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ENERGY EFFICIENCY OPTIMZATION IN WIRELESS NETWORK CODING

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ABSTRACT. In both wired and wireless networks, network coding provides important benefits such as minimized latency and energy consumption, unrepeated use of same channel, maximized bit rate, improved transmissions efficiency in used network, increased throughput and efficiently managed bandwidth. This paper provides an in-depth background on the theoretic details of network coding, adopts a more practical oriented approach to implementation, and concentrates on the impact of network coding in a dynamically changing wireless environment using distributed algorithms that do not have knowledge of the network environment. Specifically, we focus on energy optimization, which is the amount of energy required to transfer a unit of information from one source to several receivers simultaneously. The model for the proposed system is derived and simulated using the dynamic forwarding factor and transmission power efficiency. Simulation results show that our model could offer network coding benefits of a factor of $\log n$ (where n is the number of nodes), which minimizes the number of data transmissions per round.

Keywords and phrases: Sensor Networks, Network optimization, Bandwidth management, Network broadcasting
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1. INTRODUCTION

Wireless network was designed using the wired network as a blueprint. They abstract the wireless channel as a point-to-point link. But the highly stochastic nature of wireless environments makes it desirable to monitor the state of wireless links. Consider a traditional wireless network with n nodes, where all nodes transmit and receive same information and where the network configuration dynamically changes; a node wishing to transmit senses the channel and begins transmission only when it detects that the channel is free for a pre-defined interval. It then waits for an acknowledgement from the receiver. If the sender does not receive an acknowledgement within a specific duration, it assumes that there is collision

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and selects a random back-off time uniformly distributed within a contention window [1, 2], which doubles for every failed transmission to reduce the probability of collisions. Furthermore, it is unclear how to add this functionality without creating significant complexity with the potential for all implosion. The lack of an acknowledgement leads to congestion problem in such a node and the transmitter is not able to determine a successful reception in the absence of a broadcast. Network coding is a new area of research with several interesting applications in the design of practical networking systems. The notion of coding at the packet level, commonly called network coding dates back to a publication on network information flow [3]. This research which reported on the utility of network coding for multicast in wire-line packet networks attracted much interest, but the utility of network coding reaches much further and in particular, extends to include various wireless applications. In fact, wireless packet networks are the most natural settings for network coding because the very characteristics of wireless links that complicate routing, mainly, their unreliability and broadcast nature, are the very characteristics for which network coding is a natural solution [4]. In [3, 5], the deployment of networks can help to better exploit shared resources such as wireless bandwidth and conserve scarce resources such as battery life.

2. STATEMENT OF THE PROBLEM

Wireless networks are faced with numerous challenges such as throughput, dead spots, high bit error rate and inadequate mobility support. However, the unique characteristics of the wireless medium (i.e., the nature of broadcast, spatial diversity and significant redundancy) provide opportunities for new design principles. In this paper, we study a dynamically changing environment where the network topology constantly changes. Optimizing a network for energy efficiency implies minimizing the number of transmissions required to broadcast a unit of information to all receivers [6-10]. We assume two sources of variance:

(i) Nodes turn on and off independently at random, with probability p. This for instance captures nodes roaming out of range in a cellular environment. Users in a peer-to-peer network turning off their devices to save energy could cause channel erasures due to noise and interferences.

(ii) Nodes move at high speed with respect to the broadcast session duration.

The above assumptions are useful for modeling omni-directional transmissions with interference effects and single transceiver per node. In network coding, the routers mix the content of different packets and broadcast the resulting coded packets on the wireless medium. The broadcast nature of wireless medium provides opportunity to deal with unreliability, because when a node broadcasts a packet, it is likely to be received by a nearby node which can act as a node-hop to forward the packets. Redundancy can also be exploited to compress data which contributes to increasing information flow per transmission and improves the performance of the overall network.

3. BACKGROUND LITERATURE

The concept of network coding was first introduced in [3]. The authors present a simple characterization of the maximum throughput possible in a directed network for a multicast session between a source node and a given set of receivers and demonstrate that network coding could increase throughput. Using illustrations, [11] reports that the gap between the throughput and number of transmissions using network coding is $\Omega(\log n)$ where n is the number of receivers. In [12, 13], the authors show that the maximum achievable rate for multicast problems can always be achieved through linear coding. The former concentrate on linear codes and in particular raises the question of whether they can be applied to solve a wider array of network coding problems. Their proof of existence of a linear solution can be viewed as the first deterministic algorithm for network coding. However, its running time grows exponential with the size of the network. The later derive an algebraic framework for network coding which reduces the problem of finding a linear solution for a generic network coding problem to finding a non-zero point for a multivariate polynomial. While [13] extend [12] by using an algebraic framework for directed graphs with cycles, [14] consider a special case of multicasting and implement an additional structure for the problem in [13] with an efficient randomized algorithm. Their algorithm has a nice property that could be implemented in a distributed fashion without coordination between nodes in a network. The key concept is that nodes take random linear combinations of incoming packets and send them on outgoing links. When the sink receives the packets, it checks if it has received the required number of linearly independent packets and solves the equations for the original messages. In [15], energy efficient network coding design in wireless networks with multiple unicast sessions is studied. Their approach decomposes multiple unicast sessions into a superposition of multicast and unicast sessions, with coding occurring only within each session. They consider a simple XOR coding as in COPE [16] and provide an achievable rate region for a primary interference model. They also study the performance of different algorithms using packet simulation of the *Roofnet* software. There exist two main bodies of theoretical evidence related to this paper. In both cases, emphasis is placed on minimizing the speed of information dissemination expressed in terms of transmission rounds. We present results relating the specific problems of this paper and discuss the experimental results within the network coding realm.

3.1. Epidemic Algorithms for Rumour Spreading (EARS)

This research focuses on networks represented as graphs and distributed algorithms where nodes have no information about the nodes they are communicating with. Each round, a node randomly selects a communication partner among the nodes that are connected to it through an edge and either "pushes" or "pulls" information from it [5, 17]. Results in the literature have established that $\theta(n \log n)$ rounds are required to disseminate the messages. Recent research in [18] shows that using network coding over a complete graph requires $\theta(n)$ rounds, representing a significant reduction.

3.2 Broadcasting and its Applications in Radio Communication Networks

In this research, the wireless environment is modeled as a graph, where messages transmitted by a node are received by all its neighbours. A node successfully receives information *iff* exactly one of its neighbours is transmitting. Again transmissions are divided into rounds and each round is a subset of the nodes transmitting in a way scheduled to minimize conflicts, thus maximizing information spreading. The goal is to disseminate information in the smallest number of rounds. Both centralized and decentralized algorithms are possible. Research results show that the problem is *NP-hard*, i.e., there are static networks where the number of required rounds is $\theta(n \log_2 n)$ and also mobile networks with $\Omega(n)$ number of required rounds [19]. Using a similar model, the problem of minimization has been studied in [20]. However, with energy efficiency, the problem of minimum energy broadcasting in ad-hoc wireless networks has been found to be *NP-complete* [5]. Considering packet delivery ratio and overhead, network coding has been found to compare very favorably with flooding or probabilistic routing [10, 21]. In [21], the authors observe that network coding achieves a 100% delivery ratio with fewer transmissions. Minimum cost multicasting using network coding has also been examined in [22] for mobile networks. Within the scope of the network coding literature, a number of papers have proposed algorithms that employ network coding over a dynamically changing wireless environment and have analytically evaluated their performance. Rather than solving the routing problem, our paper focuses on optimizing the energy efficiency with an assessment of the benefits offered by network coding for wireless applications. We aim at network designs that will achieve the *max-flow bound* on the information transmission rate in a multicast scenario, where all available paths are explored to transmit the data. As a result, the traffic in the network is distributed to multiple links which impacts positively on load balancing [23].

3.2.1 Broadcasting in Ad-Hoc Wireless Networks

Consider n nodes, where each node is a source that wants to transmit information to all other nodes. Suppose we divide the time into rounds, the transmission policy is that during each round, each active node transmits once and transmissions occur successively without conflict. We can employ this simplified policy without loss of optimality, because we do not care about delay, but only about energy efficiency. Assuming that at the beginning of each round, nodes are placed uniformly at random on a unit-area disc of radius $\frac{1}{\sqrt{\pi}}$. This corresponds to having a uniform random mobility pattern, where the rounds are far enough in time to allow a node to move anywhere on the disc with equal probability between rounds. We use this generic mobility model to simplify the analysis of our simulation. We also assume that each node successfully broadcasts information within a fixed radius of r [24], i.e.,

$$r = \theta(\frac{1}{\sqrt{n}}) \tag{1}$$

and is fixed for all nodes. Thus, at each round, each node will have on the average, a constant number of $k = n\pi r^2$ neighbours, of which $k(1-p)$ is active. We compare the energy efficiency in the case of forwarding and network coding and underline our assumption that nodes do not know their neighbours or what information they already have. Without loss of generality, let us assume that during each round and each (possible new) position, node i will always broadcast x_i , where x represents the mobility pattern. In the case of network coding, each node transmits a random linear combination over some finite field Fq, of the symbols it has previously received. So, broadcast to all receivers can be achieved in

(i) $\frac{\theta(n \log_2 n)}{(1-p)^2}$ rounds; without network coding and

(ii) $\frac{\theta(n)}{(1-p)^2}$ rounds; with network coding

where each round has an occurrence of $(1 - p)n$ transmissions on the average and,

$$\frac{T_{nc}}{T_w} = \theta(\frac{1}{\log n}) \tag{2}$$

3.2.1.1 Proof:

Consider the first case of forwarding and given that node j would like to transmit a message to all other $(n - 1)$ nodes in a bipartite graph as follows: The left part consists of the $(n - 1)$ nodes. The right part consists of (M, V_i) where node V_i corresponds to round i, and is connected to the neighbours of node j during this round. Therefore the degree of node V_i is a random variable with average $k(1 - p)$. One question seems obvious: *How many right hand side nodes do we require ?* i.e., what number of rounds does node j need to transmit its' message to all other nodes. This analysis has been performed in the context of LT and Raptor Codes (LRC) and M should scale as $\theta(n \log n)$ [25], since node j will be active with probability $(1 - p)$ and it is easy to see that the average number of rounds required is

$$\frac{\theta(n \log_2 n)}{(1 - p)^2} \tag{3}$$

In [26], a class of erasure code is introduced to solve the problem of retrieving k data packets of interest (with high probability) in a distributed and robust way. Specifically, they show that only $\Omega(\log n)$ packets per data node can be pre-routed to randomly selected storage nodes. In our case, node j will be active on an average of $(1 - p)m$ out of m rounds. Given that node j is active, it will receive on the average $k(1 - p)$ transmissions from its active neighbours. Using standard arguments in the network coding literature and provided that the field F_q is satisfactorily large, each received transmission will bring new information to node j. Thus, it will be able to decode all n information units on the average, after $\frac{\theta(n)}{(1-p)^2}$ rounds. Note that the performance of network coding is not affected by node mobility. In contrast, mobility has a significant effect on forwarding [5]. Initially, as nodes randomly move, the information is disseminated faster than in the case of a static one. However, the assumption that nodes are unaware of what information their neighbours have, as approximately half nodes collect more information and more often, transmission do not bring new information to the receiving nodes, gives the rumour spreading algorithms an edge over networks represented as graphs.

3.2.2 Broadcasting in Cellular Networks

3.2.2.1 Proof:

In the cellular network mode, we have m base stations and n mobile phone receivers. The base-stations consist of k information units intended for transmission to all mobiles. We assume here that the transmission range is the same for all base stations. Each transmission conveys a unit of information and the coverage areas of the base stations do not overlap. In this model, base stations are always active, while nodes are mobile and may turn off. A node will be active and successfully receive information approximately $m(1-p)$ out of every m rounds. Therefore, if base stations are broadcasting using an erasure correcting node at rate $(1-p)$, then each transmission brings useful information at each node. For a node to receive k messages, we will require $\frac{k}{(1-p)^2}$ rounds. In the case of forwarding, we assume that the base station randomly selects and transmits one of the k messages. Thus, each node at each round observes one of the messages the node wants to collect and the calls correspond to the rounds. Using results in [27], we once more require on the average $\frac{n\log_2 n}{(1-p)^2}$ rounds and again realize a $\log n$ benefit.

4. SYSTEM MODEL

While most research works in the broadcasting literature consider speed of transmission measured in terms of required rounds, this paper uses energy efficiency as a measure of performance, which translates to the number of transmissions, thus, reducing network conflicts. Also we are not interested in "static" networks, but in dynamically changing topologies, where nodes do not have information about the network topology. In this section, we model the wireless environment using the dynamic forwarding factor and transmission power efficiency as follows:

4.1 The dynamic forwarding factor Let $N(v)$ be the set of direct neighbours of node v and K, a forwarding factor to be used when a node only has a single neighbour. To adapt to possible changes in the network topology and improve the efficiency, we can use the dynamic forwarding factor dv, such that

$$dv = \frac{K}{|N(v)|} \tag{4}$$

In essence, K is a cumulative forwarding factor between all nodes within a given radio range. It corresponds to the number of packets that are transmitted within the coverage area as a response to the reception of an innovative packet, independent of the node density. In this paper, dv and $N(v)$ are observed and a relationship between these variables is established. Also, using the relation [10]:

$$Q^g_{gK} = \frac{1.72029}{\sqrt{gK}})e^{-0.3210gK} \tag{5}$$

an association between the node transmission probability function Q^g_{gK} and the forwarding factor K is possible, where gK is the transmission range and g, the information vector. It is also necessary to observe the transmission ratio, i.e., transmission with network coding divided by transmission without network coding, where transmission with network coding is given by

$$T_{nc} = \theta(\log n) \tag{6}$$

and transmission without network coding by

$$T_w = \theta(n \log n) \tag{7}$$

The ratio of transmission can therefore be obtained as

$$\frac{T_{nc}}{T_w} = \theta(\frac{1}{\log n}) \tag{8}$$

where n is the number of nodes.

4.2 Transmit power efficiency

Consider a wireless environment with n nodes, in which the transmitter transmitting power P_T decays with the distance ρ , as $\frac{P_T}{\rho}$, due to pathloss and where typical values of the propagation exponent $\gamma \geq 2$. If a receiver at a distance ρ can successfully receive a signal that has power above a threshold ρ_0, then the transmitted power ρ_T must increase proportionally as $\rho_0\rho^\gamma$. Therefore, increasing the range of transmission will also increase ρ_T. On the other hand, increasing the transmission range allows the power to reach more receivers during each transmission. In the case of network coding, for each broadcast transmission to reach at least four closest neighbours (for instance), we require a total power of:

$$P_1 = \frac{n-1}{4}\rho_0 \tag{9}$$

Similarly, for i closest neighbours, we need a total power of

$$P_i = \frac{n-1}{N_i}.\frac{\rho_0}{i_\gamma} \tag{10}$$

Here, for $\gamma > 2$, the optimal strategy in terms of power efficiency is to transmit to the closest neighbour.

5. MODEL SIMULATION AND DISCUSSION OF RESULTS

The proposed model was simulated using the Visual Basic 6.0 programming toolkit. In Figs. 1 and 2, the broadcast communication scenarios for non-optimized and optimized networks, respectively, are presented. Whereas our simulation program can optimize the number of transmissions required for an information unit to reach all nodes in $n\sqrt{n}$ rounds, Fig. 2 (a case of optimized broadcast) is tailored to permit node transmissions in $0.25\sqrt{n}$ rounds. We observe from the graphs that the number of transmissions increases as the network size increases, but the transmission energy reduces with network coding than without network coding. This confirms that network coding saves the total (transmission and coding) energy cost per-source-destination and achieves a higher delivery ratio using much fewer transmissions.

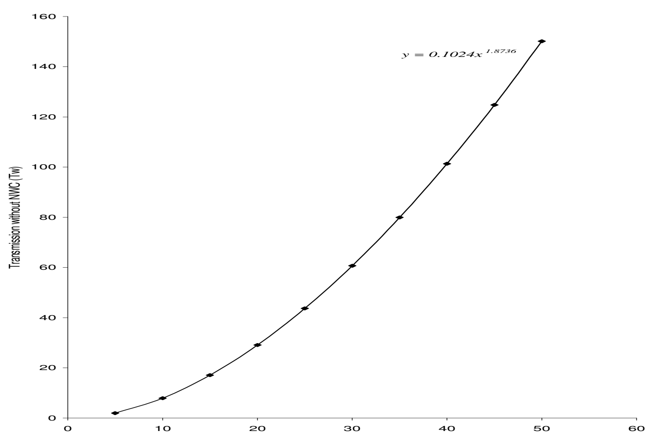

Fig. 1. A Plot of T_w vs n (without network coding)

The probability of a node transmission being innovative is presented in Fig. 3, for probabilistic routing ($g = 1$) and network coding ($g > 1$). With probabilistic routing (flooding), the node transmission probability decreases exponentially with the transmission range, but decays more rapidly with network coding. Our interest 'g' is in the order of tens to hundreds of information vectors. To achieve the probability of not being able to decode below 1%, we set $K \geq 3$ for network coding using energy efficiency optimization, and $K = 2$ for routing with optimization. In Fig. 4, the probability of node transmission is plotted as a function

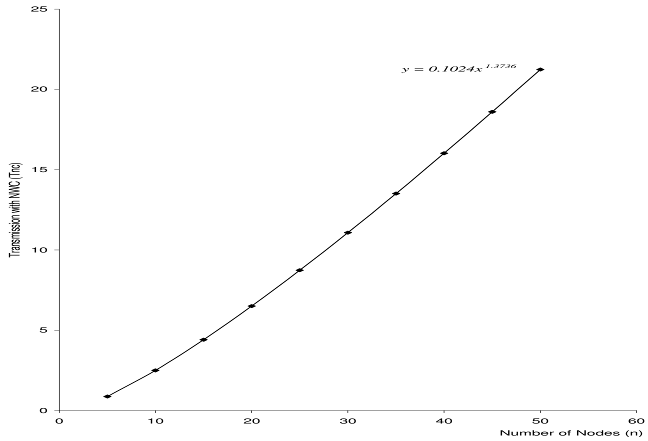

Fig. 2. A Plot of T_{nc} vs n (with network coding)

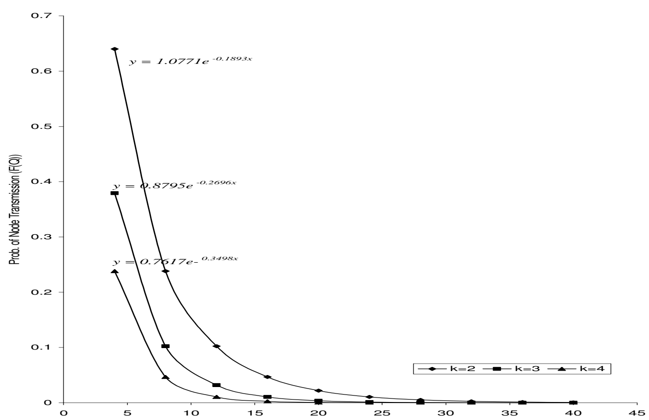

Fig. 3. A Plot of $F(Q)$ vs gK

of forwarding factor shared between nodes within a given range. This corresponds to the number of packets transmitted within a specified coverage area in response to the reception of an innovative packet, independent of the node density. As seen from the graph, the probability decays exponentially. This indicates that the broadcasting of a given message is innovative just within a given coverage area.

Shown in Fig. 5 is a graph of transmit power and the number of nodes. For each transmission range, we choose the smallest cumulative forwarding factor that will result in a performance increase by increasing the transmit power to enable more nodes to be reached by a single transmission. The intuition behind this is that, the larger the transmit range, the more "regular" the network becomes in terms of number of

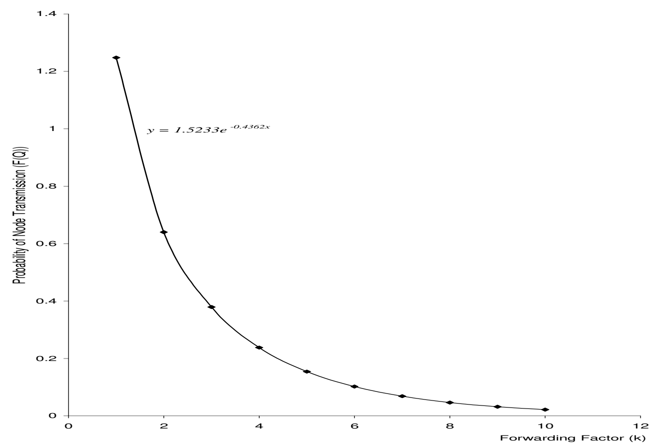

Fig. 4. A Plot of $F(Q)$ vs K

neighbours, and the closer k can be set to the optimum value. Note that nodes can tradeoff the number of transmission to transmit power, which in turn could permit less-complex MAC layer schedules.

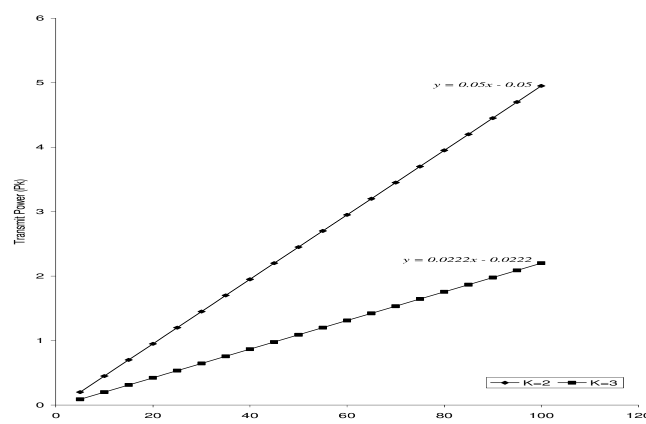

Fig. 5. A Plot of P_k vs n

Fig. 6 illustrates a tradeoff between the dynamic forwarding factor dv (different for every node) and direct neighbours of nodes. We observe from this Fig. that the performance algorithm depends on the value of k. In essence, k is a cumulative forwarding factor shared between all nodes within a given radio range. This corresponds to the number of packets transmitted within a coverage area. To optimize the energy efficiency, the dynamic forwarding factor helps the nodes adapt to irregularities without prior knowledge of the network topology/configuration.

But without optimization, a dv value that would lead to the smallest total number of successful transmissions can only be achieved through a perfect knowledge of the network topology.

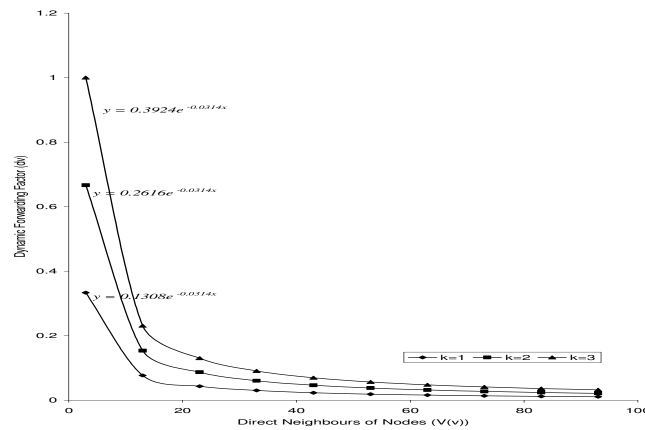

Fig. 6. A Plot of dv vs $V(v)$

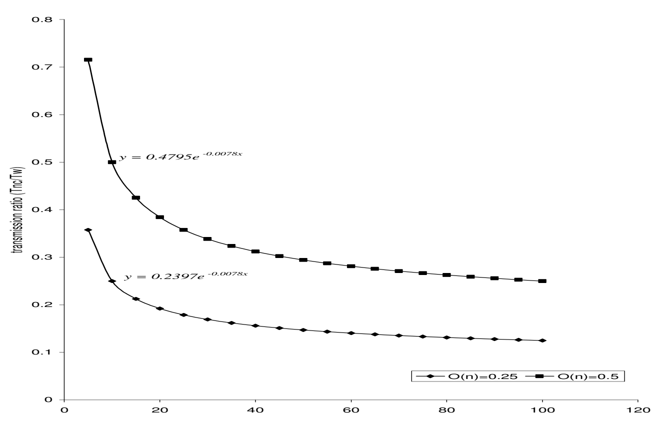

Fig. 7. A Plot of $\frac{T_{nc}}{T_w}$ vs n

Fig. 7 measures the performance of network transmissions ($\frac{T_{nc}}{T_w}$) in a dynamically changing environment, with varying network topologies. We considered two cases of trailing ordering factor of $\mathbf{O}(n) = 0.25$ and $\mathbf{O}(n) = 0.5$. It was observed that network coding with $\mathbf{O}(0.5)$ possessed a higher transmission ratio, thus providing improved performance gain and spatial diversity [28] (a more effective way to combat fading over wireless channels) in terms of number of transmissions. The graphs are also fitted with suitable trend line equations to enable the prediction of new empirical results.

4. CONCLUDING REMARKS

This paper has studied the impact of network coding in a dynamically changing environment, where the main benefits of network coding could be exploited. We adopted a practical approach to minimize the challenges faced by wireless networks during data transmission. A distributed-optimization technique that deals with omni-directional transmissions with interference effects of having single transceiver per node was employed with a derivation and simulation of a suitable system model. Results obtained show that network coding could offer logarithmic benefits in terms of energy efficiency over highly variable environments.

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