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THERMAL STABILITY ANALYSIS OF REACTIVE HYDROMAGNETIC THIRD-GRADE FLUID THROUGH A CHANNEL WITH CONVECTIVE COOLING

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ABSTRACT. This paper investigates the hydromagnetic flow of a reactive third grade fluid between two parallel plates with convective boundary conditions. Approximate solution of the strongly nonlinear boundary value problem is obtained using modified Adomian decomposition method (mADM). The rapidly convergent series solution is combined with the diagonal Pade' approximants to determine the singularity inherent in the solution. Parametric study of the fluid flow are conducted and discussed including bifurcation conditions.

Keywords and phrases: Reactive fluids, third grade fluid, Convective cooling, mADM, Pade approximant, Arrhenius kinetics
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1. INTRODUCTION

Studies on the combustible fluid flow have been on the increase in recent times due to its usefulness in many real life applications like; fuel combustion during industrial and engineering processes, bush burning, releases from automobile engines, waste burning, production of liquid steel, burning of crude oil leakages on high sea and thermal explosions in refineries to mention just a few. It is well known that excessive production of CO_2 that is not needed by plants leads to the depletion of the ozone layer which is a threat to life.

Up till today, there is no known single constitutive equation that can adequately describe the complex rheological properties of all non-Newtonian fluids. In view of this, the third grade fluid model has been used in studies [1-6] for combustible non-Newtonian fluids due to its ability to predict the shear thickening/thinning of the fluid.

Due to the effect of magnetic field on fluid dynamics (for instance, it is very useful in controlling fluid in thermal engineering at a very

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high temperature). Several important studies have been conducted on hydromagnetic fluids with or without reactive term by many researchers in literature some of which can be found in ref. [6-9, 12-14].

In the present investigation, attention is focused on the influence of magnetic field on the flow thermal structure. Of particular interest, is the effect of magnetic field on the development of thermal instability which was not accounted for in previously obtained result [5]. The present study is important in enhancing the safety of lives and properties that is unavoidable in so many petro-chemical engineering applications. For instance, when working with combustible fluids of high flash points. To achieve the set objective, a new modification of the Adomian decomposition method together with Pade approximation of the solution will be used to obtain the solution of the nonlinear differential equation. Similar approach has been used in literature [10-22] to obtain solution to several nonlinear problems.

The rest of the paper is organized as follows. Section 2 presents the formulation of the problem. In section 3, the method of solution is described while section 4 deals with the discussion of results based on the physics of the problem. Finally, section 5 concludes the paper.

2. MATHEMATICAL MODEL

Consider the steady flow of a viscous incompressible combustible internal heat generating fluid in the x-direction between impermeable boundaries at $y = \pm h$. The fluid is assumed to be under the influence of an external uniform magnetic field placed across the channel subjected to convective cooling at the boundaries as shown in the figure below. Neglecting the reactant consumption, the

dimensionless governing equations from the momentum and energy equations are [5]:

$$0 = A + u'' + 6\gamma u''(u')^2 - H^2 u, \tag{1}$$

$$0 = \theta'' + \lambda e^{\frac{\theta}{1+\epsilon\theta}} + \alpha[(u')^2(1 + 2\gamma(u')^2) + H^2 u^2], \tag{2}$$

together with the following boundary conditions

$$u'(0) = 0, u(1) = 0, \theta'(0) = 0, \theta'(1) = Bi\theta(1). \tag{3}$$

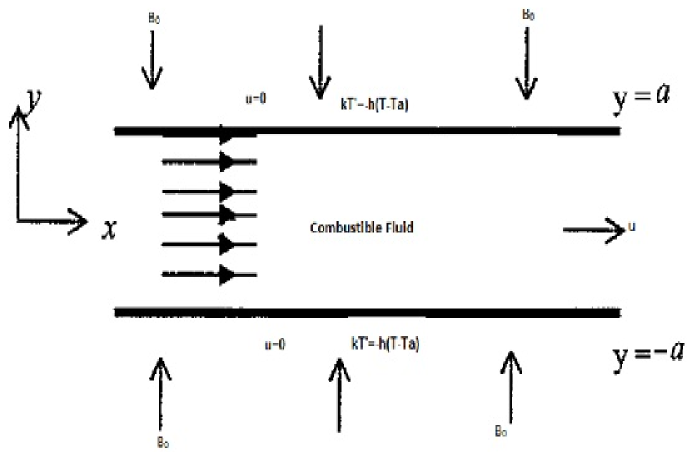

Fig. 1. Problem geometry

Additional term in the momentum equation is the Hartmann number H^2that measures the magnetic field intensity while the last terms in the energy equation is the Ohmic heating or Joule dissipation term.
The following dimensionless variables have been used for the above expressions

$$y=\frac{\overline{y}}{a},u=\frac{\overline{u}}{U},\epsilon=\frac{RT_a}{E},\gamma=\frac{\beta_3U^2}{a^2\mu},\theta=\frac{E(T-T_a)}{RT_a^2},\lambda=\frac{QEa^2C_0e^{\frac{E}{RT_a}}}{kRT_a^2}$$

$$\alpha=\frac{EU^2\mu}{kRT_a^2},A=-\frac{a^2}{U\mu}\frac{dp}{dx},Bi=\frac{ah}{k},H^2=\frac{\sigma B_0^2a^2}{\mu}$$

where $\overline{u}$ is the fluid velocity, a is the channel half width, T_a is the fluid reference temperature, h is the heat transfer coefficient, T the fluid temperature, $\overline{p}$ is the pressure, β_3 is the material coefficient, k the thermal conductivity, μ is the dynamic viscosity, C_0 is the initial concentration of the reactant specie, R is the universal gas constant, Q is the heat of reaction,A is the rate constant, E is the activation energy, γ is the dimensionless third grade material parameter, λ is the Frank-Kameneskii parameter, α viscous heating parameter, θ is the dimensionless temperature, u is the dimensionless velocity, ϵ is the activation energy and H^2 is the Hartmann Number, Bi is the Biot number, B_0 is the intensity of magnetization and k is the thermal conductivity coefficient.

3. ADOMIAN DECOMPOSITION METHOD OF SOLUTION

To obtain the approximate solution of the velocity and temperature profiles, the differential equations (1) - (2) is converted to integral equations. Using the boundary conditions, one obtains

$$u(y) = a_0 + \int_0^y \int_0^y \{(H^2 u - A - 6\gamma u''(u')^2) dY dY, \qquad (4)$$

$$\theta(y) = b_0 - \int_0^y \int_0^y \{(\lambda e^{\frac{\theta}{1+\epsilon\theta}} + \alpha[(u')^2(1 + 2\gamma(u')^2) + H^2 u^2]) dY dY, \qquad (5)$$

Where the constants a_0 and b_0 are to be determined later using the other boundary conditions. Adomian decomposition method assumes series solutions in the form:

$$u = \sum_{n=0}^{\infty} u_n(y), \theta = \sum_{n=0}^{\infty} \theta_n(y), \qquad (6)$$

Using (6) in (4) - (5) leads to

$$u_0(y) = a_0 - \int_0^y \int_0^y \{(A) dY dY$$

$$u_{n+1}(y) = \int_0^y \int_0^y \{(H^2 u_n - 6\gamma B_n) dY dY; n \geq 0, \qquad (7)$$

together with the modified algorithm

$$\theta_0(y) = b_0$$

$$\theta_1(y) = -\int_0^y \int_0^y \{\lambda C_0 + \alpha(u')^2\} dY dY$$

$$\theta_2(y) = -\int_0^y \int_0^y \{\lambda C_1 + 2\gamma\alpha(u')^4\} dY dY$$

$$\theta_{n+1}(y) = -\int_0^y \int_0^y \{\lambda C_n + \alpha H^2 u^2\} dY dY, n \geq 2, \qquad (8)$$

the approximate solutions are given by the partial sum

$$u = \sum_{n=0}^{k} u_n(y), \theta = \sum_{n=0}^{k} \theta_n(y). \qquad (9)$$

The nonlinear terms in (4) and (5) are identified as

$$B = u''(u')^2, C = e^{\frac{\theta}{1+\epsilon\theta}}, \qquad (10)$$

are decomposed into Adomian polynomials

$$B_0 = u_0''(u_0')^2$$

$$B_1 = 2u_0''u_1'u_0' + u_1''(u_0')^2$$
$$B_2 = u_0''(u_1')^2 + 2u_0''u_0'u_2' + 2u_1''u_0'u_1' + u_2''(u_0')^2, \qquad (11)$$

with other terms and

$$C_0 = e^{\frac{\theta_0}{1+\epsilon\theta_0}}$$
$$C_1 = \frac{\theta_1}{(1+\epsilon\theta_1)^2} e^{\frac{\theta_0}{1+\epsilon\theta_0}}$$
$$C_2 = \frac{(1-2\epsilon-2\epsilon^2\theta_0)\theta_1^2 + 2(1+\epsilon\theta_2\theta_0^2)}{2(1+\epsilon\theta_1)^4} e^{\frac{\theta_0}{1+\epsilon\theta_0}}, \qquad (12)$$

respectively.

3.1 PADE APPROXIMANT

It is well known that power series solution may not be useful as n goes to ∞ and could not provide adequate information on the behaviour of the solution especially at the critical points. Hence it is necessary to obtain the continuation of the series solution (9) by using Pade-approximant. The solution continuation using Pade approximant has been shown to be more reliable and convergent on the entire real axis if the underlying series solution is free of singularities on the real axis. Nowadays, the Pade approximant is built in many symbolic packages like Mapple, Mathematica, Matlab etc. Using Mathematica built-in Pade-approximant procedure, the diagonal form of the series solution (9) is evaluated at $y = 1$ so as to obtain expressions for the two unknown constants in the form

$$u(1) = 0, \theta'(1) = Bi\theta(1). \qquad (13)$$

Taking the diagonal Pade approximants [M/M] of (13) at various values of M leads to an eigenvalue problem. As a criterion for convergence of the method, the unknown constants are evaluated at specific parameter values as shown in Tables 1-2. While the critical values of the Frank-Kameneskii parameter (λ_c) for the non-existence of solution or thermal runaway is presented in Table 3.

4. RESULTS AND DISCUSSION

In this section, the mADM - Pade approximant solution of the hydromagnetic fluid flow is presented. The rapid convergence of the two solutions is shown in Tables 1-2. Table 3 shows that an increase in the Biot number stabilizes the fluid flow, this is in perfect agreement with previously obtained result in [5]. Interestingly, an increase in the magnetic field intensity is observed to delay the development of thermal runaway thereby stabilizing the flow. This is due to fact that the convective cooling suppresses the additional heat generated due to Ohmic heating of the fluid. Figure 2 shows the velocity profile for variations in the magnetic field intensity. As observed from the graph, maximum flow occurs at minimum value of the Hartmann's number. Therefore, an increase in the magnetic field intensity leads to a decrease in the velocity. This is physically true due to the retarding effect of Lorentz forces present in the magnetic field when applied perpendicularly to the flow channel.

Table 1: Numerical values of a_0

$Pade$	H	γ	u_l	u_m
2/2	1	0.1	-9.91861456	0.3315775
3/3	1	0.1	-9.91861456	0.3315775
5/5	1	0.1	-9.91861456	0.3315775
10/10	1	0.1	-9.91861456	0.3315775
20/20	1	0.1	-9.91861456	0.3315775

Table 2: Numerical values of b_0 for $Bi = 10, \alpha = 1, \epsilon = 0.1 = \gamma$

$Pade$	H	λ	θ_L	θ_U
2/2	1	0.5	0.584186	4.10862
3/3	1	0.5	0.584185	4.11214
4/4	1	0.5	0.584186	3.69915
5/5	1	0.5	0.584186	3.69868
8/8	1	0.5	0.584186	3.73301
10/10	1	0.5	0.584186	3.73300
30/30	1	0.5	0.584186	3.73300
50/50	1	0.5	0.584186	3.73300

Table 3: Effect of different parameters on the development of thermal runaway

$Pade$	H	γ	ϵ	Bi	α	λ_c
2/2	1	0.1	0.1	10	1	0.749345825095
2/2	2	0.1	0.1	10	1	0.760652140485
2/2	3	0.1	0.1	10	1	0.771113308470
2/2	1	0.1	0.1	10	1	0.749345825095
2/2	1	0.1	0.1	20	1	0.825329078304
2/2	1	0.1	0.1	200	1	0.905752170696

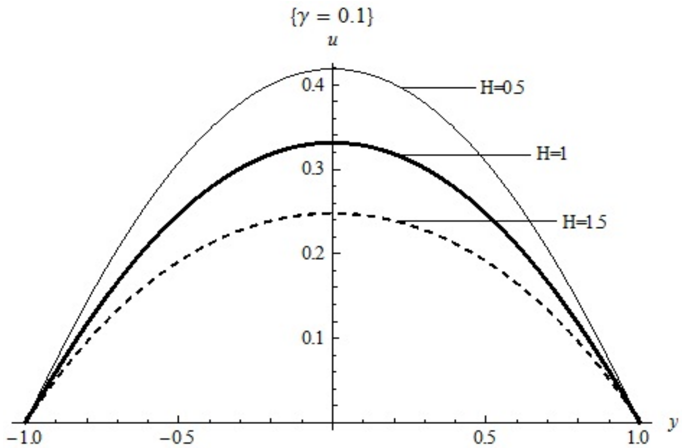

Fig. 2. Fluid velocity profile with variations in Magnetic field intensity

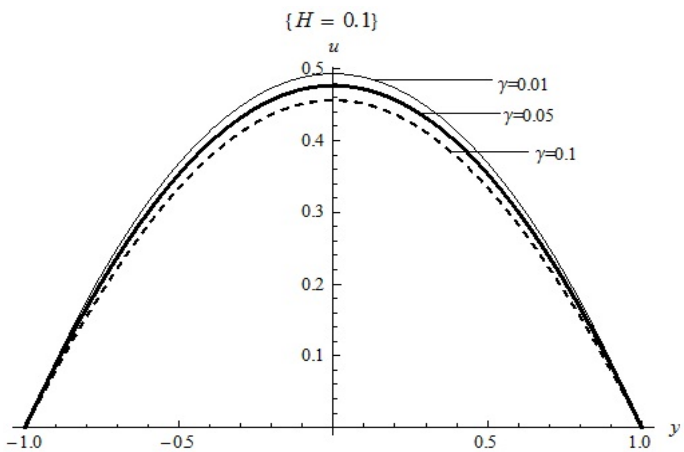

Fig. 3. Fluid velocity profile with variations in third grade material parameter

Figure 3 shows the velocity profile for variations in the non-Newtonian material parameter. As observed from the graph, an increase in the material parameter decreases the velocity maximum within the channel. This is due to the fact that fluid thickening leads to reduction in the degree of freedom of the fluid particles. Figure 4 shows the temperature profile for variations in the Hartmann's number. The result shows that an increase in the Hartmann's number lead to an increase in the fluid temperature due to the contribution of Joule heating of the fluid.In figure 5, the variation of temperature with Frank-Kameneskii parameter is presented. From the result, it is noticed that an increase in the Frank-Kameneskii parameter leads to increase in the fluid temperature, due to

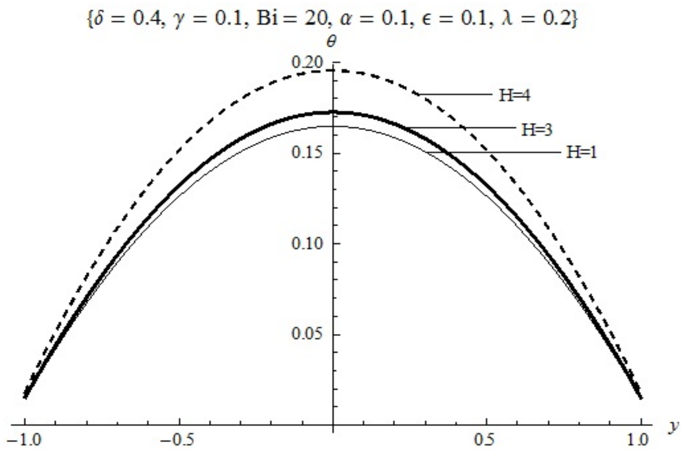

Fig. 4. Fluid temperature profile with variations in magnetic field intensity

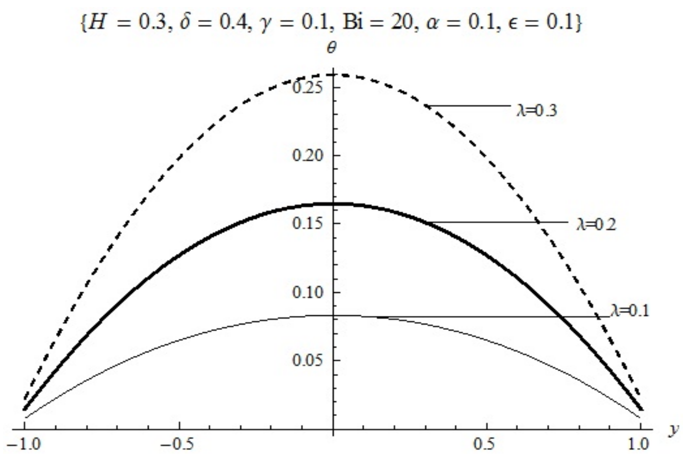

Fig. 5. Fluid temperature profile with variations in Frank-Kameneskii parameter

increase in the heat liberated during the exothermic chemical reaction. Figure 6 presents the temperature profile for variations in the viscous heating parameter. The graph shows that an increase in the viscous heating parameter increases the fluid temperature due to conversion of kinetic energy in the moving fluid to internal energy. In figure 7, the plot of temperature against the non-Newtonian material parameter is presented. The graph shows that an increase in the material parameter decreases the fluid temperature due to increase in the fluid viscosity. Figure 8 shows the temperature profile with variations in convective cooling parameter. It is observed that an increase in the Biot number decreases

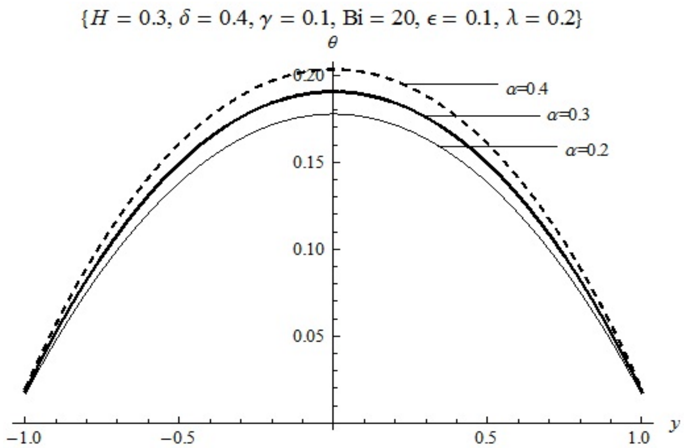

Fig. 6. Fluid temperature profile with variations in viscous heating parameter

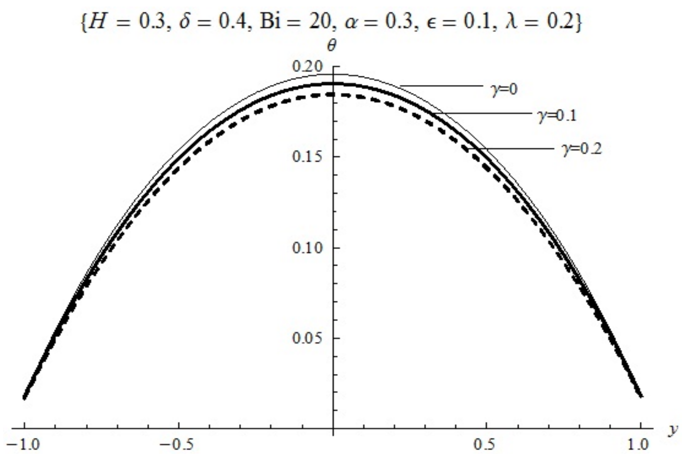

Fig. 7. Fluid temperature profile with variations in third grade material effects

the fluid temperature. This is true since increase in Biot number implies a decrease in the fluid thermal conductivity. Finally, as observed in Figure 9 the problem has two solutions whenever $\lambda < \lambda_c$ and a single solution at $\lambda = \lambda_c$ after which solutions ceases to exist whenever$\lambda > \lambda_c$.

5. CONCLUSION

The mADM-Pade approximant is used to examine the hydromagnetic flow of reactive third grade fluid though parallel plates with convective

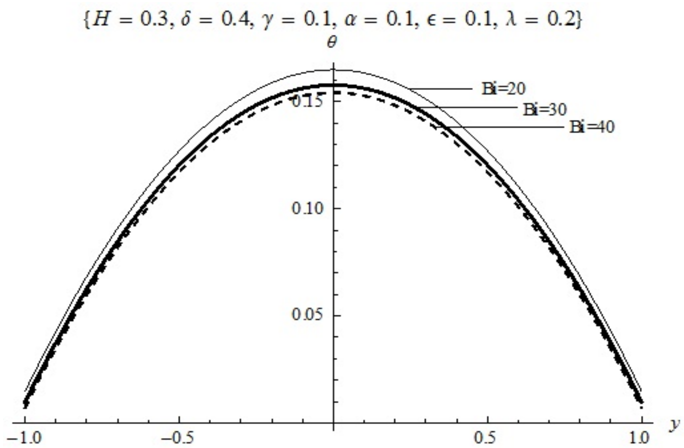

Fig. 8. Fluid temperature profile with variations in Biot number

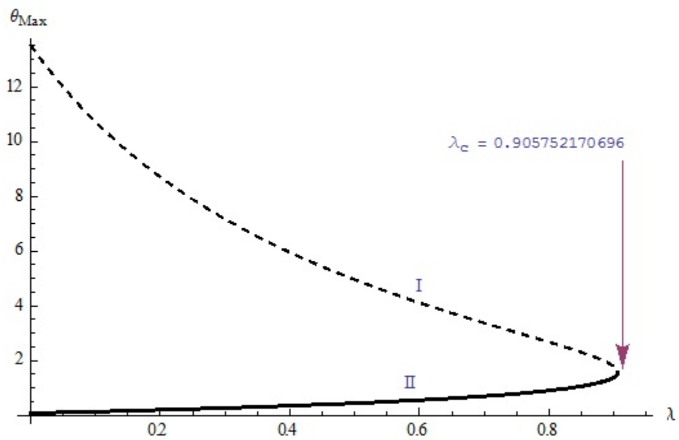

Fig. 9. A slice of approximate bifurcation diagram

cooling. An increase in the Hartmann's number and third grade material effect decreases the velocity profiles. However, an increase in the Hartmann's number, Frank-Kameneskii parameter and viscous heating parameters increases the temperature distribution within the channel while increase in the symmetrical convective cooling and non-Newtonian material parameters decreases the temperature profiles. Additionally, the result of the computation shows that increase magnetic field intensity across the channel has stabilizing effect on the fluid flow. Comparisons made with existing results in literature shows that the combination of mADM-Pade is a promising efficient and effective method in solving nonlinear problems with blow-up solutions.

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