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INCLINED MAGNETIC FIELD AND EFFECTIVE PRANDTL NUMBER EFFECTS ON UNSTEADY MHD OSCILLATORY FLOW PAST AN INCLINED SURFACE WITH CONSTANT SUCTION AND CHEMICAL REACTION

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ABSTRACT.This paper investigates an unsteady convection flow of a viscous incompressible electrically conducting fluid in a porous medium past an inclined porous plate in the presence of an inclined magnetic field, radiation absorption, chemical reaction and constant suction. The porous surface absorbs the fluid with a constant velocity and the free stream velocity of the fluid is assumed to oscillate about a mean constant value. Perturbation techniques are adopted to solve the dimensionless governing equations. The influence of the various parameters on the fluid velocity, temperature, concentration fields, skin friction coefficient, Nusselt number and Sherwood number are discussed and results are illustrated graphically and in tabular form. It is found that the velocity profile decreases with increase in the angle of inclination of the magnetic field and of the flow surface. The optimal velocity increase occurs when the angle of inclination are relatively small with the inclination angle of the magnetic field being greater or equal to the angle of inclination of the flow surface.

Keywords and phrases: Convective flow, MHD, inclined magnetic field, inclined plate, porous medium
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1. INTRODUCTION

The study of heat and mass transfer by MHD oscillatory flow of fluid through inclined plates was done by several researchers in recent years. The interest to investigate MHD oscillatory flow was prompted by its vast applications in areas such as chemical processing industries, food preservation, polymer technology, aerospace and turbo technology. Oscillations in a convection flow enhances

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velocity profiles and if associated with a heat sink then such heat can be transported faster by conduction due to both radial and axial gradients. The effect of oscillations of MHD fluid flow under varying conditions have been investigated by many authors. Makinde and Mhone [11] investigated oscillatory flow in a channel with porous medium. Das *et al.* [9] studied oscillatory suction in an MHD flow past a vertical porous plate through a porous medium. Heat transfer in MHD oscillatory flow in a channel filled with porous medium was studied by Muthuraj and Srinivas [14]. Dar and Elangovan [8] investigated the effect of thermal diffusion, radiation and inclined magnetic field effects on oscillatory flow in an asymmetric channel in presence of heat source and chemical reaction.

Thermal radiation is a heat transfer phenomenon where heat is transferred through the fluid particles. The transfer of heat through radiation have application in engineering. For instance, in space technology, polymer industry for the extrusion of polymers, nuclear and industrial engineering such as design of gas turbines, propulsion devices for aircraft missiles and satellites. Ibrahim *et al.* [10] studied the effect of radiation absorption on the unsteady MHD free convection flow past a semi-infinite permeable moving plate with heat source and chemical reaction and suction. Further, Roja *et al.* [17] investigated the effect of thermal radiation on MHD free convection flow for a micro-polar fluid over a vertical moving porous plate in the presence of thermal diffusion chemical reaction and heat generation. Following studies carried out by Mgyari [12] and Magyari and Pantokratoras [13] it has been observed that the effect of thermal radiation in the linearized Rosseland approximation is quite trivial as it has been revealed that it always reduces to a rescaling of the Prandtl number by a factor involving the radiation parameter. Further in their analysis they observed that solutions involving the radiation parameter for optically thick media in linearized Rosseland approximation depends on the combined effect of the radiation parameter and the Prandtl number, called the effective Prandtl number rather than dependence on the effect of the radiation parameter R and Prandtl number P_r independently.

A magnetic field applied to a moving electrically conducting fluid produce Lorentz force which retards the fluid flow. The effect of magnetic field on an electrically conducting fluid finds applications in many industries and in engineering. For instance, micro-MHD pumps and MHD generators are known areas of application. Several authors have investigated the effects of aligned magnetic field.

Sandeep and Sugunamma [19] examined the inclined magnetic field and radiation effects of unsteady MHD free convection flow past a moving vertical plate in a porous medium. Later, Raju *et al.* [15] explored the effects of radiation, inclined magnetic field and cross diffusion on the flow over a stretching surface. The following year, Salawa and Dada [18] investigated the effects of inclined magnetic field with dissipation in non-Darcy medium. More recently, Acharya *et al.*[1] explored the influence of inclined magnetic field on the flow of condensed nanomaterial over a slippery surface. In all cases, it was observed that the effect of increasing the inclination angle of the magnetic field strengthens the magnetic field, decreases the velocity profile of the flow and improves the heat transfer rate.

Flow past inclined surface has gained plentiful impacts in the analysis of boundary layer flow as change in the inclination angle impacts on the size of the velocity and temperature profiles. The study of flow over an inclined plate was studied by a considerable number of researchers which include among others [Alam *et al.* [2], Alam *et al.* [3], Uddin [21], Buzuzi *et al.* [5], Buzuzi and Buzuzi [6], Buzuzi *et al.* [7]]. These studies revealed that the effect of increasing the angle of inclination of the flow surface is to decrease the velocity profile and increases the temperature profile.

In all of the studies mentioned so far, researchers examined fluid flow involving either aligned magnetic field or inclined flow surface but not both. Very few researchers have explored the effect of both aligned magnetic field angle and flow inclination angle. Reddy *et al.* [16] investigated the heat and mass transfer flow of viscous incompressible fluid past an inclined porous plate with aligned magnetic field and diffusion-thermo effects. Also, Sivaraj and Sheremet [20] presented a mathematical model to analyze an MHD natural convection in an inclined square porous cavity with a heat conducting solid block. In both cases, the researchers did not focus on the combined effect of the aligned magnetic field and the flow surface inclination angle but rather they investigated them separately.

To the author's ultimate knowledge, no studies have been reported on the problem of radiation effects on unsteady MHD oscillatory flow past an inclined surface coupled with inclined magnetic field with constant suction and chemical reaction. Whereas a few authors have carried out studies on the effect of both the aligned magnetic field angle and inclination angle on fluid flow, no researchers has investigated the combined effect of both the aligned magnetic field angle and the flow velocity inclination angle, hence motivation for the current study.

The rest of the paper is organized as follows. In Section 2 we present the model formulation. Section 3 is concerned with the analytical method of solution. Section 4 provides the results and a thorough discussion of the results. Finally, Section 5 presents concluding remarks.

2. MATHEMATICAL ANALYSIS

The flow geometry is portrayed in Figure 1. We consider a two

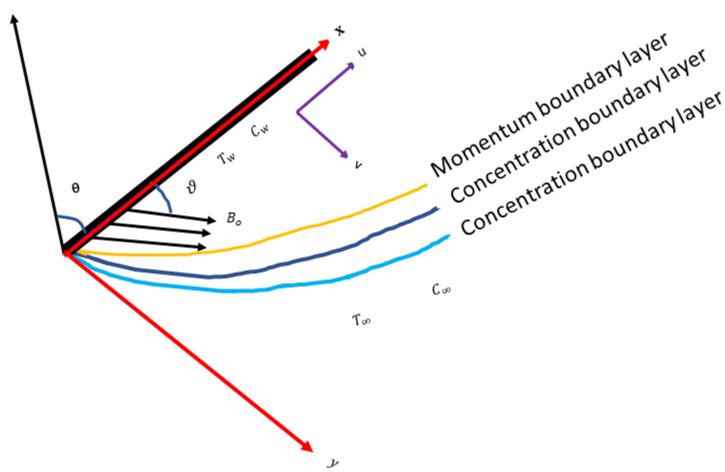

FIGURE 1. Flow geometry

dimensional unsteady flow of oscillatory, viscous, incompressible electrically conducting fluid past inclined permeable plate subjected to an inclined magnetic field in the presence of thermal diffusion, chemical reaction and constant suction velocity. The fluid is assumed to be a gray, absorbing-emitting but non scattering medium. Let the x^*-axis be directed along the porous plate and the y^*-axis be normal to the plate. The plate is inclined at an angle θ from the vertical. The inclined porous plate is semi-infinite hence the flow variables are functions of y^* and t^* only. An inclined magnetic field of uniform strength B_o is applied at an angle ϑ to the x^* direction. We assume a homogenous first order chemical reaction between the fluid and the species concentration. Under the above assumptions and taking into account the usual Boussinesq's approximation the MHD unsteady convective boundary layer equations of continuity, momemntum, energy and concentration that describe the physical situation are (see [7, 10, 16])

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{\mu}{\rho}\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_t(T^* - T_\infty)\cos\theta \tag{2}$$

$$+ g\beta_m(C^* - C_\infty)\cos\theta - \frac{\mu}{\rho K^*}u^* - \frac{\sigma {B_o}^2\sin^2\vartheta}{\rho^*}u^*,$$

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y^*}, \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - {k^*}_l(C^* - C_\infty), \tag{4}$$

where u^* and v^* are the components of dimensional velocities along the x^* and y^* directions respectively, ρ is the fluid density, and μ is the viscosity constant, β_t the coefficient of thermal expansion of fluid, and β_m is the coefficient of concentration expansion, K^* the permeability of the porous medium, σ is the electrical conductivity of the fluid, κ is the thermal diffusivity, c_p is the specific heat at constant pressure, B_o is the magnetic induction, θ is plate angle of inclination, ϑ is the angle of inclination of the magnetic field, T^* is the temperature of the fluid in the boundary layer, C^* is the concentration of fluid in the boundary layer, g is gravitational acceleration, k_l^* is the chemical reaction parameter, D is the coefficient of molecular diffusivity, q^r is the radiative heat flux.

According to (Brewster[4]), the radiative heat flux is given by

$$q^r = -\frac{4\sigma}{2k_1}\frac{\partial T^4}{\partial y'},$$

where, σ, and k_1 are the Stefan-Boltmann constant and mean absorption coefficient respectively. We assume sufficiently small temperature difference such that T^4 is expressed as a linear function of the temperature such that

$$T^4 \approx 4{T^4}_\infty T^* - 3T_\infty^{*4}$$

and

$$q^r = -\frac{16\sigma T_\infty^{*3}}{3k_1}\frac{\partial T^*}{\partial y'}.$$

The corresponding boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned} u^* &= 0,\ T^* = T_w,\ C^* = C_w \text{ at } y^* = 0, \\ u^* &= U_0(1 + e^{i\omega^* t^*}),\ T \to T_\infty,\ C \to C_\infty, \text{as } y^* \to \infty, \end{aligned} \quad (5)$$

where C_w and T_w are wall dimensional concentration and temperature respectively. C_∞ and T_∞ are the free stream dimensional concentration and temperature respectively and U_o and ω^* are the scale of free stream velocity and frequency of oscillation respectively. Since the surface absorbs fluid with constant velocity we deduce from the continuity equation (1) that

$$V^* = -V_0, \quad (6)$$

where V_0 is the scale of function velocity and the negative sign informs us that the suction is towards the plate.

We introduce the following non-dimensional quantities to enable us to write the governing equations in non-dimensional form:

$$y = \frac{V_0 y^*}{\nu},\ t = \frac{V_0^2 t^*}{\nu},\ T = \frac{T^* - T^*_\infty}{T^*_w - T^*_\infty},\ C = \frac{C^* - C^*_\infty}{C^*_w - C^*_\infty},\ u = \frac{u^*}{U_0},$$
$$P_r = \frac{\nu\rho C_p}{\kappa},\ \omega = \frac{\nu}{V_0^2}\omega^*,\ K = \frac{K^* V_0^2}{\nu^2},\ S_c = \frac{\nu}{D},$$

$$G_m = \nu\beta_m g\frac{(C^*_w - C^*_\infty)\cos\theta}{U_0 V_0^2},\ G_t = \nu\beta_t g\frac{(T^*_w - T^*_\infty)\cos\theta}{U_0 V_0^2},$$

$$M = \frac{\sigma\nu B_0^2}{\rho V_0^2},\ k_l = \frac{{k_l}^*\nu}{V_0^2},\ R = \frac{4\sigma {T^*}^3_\infty}{\kappa k_1}, \quad (7)$$

where P_r is the Prandtl number, G_m is the mass Grashof number, G_t is the thermal Grashof number, S_c is the Schmidt number, R is the radiation parameter, M is the magnetic field parameter and, k_l^* is the chemical reaction parameter, β is the dimensional viscosity ratio and μ is the viscosity of fluid. In view of equations (6)-(7) the non-dimensional form the governing equations (1)-(4) read:

$$\begin{aligned} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} &= \frac{\partial^2 u}{\partial y^2} + (G_t T + G_m C)\cos\theta \\ &- \left(\frac{1}{K} + M\sin^2\vartheta\right)u, \end{aligned} \quad (8)$$

$$\left(\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y}\right) = \frac{1}{Pr_{eff}}\frac{\partial^2 T}{\partial y^2}, \quad (9)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - k_l C, \tag{10}$$

where $Pr_{eff} = \dfrac{3RP_r}{3R+4}$ is the effective Prandtl number (see[13]). The corresponding boundary conditions (5) in dimensionless form are given by

$$\begin{aligned} &u = 0,\ T = 1,\ C = 1 \text{ at } y = 0,\\ &u = 1 + e^{i\omega t},\ T = 0,\ C = 0 \text{ as } y \to \infty. \end{aligned} \tag{11}$$

3. METHOD OF SOLUTION

In order to reduce the basic governing equations (8)-(10) to a system of ordinary differential equations in non-dimensional form we adopt a perturbation approach whereby we represent the linear velocity, temperature and concentration in the neighbourhood of the plate as

$$\begin{aligned} u(y,t) &= u_0(y) + e^{i\omega t}u_1(y),\\ T(y,t) &= T_0(y) + e^{i\omega t}T_1(y),\\ C(y,t) &= C_0(y) + e^{i\omega t}C_1(y). \end{aligned} \tag{12}$$

Substituting (12) in (8)-(10), and equating the harmonic and non-harmonic terms gives the following equations for $(u_0,\ T_0,\ C_0)$ and $(u_1,\ T_1,\ C_1)$,

$$i\omega u_1 - u_1' = -G_t T_1 \cos\theta - G_m C_1 \cos\theta + u_1'' - \left(\frac{1}{K} + M\sin^2\vartheta\right) u_1, \tag{13}$$

$$-u_0' = G_t T_0' \cos\theta + G_m C_0 \cos\theta + u_0'' - \left(\frac{1}{K} + M\sin^2\vartheta\right) u_0, \tag{14}$$

$$i\omega C_1 - C_1' = \frac{C_1''}{S_c} - k_l C_1 \tag{15}$$

$$-C_0' = \frac{C_0''}{S_c} - K_l C_0, \tag{16}$$

$$i\omega T_1 - T_1' = \left(\frac{1}{Pr_{eff}}\right) T_1'', \tag{17}$$

$$T_0 = -\left(\frac{1}{Pr_{eff}}\right) T_0'', \tag{18}$$

$$u_0 = 0,\ u_1 = 0,\ T_0 = 1,\ T_1 = 0,\ C_0 = 1,\ C_1 = 0 \text{ at } y = 0,$$

$$u_0 = 1,\ u_1 = 1,\ T_0 = 0,\ T_1 = 0,\ C_0 = 0,\ C_1 = 0 \text{ as } y \to \infty. \tag{19}$$

The solution of the equations (13)-(18) subject to the boundary conditions (19) are:

$$u_0 = 1 + d_1 e^{-k_1 y} + d_2 e^{-k_2 y} + d_3 e^{-k_4 y},$$

$$u_1 = 1 - e^{-k_3 y},$$

$$T_0 = e^{-k_1 y},\ T_1 = 0,\ C_0 = e^{-k_2 y},\ C_1 = 0. \tag{20}$$

The velocity, temperature and concentration equations become

$$u(y,t) = 1 + d_1 e^{-k_1 y} + d_2 e^{-k_2 y} + d_3 e^{-k_4 y} + \left(1 - e^{-k_3 y}\right) e^{i\omega t},$$

$$T(y,t) = e^{-k_1 y},\ C(y,t) = e^{-k_2 y}. \tag{21}$$

where,

$$k_1 = Pr_{eff},\ k_3 = \frac{1 + \sqrt{1 + 4\left(\frac{1}{K} + M\sin^2\vartheta + i\omega\right)}}{2},$$

$$k_2 = \frac{S_c + \sqrt{S_c^2 + 4k_l}}{2},\ k_4 = \frac{1 + \sqrt{1 + 4\left(\frac{1}{K} + M\sin^2\vartheta\right)}}{2},$$

$$d_1 = \frac{-G_t\cos\theta}{k_1^2 - k_1 - \left(\frac{1}{K} + M\sin^2\vartheta\right)},\ d_2 = \frac{-G_m\cos\theta}{k_2^2 - k_2 - \left(\frac{1}{K} + M\sin^2\vartheta\right)},$$

$$d_3 = -(1 + d_1 + d_2).$$

Of interest to engineers are three flow characteristics, namely, the skin friction, Nusselt number and the Sherwood number. The skin friction coefficient C_f, given by

$$C_f = \frac{\tau_w^*}{U_0 V_0} = \frac{\partial u}{\partial y}|_{y=0} = -d_3 k_4 - d_1 k_1 - d_2 k_2 + k_3 e^{i\omega t},$$

The local surface heat flux, q_w^* and the local Nusselt number, Nu are given by

$$q_w = -k\frac{\partial T^*}{\partial y^*}|_{y^*=0},\ Nu = \frac{q_w^*}{T_w^* - T_\infty^*}.\frac{x}{k}$$

respectively, where k is the effective thermal conductivity. The heat transfer coefficient $NuRe^{-1}$ is given by

$$NuRe^{-1} = \frac{q_w^*.x}{(T_w^* - T_\infty^*)k.Re} = -\frac{\partial T}{\partial y}|_{y=0} = k_1 = Pr_{eff}$$

The Sherwood number Sh and the mass transfer coefficient $ShRe^{-1}$ are given by

$$Sh = \frac{x\frac{\partial C^*}{\partial y^*}|_{y^*=0}}{C_w^* - C_\infty^*},\ ShRe^{-1} = -\frac{\partial C}{\partial y}|_{y=0} = k_2,$$

where $Re = \frac{V_0 x}{\nu}$ is the Reynolds number.

4. RESULTS AND DISCUSSION

The problem of an inclined magnetic field and effective Prandtl number effects on unsteady oscillatory flow past a inclined surface with concentration and constant suction is studied. In order to deduce the physical interpretation of the problems, calculations were carried out to determine the effects of various parameters on the flow. In this paper we investigate the effects of pertinent parameters such as angle of inclination of the magnetic field (ϑ), angle of inclination of the surface (θ), frequency of oscillation of the fluid (ω), permeability parameter of the porous medium (K), magnetic parameter (M), Schmidt number (S_c), the effective Prandtl number (Pr_{eff}), chemical reaction parameter (k_l), Grashof number for heat transfer (G_t) and Grashof number for mass transfer (G_m) on the velocity, temperature and concentration profiles of the flow field.

Results are computed for the different physical parameters and presented graphically in Figures 2 - 9 and in tabular form in Tables 1 - 3. In plotting the graphs, the following flow conditions were fixed unless specified, $\vartheta = \pi/2,\ \theta = \pi/2,\ \omega = \pi/2,\ K = 1,\ Pr_{eff} = 0.19,\ S_c = 0.2,\ ,\ t = 1,\ k_l = 1, M = 1,\ G_t = 1$ and $G_m = 2$. The effect of the different parameters on the velocity profile are shown in Figures 2 - 7. Figure 2 displays the effect of varying the angle ϑ between B_0 and the flow direction. It is noticed that by increasing the angle of inclination of the magnetic field ϑ, the intensity of the buoyancy force is enhanced. The increased magnetic field generates opposing force to the flow, called Lorentz force which reduces the thickness of the velocity boundary layer and causes decrease in the velocity profiles.

The effect of varying the angle of inclination of the surface θ on the velocity profile is illustrated in Figure 3. It is observed that the velocity profile decreases as the angle of inclination θ increases. As the angle of inclination increases from 0 to $\pi/2$ the effect of the buoyancy forces decreases. The decrease in the buoyancy forces is due to the multiplication of the term $cos\theta$ in the buoyancy force term since $cos\theta$ decreases as θ increases from 0 to $\pi/2$. Figure 4 portrays the combined effect of ϑ and θ on the velocity profile. It is shown that velocity is enhanced if both inclination angles are small. Moreover, it is observed that for a given combination of ϑ and θ velocity profiles are more pronounced when $\vartheta \geq \theta$. The velocity profiles are maximal when $\vartheta = \theta = 0$ and minimal when $\vartheta = \theta = \pi/2$. The effects of the frequency of oscillation ω is illustrated in Figure 5. It is deduced that as the frequency of fluid oscillation is increased the velocity profile is notably reduced. Figure 6 displays the effects of the effective Prandtl number Pr_{eff}, magnetic parameter M and permeability parameter K. It is noticed that the effect of increasing the value of the effective Prandtl number is to decrease the velocity profile. The effect of the permeability parameter is to increase the velocity profile. Physically, if a medium is porous and pores are large enough then resistance of the porous medium to fluid flow can be ignored allowing the fluid to flow freely and so the velocity profile increases. The effect of increasing the magnetic parameter M is to reduce the velocity profile. The magnetic field has a considerable effect on the velocity profile of an electrically conducting fluid. The magnetic field generates a driving force, called the Lorentz force which resists flow of fluid, hence reduces the velocity boundary layer thickness.

Figure 7 depicts the velocity profile for a combination of different values of the chemical reaction parameter k_l, Schmidt number S_c, thermal Grashof number G_t and mass Grashof number G_m. It is observed that the effect of increasing G_t or G_m is to increase the velocity profile. G_t is the relative strength of thermal buoyancy force to viscous force and G_m is the measure of solutal buoyancy force to viscous force. Therefore as G_t and G_m increases the thermal and solutal force gets stronger while viscous forces decrease and consequently the velocity profile grows. The effect of the chemical reaction parameter k_l and the Schmidt number S_c is to decrease the velocity profile.

Figure 8 displays the temperature profiles for different values of the effective Prandtl number Pr_{eff}. It is noticed that the temperature profile is reduced when the effective Prandtl number Pr_{eff}

is increased. Prandtl number relates viscous forces to thermal conductivity. When the effective Prandtl number is increased the viscous forces become stronger and the thermal conductivity becomes weaker and consequently the thermal boundary layer become thinner and hence temperature profile is reduced.

The effect of varying the chemical reaction parameter k_l and Schmidt number S_c on the concentration profiles is illustrated in Figure 9. It is viewed that the effect of increasing the chemical reaction parameter or the Schmidt number is to decrease the concentration profile. Since $k_l > 0$ the chemical reaction is of the destructive type and as k_l is increased the consumption of species rate is improved and consequently the thickness of the boundary layer decreases. The Schmidt number is the ratio of momentum diffusivity to chemical molecular diffusivity and as S_c is increased the chemical molecular diffusion is effectively lowered hence reducing the concentration profile.

Tables 1- 3 displays the effects of various parameters on the skin friction C_f, Nusselt number Nu and Sherwood number Sh. Table 1 depicts the effect of various flow parameters on the skin friction. Increasing the value of the thermal Grashof number, mass Grashof number and permeability parameter causes the skin friction to increase. The increase in the mentioned parameters accelerates the flow velocity which causes the momentum boundary layer to thicken and consequently increase the value of the skin friction coefficient. On the contrary the skin friction decreases with increase in the effective Prandtl number, magnetic parameter, angle of inclination of the magnetic field, angle of inclination of the flow surface, Schmidt number, chemical reaction parameter and frequency of oscillation. The decrease in the skin friction is caused by the velocity flow being reduced by the presence of the flow parameters which enhances the momentum boundary layer thickness.

Table 2 portrays the effect of the effective Prandtl number Pr_{eff} on the heat transfer coefficient $NuRe^{-1}$. It is observed that an increase in the effective Prandtl number enhances heat transfer rate.

Table 3 depicts the influence of the Schmidt number S_c and chemical reaction parameter k_l on the Sherwood number Sh. It is noticed that Sherwood number increases as the Schmidt number or chemical reaction parameter increases. The Schmidt number measures the relative effectiveness of the momentum and mass transport by diffusion in the hydrodynamic and concentration boundary layers.

Physically it relates to the relative thickness of the velocity boundary layer and the mass transfer boundary layer. The higher value of S_c has a low mass diffusivity which results in reducing the concentration boundary layer. The chemical reaction parameter has the effect of reducing the heat transfer coefficient and increase the mass transfer rate. Increase in k_l means that more interaction of species concentration with the momentum boundary layer and less interaction with the thermal boundary layer and has significant impact on the Sherwood number.

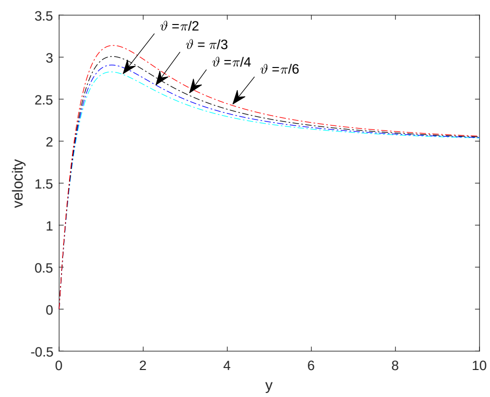

FIGURE 2. Velocity profile for different values of ϑ.

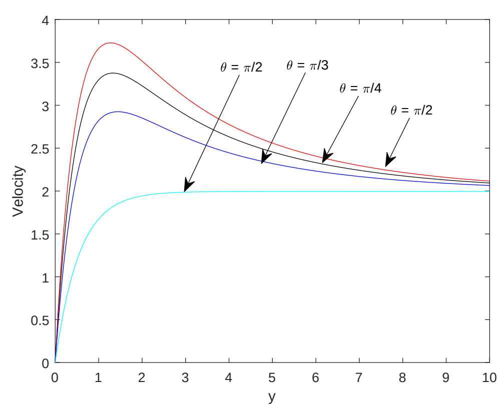

FIGURE 3. Velocity profile for different values of θ.

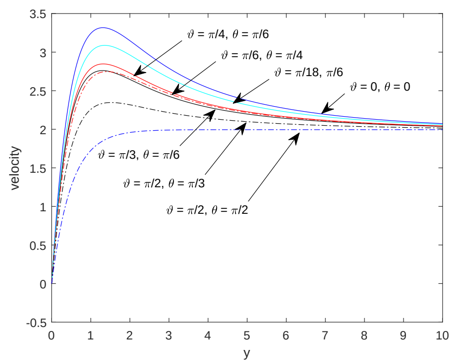

FIGURE 4. Velocity profile for different values of ϑ and θ.

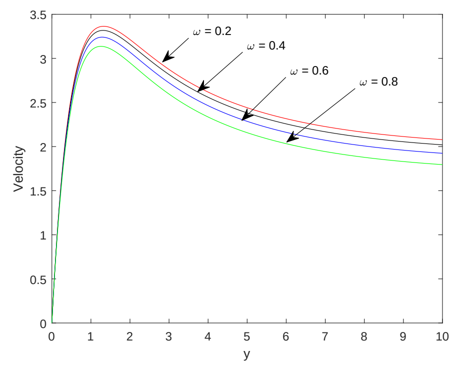

FIGURE 5. Velocity profile for different values of ω

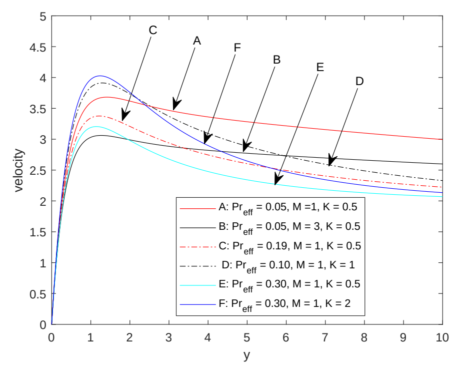

FIGURE 6. Velocity profile for different values of Pr_{eff}, M and K.

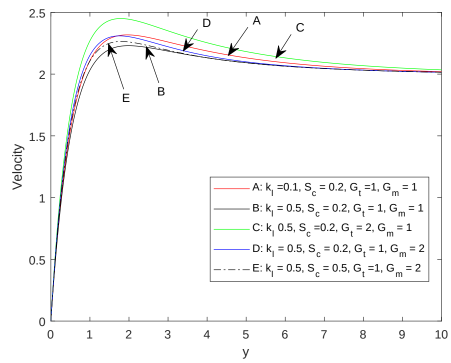

FIGURE 7. Velocity profile for different values of k_l, S_c, G_t and G_m.

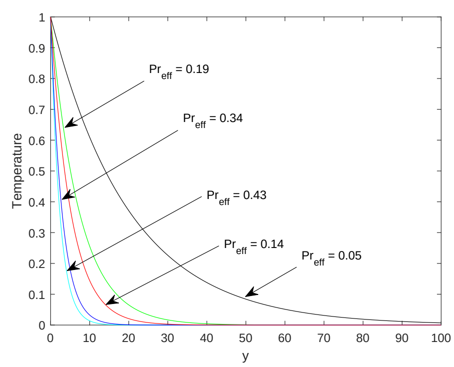

FIGURE 8. Temperature profile for different values of the effective Prandtl number, Pr_{eff}

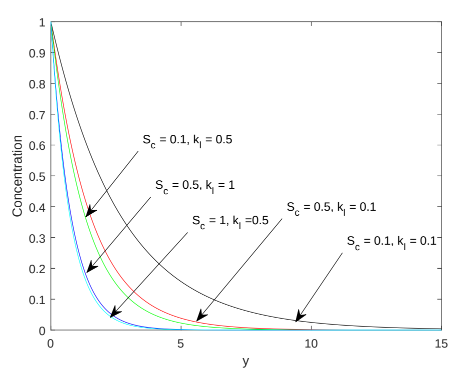

FIGURE 9. Concentration profile for different values of S_c and k_l.

TABLE 1. Numerical values of skin friction coefficient at the plate surface for parameters $Pr_{eff}, G_t, G_m, K, \vartheta, \theta, S_c, k_l,$ and ω.

Pr_{eff}	G_t	G_m	M	K	ϑ	θ	S_c	k_l	ω	C_f
0.19	1	2	1	1	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/2$	1.4503
0.33	1	2	1	1	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/2$	1.4001
0.30	1	2	1	1	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/2$	1.3820
0.19	2	2	1	1	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/2$	2.1460
0.19	1	3	1	1	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/2$	1.8823
0.19	1	2	2	1	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/2$	1.2917
0.19	1	2	1	2	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/2$	1.7004
0.19	1	2	1	1	$\pi/3$	$\pi/4$	0.2	0.5	$\pi/2$	1.3636
0.19	1	2	1	1	$\pi/4$	$\pi/3$	0.2	0.5	$\pi/2$	0.9936
0.19	1	2	1	1	$\pi/4$	$\pi/4$	1	0.5	$\pi/2$	1.2325
0.19	1	2	1	1	$\pi/4$	$\pi/4$	0.2	1	$\pi/2$	1.3200
0.19	1	2	1	1	$\pi/4$	$\pi/4$	0.2	0.5	$\pi/4$	1.5281

TABLE 2. Effect of Pr_{eff} on the heat transfer coefficient.

Pr_{eff}	$NuRe^{-1}$
0.05	0.0495
0.06	0.0558
0.19	0.1936
0.07	0.0698
0.30	0.3043

TABLE 3. Effect of S_c and k_l on the mass transfer coefficient.

S_c	k_l	$ShRe^{-1}$
0.2	0.1	0.4317
0.5	0.1	0.6531
0.2	0.5	0.8141
1	0.1	1.0916
0.2	1	1.1050

4. CONCLUDING REMARKS

In the present investigation, the effects of inclined magnetic field, inclined flow surface, effective Prandtl number and chemical reaction on unsteady MHD oscillatory flow with constant suction are presented and discussed. The governing equations were developed and transformed into a system of nonlinear differential equations using perturbation techniques. The findings of the current numerical study are summmarized as follows:

(1) The velocity profile increases with increase in the thermal Grashof number, mass Grashof number and chemical reaction parameter.
(2) The velocity profile decreases with increase in the angle of inclination of the magnetic field, angle of inclination of the flow surface, frequency of oscillation, magnetic parameter, effective Prandtl number, Schmidt number or chemical reaction parameter.
(3) The temperature profile falls with increase in the effective Prandtl number.
(4) The concentration profile decreases with increase in either the chemical reaction parameter or Schmidt number.
(5) The value of the skin friction increases with increase in either the permeability parameter, mass Grashof number or thermal Grashof number. On the contrary the skin friction decreases with increase in the chemical reaction parameter, magnetic parameter, inclination angle of the magnetic field, inclination angle of the flow surface, the effective Prandtl number, Schmidt number or frequency of oscillation.
(6) The Nusselt number increases with increase in the effective Prandtl number.
(7) Sherwood number increases with increase in the Schmidt number or the chemical reaction parameter.

Most of the above findings on the effect of the various parameters on fluid flow agrees with previously published work. The findings which stand out in the current investigation are that for optimal velocity profile the angles of inclination ϑ and θ need be relatively small and for any given combination $\vartheta \geq \theta$.

In view of the above summary, the usefulness and industrial and scientific application of the current investigation cannot go unnoticed. The study finds applications in areas where cooling of equipments is needed. The cooling process takes place as heat is removed from equipments such as reactors, towers, outer space modules and closed cabins where the convective fluid involved is oscillatory. Application can also be found in cases where electronic devices may need cooling and the chips in their equipment are not disturbed by the oscillation of the fluid. Also the knowledge of aligned magnetic field angle, flow surface inclination, effective Prandtl number and other flow parameters will assist in determing the flow configuration that will give optimal results. For instance, varying the aligned magnetic field angle, the surface inclination angle or both will inevitably alter the flow rate, mass transfer rate and heat transfer rate.

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