(1-t_n)\langle x_n - \alpha p, w_n - \alpha p\rangle \\
&= [t_n^2 + (1-t_n)^2]\|x_n - \alpha p\|^2 \\
&\quad +(1-t_n)^2\|\gamma_n A^*(Ax_n - By_n)\|^2 \\
&\quad -2\gamma_n(1-t_n)^2\langle Ax_n - A\alpha p, Ax_n - By_n\rangle \\
&\quad +k(1-t_n)^2\|(I-T_1)w_n\|^2 \\
&\quad +2t_n(1-t_n)\langle x_n - \alpha p, (T_1 - I)w_n\rangle \\
&\quad +2t_n(1-t_n)\langle x_n - \alpha p, w_n - \alpha p\rangle
\end{aligned}
$$

$$
\begin{aligned}
&= \|x_n - \alpha p\|^2 + (1-t_n)^2\|\gamma_n A^*(Ax_n - By_n)\|^2 \\
&\quad +k(1-t_n)^2\|(I-T_1)w_n\|^2 \\
&\quad -2\gamma_n(1-t_n)\langle Ax_n - A\alpha p, Ax_n - By_n\rangle \\
&\quad +2t_n(1-t_n)\langle w_n - \alpha p, (T_1 - I)w_n\rangle \\
&\quad +2t_n(1-t_n)\langle \gamma_n A^*(Ax_n - By_n), (T_1 - I)w_n\rangle \\
&\leq \|x_n - \alpha p\|^2 + (1-t_n)^2\|\gamma_n A^*(Ax_n - By_n)\|^2 \\
&\quad +k(1-t_n)^2\|(I-T_1)w_n\|^2 \\
&\quad -2\gamma_n(1-t_n)\langle Ax_n - A\alpha p, Ax_n - By_n\rangle \\
&\quad -t_n(1-t_n)(1-k)\|(T_1 - I)w_n\|^2 \\
&\quad +2t_n(1-t_n)\|\gamma_n A^*(Ax_n - By_n)\|\|(T_1 - I)w_n\| \\
&\leq \|x_n - \alpha p\|^2 \\
&\quad +[t_n^2 + (1-t_n)^2]\|\gamma_n A^*(Ax_n - By_n)\|^2 \\
&\quad +(1-t_n)(k+1-2t_n)\|(I-T_1)w_n\|^2 \\
&\quad -2\gamma_n(1-t_n)\langle Ax_n - A\alpha p, Ax_n - By_n\rangle \\
&= \|x_n - \alpha p\|^2 \\
&\quad +[t_n^2 + (1-t_n)^2]\gamma_n^2\langle Ax_n - By_n, AA^*(Ax_n - By_n)\rangle \\
&\quad +(1-t_n)(k+1-2t_n)\|(I-T_1)w_n\|^2 \\
&\quad -2\gamma_n(1-t_n)\langle Ax_n - A\alpha p, Ax_n - By_n\rangle \\
&\leq \|x_n - \alpha p\|^2 \\
&\quad +[t_n^2 + (1-t_n)^2]\gamma_n^2\lambda_A\langle Ax_n - By_n, Ax_n - By_n\rangle \\
&\quad -(1-t_n)(2t_n - k - 1)\|(I-T_1)w_n\|^2 \\
&\quad -2\gamma_n(1-t_n)\langle Ax_n - A\alpha p, Ax_n - By_n\rangle \\
&\quad = \|x_n - \alpha p\|^2 \\
&\quad +[t_n^2 + (1-t_n)^2]\gamma_n^2\lambda_A\|Ax_n - By_n\|^2 \\
&\quad -(1-t_n)(2t_n - k - 1)\|(I-T_1)w_n\|^2 \\
&\quad -2\gamma_n(1-t_n)\langle Ax_n - A\alpha p, Ax_n - By_n\rangle \qquad (3.2)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\|y_{n+1} - \alpha q\|^2 \leq\; & \|y_n - \alpha q\|^2 + [t_n^2 + (1-t_n)^2]\gamma_n^2\lambda_B\|Ax_n - By_n\|^2 \\
& -(1-t_n)(2t_n - k - 1)\|(I-T_2)z_n\|^2 \\
& +2\gamma_n(1-t_n)\langle By_n - B\alpha q, Ax_n - By_n\rangle \qquad (3.3)
\end{aligned}
$$

Let $\Gamma_n(x, y) = \|x_n - \alpha p\|^2 + \|y_n - \alpha q\|^2$. Since $(p, q) \in \Gamma$, we have that $Ap = Bq$ and $A(\alpha p) = B(\alpha q)$. Hence, on addition of (3.2) and

(3.3), we have

$$\begin{aligned}
\Gamma_{n+1}(x,y) &\leq \Gamma_n(x,y)\\
&\quad +[t_n^2+(1-t_n)^2]\gamma_n^2(\lambda_A+\lambda_B)\|Ax_n-By_n\|^2\\
&\quad -2\gamma_n(1-t_n)\langle Ax_n-By_n, Ax_n-By_n\rangle\\
&\quad +2\gamma_n(1-t_n)\langle Ax_n-By_n, Ax_n-By_n\rangle\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_1)w_n\|^2\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_2)z_n\|^2\\
&= \Gamma_n(x,y)\\
&\quad +[t_n^2+(1-t_n)^2]\gamma_n^2(\lambda_A+\lambda_B)\|Ax_n-By_n\|^2\\
&\quad -2\gamma_n(1-t_n)\|Ax_n-By_n\|^2\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_1)w_n\|^2\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_2)z_n\|^2\\
&= \Gamma_n(x,y)\\
&\quad -\gamma_n\{2(1-t_n)-[t_n^2+(1-t_n)^2]\gamma_n(\lambda_A+\lambda_B)\}\\
&\quad \times\|Ax_n-By_n\|^2\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_1)w_n\|^2\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_2)z_n\|^2\\
&\leq \Gamma_n(x,y)\\
&\quad -\gamma_n\{2(1-\beta)-[\beta^2+(1-t)^2]\gamma_n(\lambda_A+\lambda_B)\}\\
&\quad \times\|Ax_n-By_n\|^2\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_1)w_n\|^2\\
&\quad -(1-t_n)(2t_n-k-1)\|(I-T_2)z_n\|^2 \qquad (3.4)
\end{aligned}$$

It is clear from (3.4) that

$$\|x_{n+1}-\alpha p\|^2+\|y_{n+1}-\alpha q\|^2 \leq \|x_n-\alpha p\|^2+\|y_n-\alpha q\|^2 \qquad (3.5)$$

Thus, $\{\Gamma_n(x,y)\}$ is monotone decreasing and bounded below by zero. Hence, it converges to some finite limit $\gamma(p,q)$. Next, considering carefully selected terms of (3.4) and taking limits, we have following:

$$\lim_{n\to\infty}\|Ax_n-Bx_n\|=0 \qquad (3.6)$$

$$\lim_{n\to\infty}\|(I-T_1)(x_n-\gamma_nA^*(Ax_n-By_n))\|=0 \qquad (3.7)$$

$$\lim_{n\to\infty}\|(I-T_2)(y_n+\gamma_nB^*(Ax_n-By_n))\|=0 \qquad (3.8)$$

Obviously, $\{x_n\}$ and $\{y_n\}$ are bounded because $\|x_n-p\|\leq\Gamma_n(p,q)$, and $\|y_n-q\|\leq\Gamma_n(p,q)$. Let p^* and q^* be the respective weak

cluster points of $\{x_n\}$ and $\{y_n\}$. Then there exists a subsequence $\{x_{n_k}, y_{n_k}\}$ of $\{x_n, y_n\}$ such that $x_{n_k} \rightharpoonup p^*$ and $y_{n_k} \rightharpoonup q^*$. For purpose of simplicity, let $w_{n_k} = x_{n_k} - \gamma_{n_k} A^*(Ax_{n_k} - By_{n_k})$ and $z_{n_k} = y_{n_k} + \gamma_{n_k} B^*(Ax_{n_k} - By_{n_k})$.
Since T_1 and T_2 are uniformly continuous, it follows from (3.6) that

$$\lim_{k\to\infty} \|T_1 w_{n_k} - T_1 x_{n_k}\| = 0 \tag{3.9}$$

$$\lim_{k\to\infty} \|T_2 z_{n_k} - T_2 y_{n_k}\| = 0 \tag{3.10}$$

Clearly,

$$x_{n_k} - T_1 x_{n_k} = (w_{n_k} - T_1 w_{n_k}) + (T_1 w_{n_k} - T_1 x_{n_k}) + (x_{n_k} - w_{n_k}).$$

Taking norm of both sides and simplifying yields

$$\begin{aligned} \|x_{n_k} - T_1 x_{n_k}\| &\leq \|(w_{n_k} - T_1 w_{n_k})\| + \|(T_1 w_{n_k} - T_1 x_{n_k})\| \\ &\quad + \|\gamma_{n_k}\| \|A^*\| \|Ax_n - By_n\|. \end{aligned}$$

On taking limits of both sides as $k \to \infty$ and applying (3.6), (3.7) and (3.9), we deduce that

$$\lim_{k\to\infty} \|x_{n_k} - T_1 x_{n_k}\| = 0 \tag{3.11}$$

Similar analysis yields that

$$\lim_{k\to\infty} \|y_{n_k} - T_2 y_{n_k}\| = 0 \tag{3.12}$$

Since $(I - T_1)$ and $(I - T_2)$ are demiclosed at origin, it follows from demiclosedness principle that $p^* = T_1 p^*$ and $q^* = T_2 q^*$. Again, since $\{x_{n_k}\} \rightharpoonup p^*$ and $\{y_{n_k}\} \rightharpoonup q^*$, where A and B are bounded linear operators, we have that $Ax_{n_k} - By_{n_k} \rightharpoonup Ap^* - Bq^*$ and by the weak lower semi continuity of squared norm,

$$\|Ap^* - Bq^*\| \leq \liminf_{k\to\infty} \|Ax_{n_k} - By_{n_k}\| = 0$$

Hence, $Ap^* = Bq^*$ which in turn implies that $(p^*, q^*) \in \Gamma$.
Therefore, we have so far obtained that for each $(p, q) \in \Gamma$, the sequence $\{\|x_n - p\|^2 + \|y_n - q\|^2\}$ has limit and each cluster point of the sequence $\{(x_n, y_n)\} \in \Gamma$. Suppose that $H = H_1 \times H_2$ endowed with the usual norm in $\mathbb{R}^2$, $W = \Gamma$, $\mu_n = \{(x_n, y_n)\}$ and $\mu = (p^*, q^*)$, then we can invoke the celebrated Opial's lemma to conclude that there exists $(p^*, q^*) \in \Gamma$ such that $x_n \rightharpoonup p^*$ and $y_n \rightharpoonup q^*$. Therefore, the sequence $\{(x_n, y_n)\}$ generated by the iterative algorithm (3.1) converges weakly to a solution of the split equality problem (1.1). $\square$

Theorem 3.2: Suppose that the conditions of theorem 3.1 are satisfied with T_1 and T_2 being demicompact also,then for any initial point (x_1, y_1), the coupled iterative sequence $\{(x_n, y_n)\}$ generated by algorithm (3.1) strongly converges to a solution of split equality problem(1.1)

Proof. We have obtained from theorem 3.1 that $\{(x_n, y_n)\}$ is bounded, and that $\lim_{k\to\infty} \|x_{n_k} - T_1 x_{n_k}\| = 0$ and $\lim_{k\to\infty} \|y_{n_k} - T_2 y_{n_k}\| = 0$. Then, demicompactness of T_1 and T_2 guarantees that there exists some subsequences, $\{x_{n_k}\}$ and $\{y_{n_k}\}$ of $\{x_n\}$ and $\{y_n\}$ respectively such that $\{x_{n_k}\}$ and $\{y_{n_k}\}$ converges strongly to $p^* \in H_1$ and $q^* \in H_2$ respectively. Hence, $Ax_{n_k} - By_{n_k} \rightharpoonup Ap^* - Bq^*$. As shown above, this yields that $Ap^* = Bq^*$ which in turn implies that $(p^*, q^*) \in \Gamma$. Having shown in the proof of theorem 3.1 that

$$\lim_{n\to\infty} \|(x_n, y_n) - (x^*, y^*)\|^2 \textit{ exists and}$$

$$\lim_{k\to\infty} \|(x_{n_k}, y_{n_k}) - (x^*, y^*)\|^2 = 0$$

we may conclude from lemma 2.2 that $(x_n, y_n) \to (x^*, y^*) \in \Gamma$. Therefore, the sequence $\{(x_n, y_n)\}$ generated by the iterative algorithm (3.1) converges strongly to a solution of the split equality problem (1.1).

Example 3.3 Let T be an arbitrary $\kappa-$strictly pseudocontractive mapping with nonempty fixed point set and let $p \in F(T)$. Then, for all $x, y \in D(T)$, we have that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + \kappa\|x - Tx - (y - Ty)\|^2.$$

Suppose, in particular that $y = p$, then direct substitution in the above inequality yields

$$\|Tx - Tp\|^2 \leq \|x - p\|^2 + \kappa\|x - Tx - (p - Tp)\|^2.$$

This in turn implies that

$$\|Tx - p\|^2 \leq \|x - p\|^2 + \kappa\|x - Tx\|^2.$$

Hence, T is demicontractive as well as $\alpha-$demicontractive with $\alpha = 1$.

Therefore, every $\kappa-$strictly pseudocontractive mapping with nonempty fixed point set is $\alpha-$demicontractive.

Next, we show that there exists at least one $\alpha-$demicontractive mapping that is not $\kappa-$strictly pseudocontractive. To this end, let us consider $T : \left[\frac{1}{2}, 1\right] \to \mathbb{R}$ defined for all $x \in D(T)$ by

$$Tx = 2x^2 - x + \frac{1}{2}.$$

Clearly, $\frac{1}{2} \in F(T)$. For $\alpha = 4$ and $x \in D(T)$, we have that $(\alpha p - x) = (2 - x) \geq 1$.

$$\begin{aligned} Tx - x &= 2x^2 - 2x + 1/2 \\ &= (2x-1)(x-1/2) \\ &\geq 0. \end{aligned}$$

Hence, $\|Tx - x\| < 1$ and for an arbitrary $\lambda \in [0,1)$, we have that

$$\begin{aligned} \langle x - Tx, x - \alpha p\rangle &= \langle Tx - x, \alpha p - x\rangle \\ &\geq \langle Tx - x, 1\rangle \\ &= \|Tx - x\| \\ &\geq \|Tx - x\|^2 \\ &> \lambda\|Tx - x\|^2. \end{aligned}$$

Therefore, T is 4$-$demicontractive. However, if $x, y \in D(T)$ with $x = \frac{11}{20}$ and $y = \frac{3}{5}$. Then,

$$\begin{aligned} \|x-y\|^2 + \kappa\|x - Tx - (y - Ty)\|^2 &< \|x-y\|^2 \\ &\quad + \|x - Tx - (y - Ty)\|^2 \\ &= \left\|\frac{11}{20} - \frac{3}{5}\right\|^2 \\ &\quad + \left\|\frac{11}{20} - \frac{111}{200} - (\frac{3}{5} - \frac{31}{50})\right\|^2 \\ &= \left\|\frac{-1}{20}\right\|^2 \\ &\quad + \left\|\left(\frac{-1}{200}\right) - \left(\frac{-1}{50}\right)\right\|^2 \\ &= \left\|\frac{-1}{20}\right\|^2 + \left\|\frac{3}{200}\right\|^2 \\ &= \frac{1}{400} + \frac{9}{40000} \\ &= 109/40000 \\ &< 169/40000 \\ &= (-13/200)^2 \\ &= \left\|\left(\frac{111}{200}\right) + \left(\frac{31}{50}\right)\right\|^2 \\ &= \|Tx - Ty\|^2. \end{aligned}$$

Therefore, T is not $\kappa-$strictly pseudocontractive $\square$

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