Journal of the
Nigerian Mathematical Society

Vol. 39, Issue 1, pp. 21-38, 2020

THE EFFECT OF VARIABLE VISCOSITY ON A REACTIVE HEAT GENERATING FLUID FLOW OVER A CONVECTIVE SURFACE

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ABSTRACT. The study examines the effect of variable viscosity on a reactive fluid moving with the strong effect of internal energy running through a convective surface with the impact of appreciable thermal radiation. The governing equations guiding the energy heat transfer and the fluid motion are solved by adopting the series solution of modified decomposition method named after George Adomian. The amount of entropy formation across the convective surface and other thermophysical parameters affecting the system of the fluid flow are examined and illustrated.

Keywords and phrases: Variable viscosity, Heat generating, Convective cooling, Reactive fluid and Radiative heat.
2010 Mathematical Subject Classification: 76A05, 80A20, 80A35.

1. INTRODUCTION

More researchers have paid renewed interest and attentions to the flow of reactive fluid because of their formidable operations in technical and manufacturing procedures as discussed extensively in [1] - [6]. Also, the problems of variable viscosity property of a fluid and energy transfer have been of vast interest and issues of discussion among researchers due to its technological and industrial applications as in [6] - [14]. Some of the applications of flow over a convective surface as discussed in [6] can be established in well arranged chemical reactors, geothermal containers, physical transformation companies, convertible exhaust schemes to mention a few. Investigations on fluid behaviour cannot be adequately described on the basis of variable viscosity properties, convective cooling, thermal radiation, heat source/sink, porosity, among the few properties.

Received by the editors August 31, 2019; Revised March 05, 2020 ; Accepted: March 26, 2020

www.nigerianmathematicalsociety.org; Journal available online at https://ojs.ictp.it/jnms/

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For instance, investigation showing the significant impact of variable viscosity property in fluid flow is shown in [8] where Poiseuille - Couette flow of a variable viscosity, incompressible, electrically and thermally conducting nanofluids amidst two boundless parallel surfaces under the impact of systematic transverse magnetic range along with the hall current has been investigated. In addition to that, [9] examined the flow and heat transfer of an electrically conducting viscoelastic fluid over a continuously spreading sheet where the fluid thickness and thermal conductivity are simulated to modify the role of temperature. Furthermore, on the behaviour of fluid flow and thermal radiation, [10] recently investigated numerically, the heat and mass transfer in a pressure induced flow of a reactive third - grade fluid with Reynolds' viscosity model through a fixed cylindrical annulus.

Of recent, [7] investigated the consequence of heat source on a variable viscosity reactive Couette fluid flow where both the heat source and variable viscosity depend on temperature . Also, [15] delivered a numerical study of an electrically conducting magnetohydrodynamic nonlinear convection flow of micropolar fluid over a slendering stretching surface such that the significance revealing the influence of viscous dissipation, Joule heating, non-uniform heat source or sink, temperature-dependent thermal conductivity and thermal radiation are deemed. However, a further study in [16] examined the effect of Arrhenius activation energy on magnetohydrodynamic mixed convection stagnation point flow of a micropolar fluid over a variable thickened surface in the attendance of Brownian motion. Other research studies on fluid viscosity are richly discussed in [11] - [14], [17] - [25].

However, the effect of thermal radiation is highly significant in fluid flow and thermal transfer as viscosity of fluid is sensitive to the temperature variation as extensively discussed in [26]. Recently, [2] also analysed the thermal transfer of a reactive hydromagnetic fluid through parallel porous channels with respect to the effect of internal energy and appreciable thermal radiation with convective boundary conditions obeying Newton's law of cooling. Other investigation showing relevance of thermal radiation on fluid flow and thermal heat transfer can further be seen in [27] - [30]. Other properties of fluid flow and heat transfer with convective boundary conditions are illustrated in [2, 6, 11, 32, 33] and that of fluid flow within porous materials are mentioned in [1, 6, 24, 26, 33, 34].

Moreover, the assessment of the impact of thermal radiation on a reactive non - Newtonian fluid within a channel is remarkably of high importance in fluid motion and heat transfer as explained in [6]. The importance is to ensure and predict the protection strategy of resources and valuables at a period of operation of such fluids especially in the design of heat exchangers, gas turbines and power plants as highlighted in [6, 26]. Therefore, the present study is designed to analyze the effect of variable viscosity that is temperature dependent on a reactive fluid flow within a channel accross a convective surface with the impact of internal energy and appreciable thermal radiation. To the best of our knowledge, no effort has been made to examine the effect of appreciable thermal radiation and variable viscosity on the flow of a reactive heat source fluid over a convective surface. The governing equations guiding the energy heat transfer and the fluid motion are solved by applying the series solution of modified Adomian decomposition method which has been established extensively in literature [2] - [5], [34] and [35] to be an effective and efficient way of getting an approximate solution that converge with few iterations.

2. MATHEMATICAL FORMULATION

Consider an incompressible steady flow of a reactive fluid with internal heat generation under the effect of appreciable thermal radiation with variable viscosity depending on temperature over a convective surface. The channel is at a distance $2z$ apart located at $y = z$ and $y = -z$. Also, the temperature dependent viscosity $(\overline{\mu})$ and heat source (m) are stated accordingly in Arrhenius kinetics as mentioned in [3, 13]:

$$\overline{\mu} = \mu_0 e^{\frac{E}{R\overline{T}}} \qquad \text{and} \quad m = Q_0\left(\overline{T} - T_0\right) \tag{1}$$

where μ_0 represents the fluid dynamic viscosity at the instance of large temperature (i.e. as $T \longrightarrow \infty$), $\overline{T}$ is the non - dimensionless fluid temperature, E represents the activation energy, R represents universal gas constant, T_0 stands for the wall temperature, Q_0 is the dimensional heat source. Hence, the differential equations regulating the flow system in dimensionless forms are stated thus:

$$-\frac{d\mathrm{p}}{d\mathrm{x}} + \frac{d}{d\overline{\mathrm{y}}}\left(\overline{\mu}\frac{d\overline{\mathrm{u}}}{d\overline{y}}\right) = 0 \tag{2}$$

$$k\frac{d^2\overline{\mathrm{T}}}{d\overline{y}^2} + \overline{\mu}\left(\frac{d\overline{\mathrm{u}}}{d\overline{y}}\right)^2 + QC_0Ae^{-\frac{E}{RT_0}} + Q_0\left(\overline{T} - T_0\right) - \frac{d\mathrm{q_r}}{d\overline{y}} = 0 \tag{3}$$

subject to the boundary conditions

$$\frac{d\overline{\mathrm{u}}}{d\overline{y}} = \frac{d\overline{\mathrm{T}}}{d\overline{y}} = 0 \quad \text{on} \quad y = 0 \quad \text{and}$$
$$\overline{u} = 0, \quad k\frac{d\overline{\mathrm{T}}}{d\overline{y}} = -h\left(\overline{T} - T_0\right) \quad \text{on} \quad y = \pm\, z \tag{4}$$

where p is the pressure, $\overline{u}$ is the non - dimensionless fluid motion, k stands for thermal conductivity, Q represents the heat of reaction term, C_0 stands for initial concentration of the reactant species and A is the reaction rate constant. In addition, h is the length, z is the channel half width and q_r denotes the radiative heat transfer.

Additionally, the impact of appreciable transfer of thermal heat in the flow regime as expressed in [2], [27] - [31] with respect to Rosseland approximation is given as:

$$q_r = -\frac{4\sigma}{3k^*}\frac{d^4\overline{\mathrm{T}}}{d\overline{\mathrm{y}}^4} \tag{5}$$

Here, σ stands for Stefan - Boltzmann constant and k^* represents the mean absorption coefficient. The well established notion with the temperature variation for the fluid flow is broadened in Taylor series about the free - stream temperature (T_∞), by setting aside the higher order terms as done in [2, 27, 28] gives the following:

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

such that

$$\frac{d\mathrm{q_r}}{d\overline{y}} = -\frac{16\sigma T_\infty^3}{3k^*}\frac{d^2\overline{\mathrm{T}}}{d\overline{y}^2} \tag{7}$$

With the introduction of equation (7) in equation (3), the energy equation becomes

$$k\frac{d^2\overline{\mathrm{T}}}{d\overline{y}^2} + \overline{\mu}\left(\frac{d\overline{\mathrm{u}}}{d\overline{y}}\right)^2 + QC_0Ae^{-\frac{E}{RT_0}} + Q_0\left(\overline{T} - T_0\right) + \frac{16\sigma T_\infty^3}{3k^*}\frac{d^2\overline{\mathrm{T}}}{d\overline{y}^2} = 0 \tag{8}$$

Moreover, the amount of entropy production in the flow system controlled with respect to appreciable radiative heat flux and other effect of viscous dissipation as discussed in [14] is given as:

$$S^m = \frac{k}{T_0^2}\left[\left(\frac{d\overline{\mathrm{T}}}{d\overline{y}}\right)^2 + \frac{16\sigma T_\infty^3}{3kk^*}\left(\frac{d\overline{\mathrm{T}}}{d\overline{y}}\right)^2\right] + \frac{1}{T_0}\left(\frac{d\overline{\mathrm{u}}}{d\overline{y}}\right)^2 \tag{9}$$

However, we introduce the following dimensionless parameters in equations (2), (4), (8) and (9) to further obtain non - dimensional boundary - valued problem.

$$y=\frac{\overline{y}}{z},\ x=\frac{\overline{x}}{z},\ u=\frac{\overline{u}}{U},\ \theta=\frac{E\left(\overline{T}-T_0\right)}{RT_0^2},\ \epsilon=\frac{RT_0}{E},\ Bi=\frac{zh}{k},$$
$$Br=\frac{E\mu_0U^2}{kRT_0^2},\ G=-\frac{z^2}{\mu_0U}\frac{\mathrm{d}p}{\mathrm{d}x},\ \mu=\frac{\overline{\mu}}{\mu_0}\mathrm{e}^{-\frac{E}{RT_0}},\ \beta=\frac{Q_0RT_0^2}{QAEC_0}\mathrm{e}^{\frac{E}{RT_0}},$$
$$\gamma=\frac{\mu_0U^2}{QAz^2C_0}\mathrm{e}^{\frac{E}{RT_0}},\ \alpha=\frac{16\sigma T_\infty^3}{3kk^*}\ \text{ and } \lambda=\frac{QEAz^2C_0}{kRT_0^2}\mathrm{e}^{-\frac{E}{RT_0}} \tag{10}$$

Hence, the following non - dimensional boundary - valued problems with the introduction of equation (10) is hereby given as:

$$\frac{d}{d\mathrm{y}}\left(\mu\frac{d\mathrm{u}}{d\mathrm{y}}\right)+G=0 \tag{11}$$
$$\frac{d^2\theta}{d\mathrm{y}^2}+\frac{\lambda}{1+\alpha}\left[\mathrm{e}^{\frac{\theta}{1+\epsilon\theta}}+\mu\gamma\left(\frac{d\mathrm{u}}{d\mathrm{y}}\right)^2+\beta\theta\right]=0 \tag{12}$$

subject to the boundary conditions

$$\frac{\mathrm{d}u}{\mathrm{d}y}=\frac{\mathrm{d}\theta}{\mathrm{d}y}=0 \quad \text{on} \quad y=0 \qquad \text{and}$$
$$u=0,\quad \frac{\mathrm{d}\theta}{\mathrm{d}y}=-Bi\theta \quad \text{on} \quad y=\pm 1. \tag{13}$$

Moreso, the rate of entropy production is expressed as follows:

$$N_s=\frac{S^mE^2z^2}{kR^2T_0^2}=[1+\alpha]\left(\frac{\mathrm{d}\theta}{\mathrm{d}y}\right)^2+\mu\frac{Br}{\Omega}\left[\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2\right] \tag{14}$$

where $\mu=\mathrm{e}^{-\frac{\theta}{1+\epsilon\theta}}$.

Here, G represents the pressure gradient, u and θ are respectively for fluid motion and temperature, U denotes the mean velocity, Bi represents the convective cooling term named after Biot and Br stands for Brinkman number. Also, ϵ, γ, α, λ, β and Ω are parameters representing the activation energy, viscous heating, thermal radiation, combustion named after Frank - Kamenettski, heat source and wall temperature.

3. METHOD OF SOLUTION

The solutions to the coupled equations (11) and (12) governing the fluid flow with the boundary conditions in (13) are secured by putting to use the established modified decomposition style formulated by Adomian (mADM) on integrating as follows:

$$\theta(y) = a_0 - \frac{\lambda}{1+\alpha}\int_0^y \int_0^y \left[\left(1+\gamma G^2 y^2\right) \mathrm{e}^{\frac{\theta}{1+\epsilon\theta}} + (\beta\theta)\right] \; dY \; dY \tag{15}$$

$$u(y) = b_0 - \int_0^y \left[(Gy)\, \mathrm{e}^{\frac{\theta}{1+\epsilon\theta}}\right] \; dY \tag{16}$$

such that $a_0 = \theta(0)$ and $b_0 = u(0)$ that will be determined by using the boundary conditions together with the rapid convergence of the series solution assumed as follows:

$$u(y) = \sum_{n=0}^{\infty} u_n(y) \qquad \text{and} \qquad \theta(y) = \sum_{n=0}^{\infty} \theta_n(y) \tag{17}$$

such that the governing coupled equations (11) and (12) are represented as follows:

$$\begin{aligned} \theta(y) = a_0 - \frac{\lambda}{1+\alpha}\int_0^y \int_0^y & \left[\left(\left(1+\gamma G^2 y^2\right) \sum_{n=0}^{\infty} A_n\right)\right. \\ & \left. + \left(\beta \sum_{n=0}^{\infty} \theta_n(y)\right)\right] \; dY \; dY \end{aligned} \tag{18}$$

$$u(y) = b_0 - G\int_0^y \left[y \sum_{n=0}^{\infty} A_n\right] \; dY \tag{19}$$

such that the non - linear term is represented as

$$\sum_{n=0}^{\infty} A_n = \sum_{n=0}^{\infty} \mathrm{e}^{\frac{\theta_n(y)}{1+\epsilon\theta_n(y)}} \tag{20}$$

whose components $A_0, \; A_1, \; A_2, \; . \; .$ are known as Adomian polynomials.

Therefore, the couple equations (18) and (19) with respective zeroth component are simultaneously solved as follows:

$$\theta_0(y) = 0 \tag{21}$$

$$u_0(y) = b_0 \tag{22}$$

$$\theta_1(y) = a_0 - \frac{\lambda}{1+\alpha}\int_0^y \int_0^y \left[\left(1+\gamma G^2 y^2\right) A_0 + \beta\, \theta_0\right]\, dY\, dY \tag{23}$$

$$u_1(y) = -G\int_0^y \left[A_0\, y\right]\, dY \tag{24}$$

$$\theta_{n+1}(y) = -\frac{\lambda}{1+\alpha}\int_0^y \int_0^y \left[\left(1+\gamma G^2 y^2\right) A_n + \beta\, \theta_n\right]\, dY\, dY \tag{25}$$

$$u_{n+1}(y) = -G\int_0^y \left[A_n\, y\right]\, dY \qquad \text{for} \quad n \geq 1 \tag{26}$$

With few iterations using a software package to estimate the solutions for the coupled equation to obtain:

$$\theta(y) = \sum_{n=0}^{k} \theta_n(y) \qquad \text{and} \qquad u(y) = \sum_{n=0}^{k} u_n(y) \tag{27}$$

Additionally, the rate of entropy generation is obtained by using the solutions of (27) in (14) and extensively discussed in the next section with figures together with Bejan number as an alternative to the irreversibility distribution ratio on domination of thermal energy with respect to the presence of appreciable radiative flux and local entropy generation noticeable on both lower and upper plates.

4. RESULTS AND DISCUSSION

Table 1. Rapid convergence of the series solution for a_0 and b_0. $\epsilon = \beta = \lambda = 0.1, \gamma = \alpha = 0.5, G = Bi = 1.$

n	a_0	b_0
0	0	0
1	0.113889	0.5
2	0.128464	0.0555436
3	0.128984	0.0556997
4	0.128975	0.0556896
5	0.128974	0.0556889
6	0.128974	0.0556889

We discuss in table and figures, the thermodynamic survey of a variable viscosity reactive hydromagnetic fluid flow with the effect of internal heat energy running through a convective surface under the control of appreciable thermal radiation.

The table 1 illustrates the accelerated convergence of the series solution for the limitation of constant values of a_0 and b_0 in equations (15) and (16) respectively which converge at the 4^{th} iteration. The convergence confirms the efficiency and accuracy of the series solution of Adomian decomposition approach.

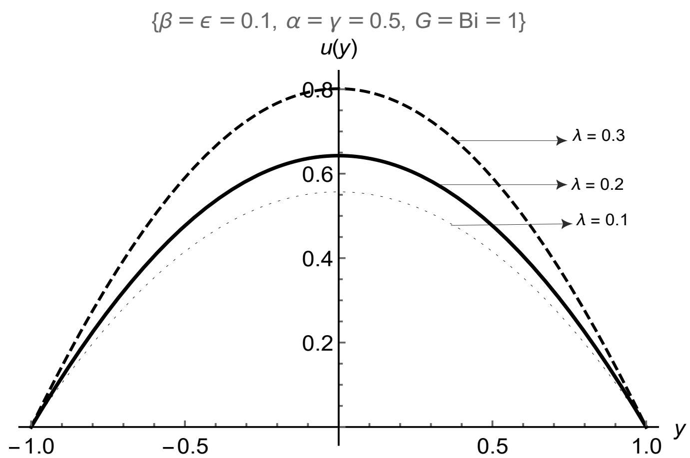

Fig. 1. Effects of λ on u(y).

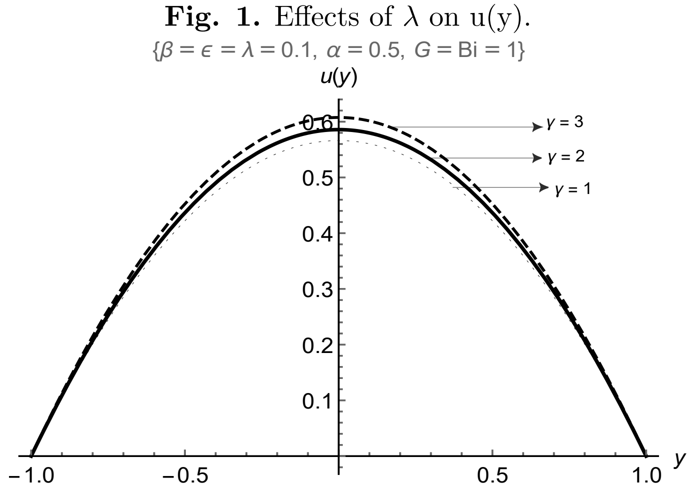

Fig. 2. Effects of γ on u(y).

Figures 1 - 5 respectively display the variations in the parameters of Frank - Kamenettski (λ), viscous heating (γ), heat source (β), thermal radiation (α) and convective cooling term (Bi) on the fluid motion. Generally, the utmost speed of the fluid flow is observed at the centreline of the passage with increase in fluid motion with respect to the increasing values of Frank - Kamenettski parameter (λ) in figure 1, viscous heating parameter (γ) in figure 2 and heat source parameter (β) in figure 3. The enhancement of fluid motion observed in figures 1 - 3 is due to increase in the viscosity associated with rise in fluid motion. Meanwhile, the reverse is noticed in figures 4 and 5 where a decrease in fluid motion is observed with rising values of thermal radiation parameter (α) in figure 4 and convective cooling term (Bi) in figure 5. The reduction in fluid motion noticed in figures 4 and 5 is due to heat energy being spread out from the centreline of the channel to both lower and upper plates accordingly.

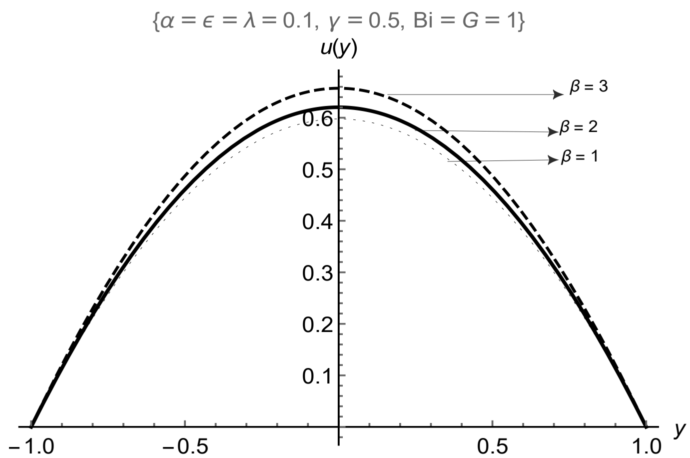

Fig. 3. Effects of β on u(y).

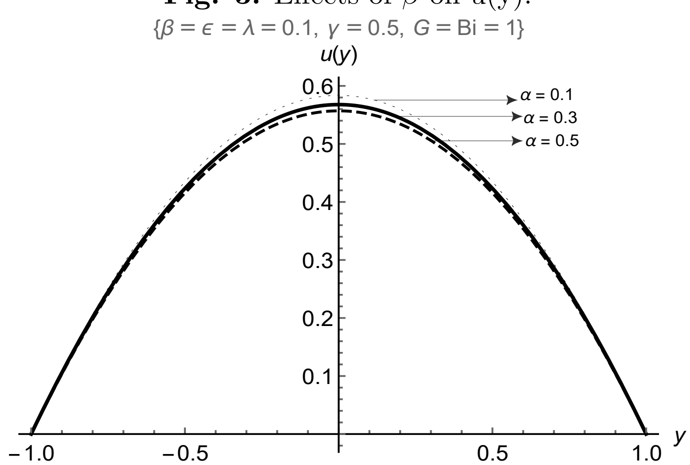

Fig. 4. Effects of α on u(y).

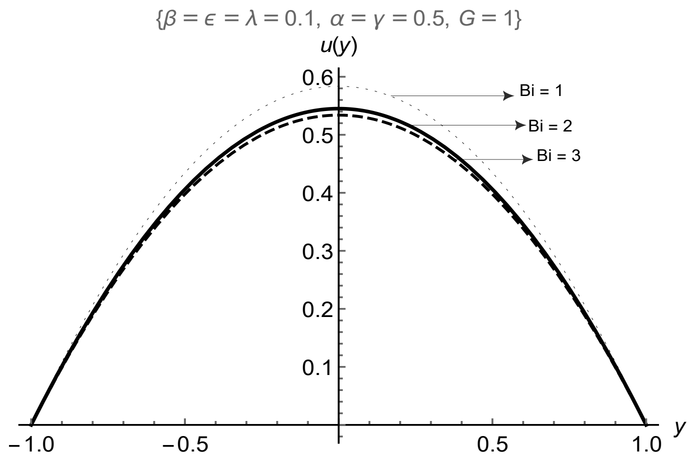

Fig. 5. Effects of Bi on u(y).

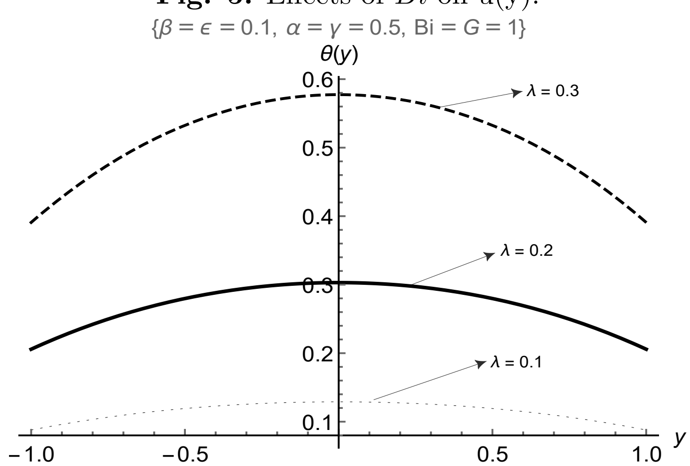

Fig. 6. Effects of λ on $\theta(y)$.

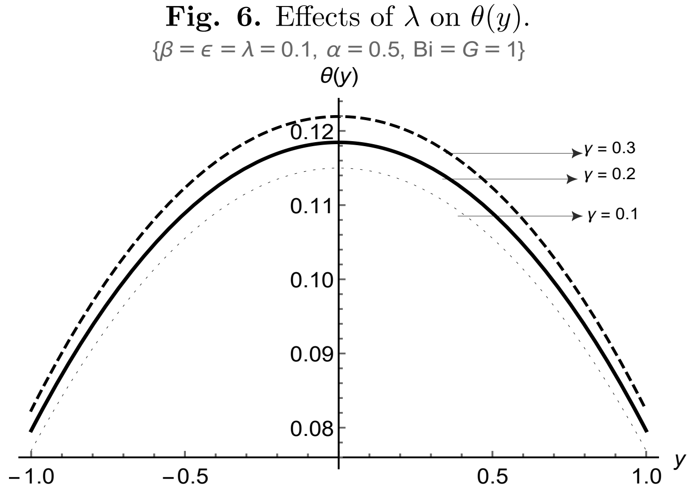

Fig. 7. Effects of γ on $\theta(y)$.

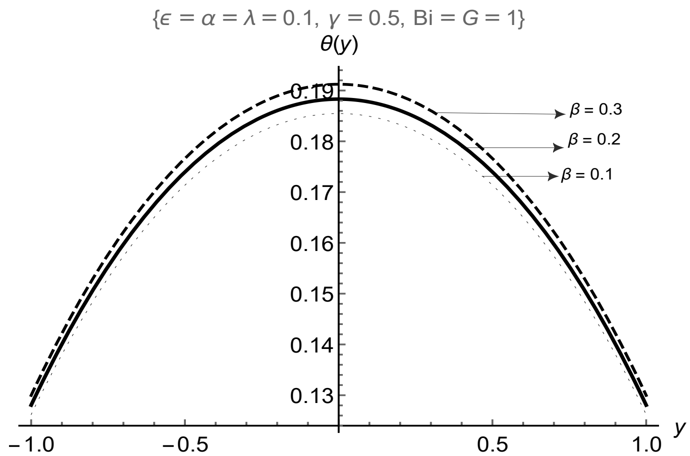

Fig. 8. Effects of β on $\theta(y)$.

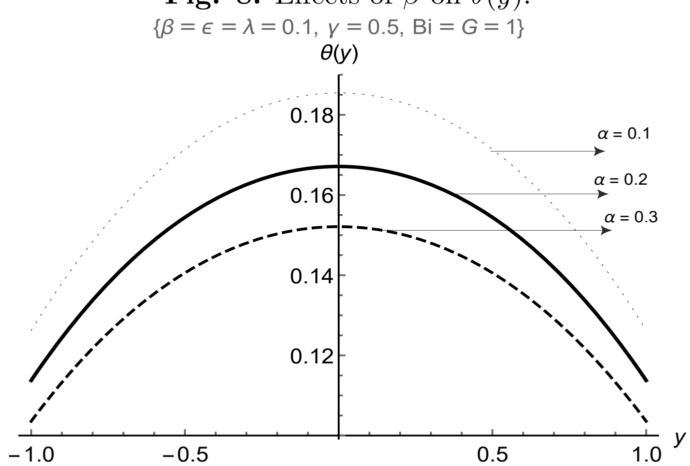

Fig. 9. Effects of α on $\theta(y)$.

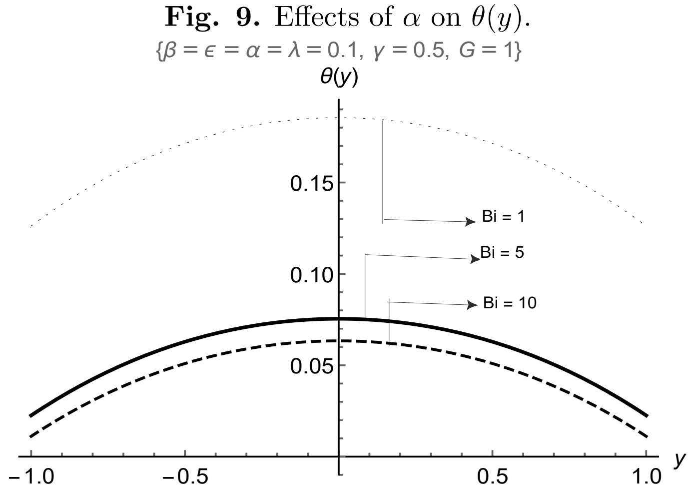

Fig. 10. Effects of Bi on $\theta(y)$.

The fluid temperature profiles for variations in the parameters of Frank - Kamenettski (λ), viscous heating (γ), heat source (β), thermal radiation (α) and convective cooling term (Bi) are respectively illustrated in figures 6 - 10. On a general note, the fluid temperature occurs at the maximum across the centreline of the flow channel towards the upper and lower plates. The fluid temperature therefore, increases with rising values of Frank - Kamenettski parameter (λ) in figure 6, viscous heating parameter (γ) in figure 7 and heat source parameter (β) in figure 8. This is physically true for the enhancement of fluid temperature due to their natural association with heat and transfer within the fluid particles. However, at the same time, the fluid temperature decreases with rising values of thermal radiation parameter (α) in figure 9 and convective cooling term (Bi) in figure 10. This is due to the absorption of heat energy being spread out across the flow channel.

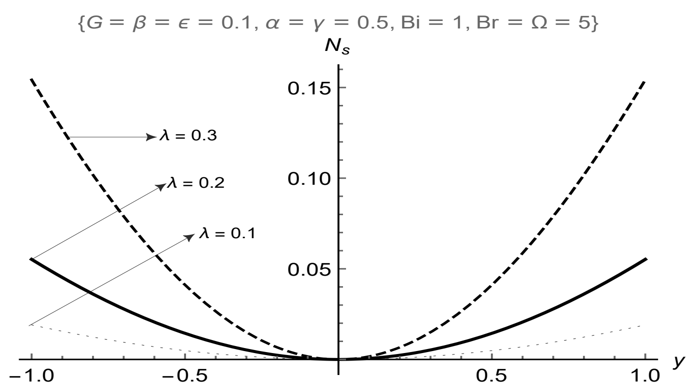

Fig. 11. Effects of λ on N_s.

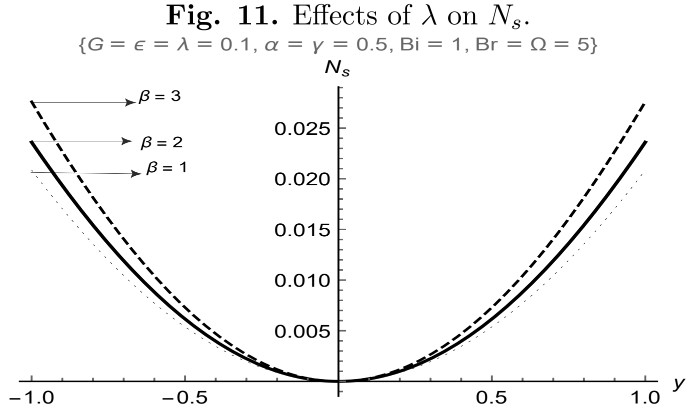

Fig. 12. Effects of β on N_s.

The rate of entropy generation within the flow regime with variations in Frank - Kamenettski parameter (λ), heat source parameter

(β), Brinkman number Br, thermal radiation parameter (α), convective cooling term (Bi) and wall temperature parameter (Ω) are respectively displayed in figures 11 - 16. The minimum irreversibility transfer occurs at the centreline and increases to the core region of the channel and rises to the maximum value of the plate surfaces. The rate of entropy generation increases with respect to rising values of Frank - Kamenettski parameter (λ) in figure 11, heat source parameter (β) in figure 12 and Brinkman number Br in figure 13 while the contrary is noticed in the case of thermal radiation parameter (α) in figure 14, convective cooling term (Bi) in figure 15 and wall temperature parameter (Ω) in figure 16 where the rate of entropy generation reduces with rising values of thermal radiation parameter (α), convective cooling term (Bi) and wall temperature parameter (Ω).

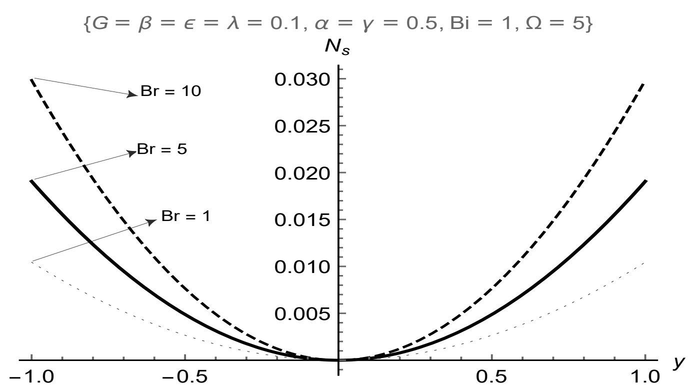

Fig. 13. Effects of Br on N_s.

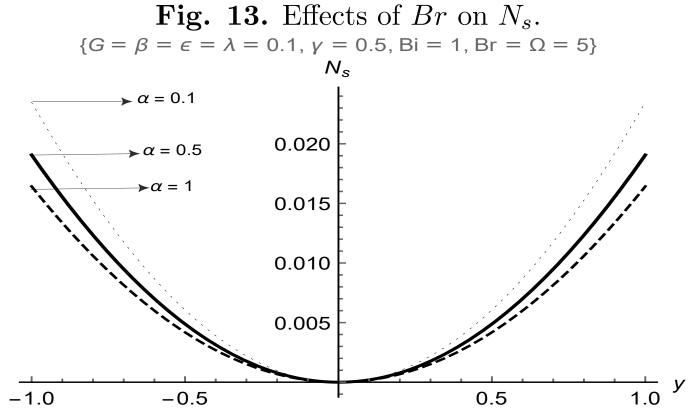

Fig. 14. Effects of α on N_s.

However, in figures 17 - 20, the variations in heat source parameter (β), viscous heating parameter (γ), convective cooling term

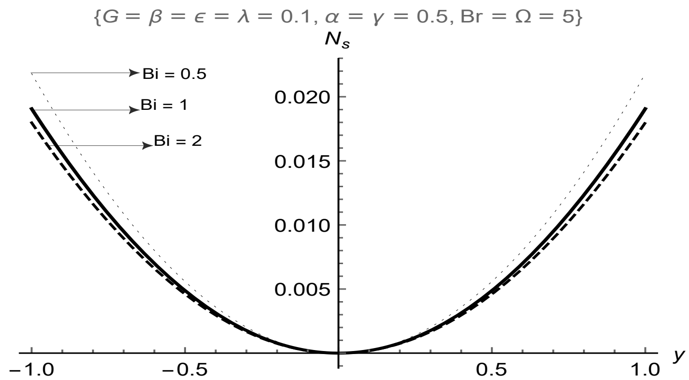

Fig. 15. Effects of Bi on N_s.

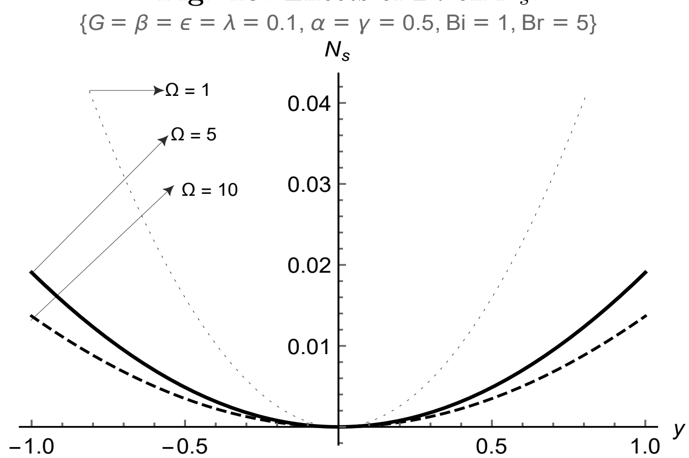

Fig. 16. Effects of Ω on N_s.

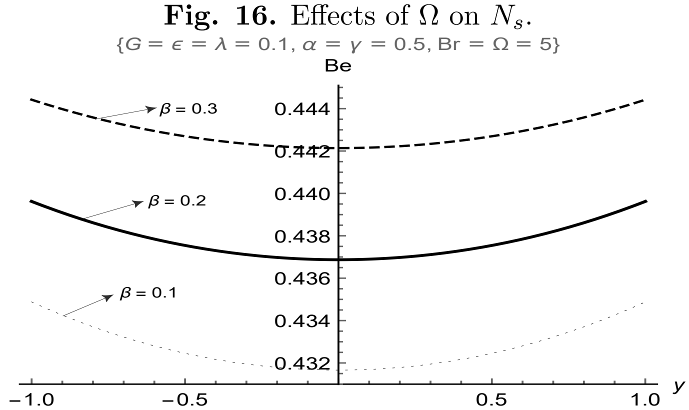

Fig. 17. Effects of β on Be.

(Bi) and thermal radiation parameter (α) are respectively illustrated against Bejan number. Bejan number increases with respect to variations in the values of heat source parameter (β) in figure 17 and viscous heating parameter (γ) in figure 18 while a decrease is noticed in the variations of convective cooling term (Bi) in figure 19 and thermal radiation parameter (α) in figure 20.

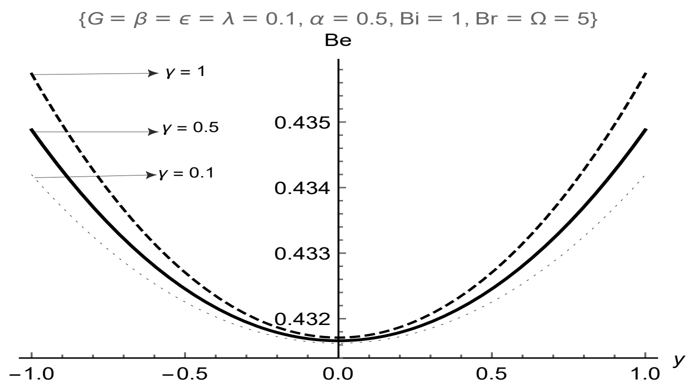

Fig. 18. Effects of γ on Be.

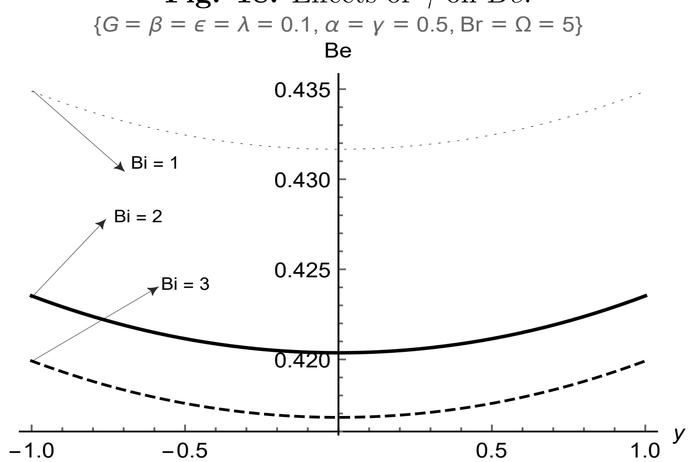

Fig. 19. Effects of Bi on Be.

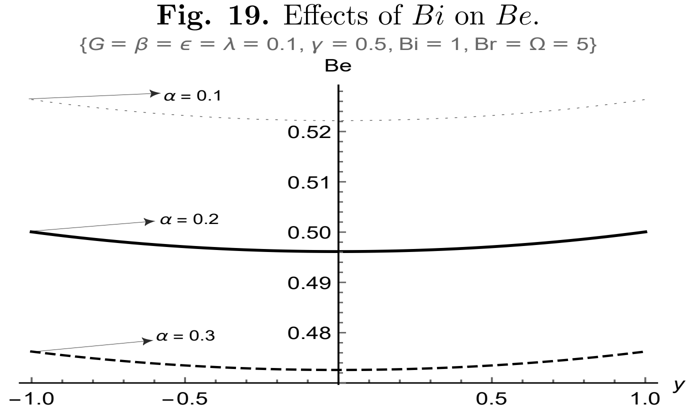

Fig. 20. Effects of α on Be.

5. CONCLUSION

The investigation on the thermodynamic survey of a variable viscosity reactive hydromagnetic fluid flow with the influence of heat source across the flow channel over a convective surface under the influence of appreciable thermal radiation. The governing equations guiding the energy heat transfer and the fluid motion are obtained seeking the series solution of modified Adomian decomposition method. The results obtained indicate the significance of all the thermophysical parameters. The revealed results will be of great attention to lubrication companies in increasing the rate of productivity of hydromagnetic materials used in engineering system.

ACKNOWLEDGEMENTS

The author would like to thank the anonymous referee whose comments improved the original version of this manuscript.

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