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A FIVE-STEP EXTENDED BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLUTIONS OF SEMI-EXPLICIT INDEX-1 DAE SYSTEMS

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ABSTRACT. A five-step Extended Block Backward Differentiation Formulae (EBBDF) for the solutions of semi-explicit index-1 systems of Differential Algebraic Equations (DAEs) is presented. The processes compute the solutions of DAEs in a block by block fashion by some discrete schemes obtained from the associated continuous scheme which are combined and implemented as a set of block formulae. Numerical results revealed this method to be efficient and very accurate, and particularly suitable for semi implicit index one DAEs.

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1. INTRODUCTION

There are many physical problems which are naturally described by a system of Differential Algebraic Equations (DAEs). These problems have a wide range of applications in various branches of science and engineering.

These include mechanical or multibody systems, chemical processes, optimal control, electric circuit design and dynamical systems. A system of ordinary differential equations (ODEs) with algebraic constraints which can be written in the form

$$\left.\begin{aligned} y' = f_1(y(t), z(t)),\ \ y(t_0) = y_0 \\ f_2(y(t), z(t)) = 0,\ \ z(t_0) = z_0 \end{aligned}\right\} \qquad (1)$$

is called differential algebraic equation.

Definition 1: The index along the solution path is defined as the minimum number of differentiations of the system (1) that is required to reduce the system to a set of ODEs .

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Numerical solutions for DAEs were first introduced by Gear by applying numerical methods for ODEs to DAEs [1]. Runge-Kutta methods [2] and BDF [3], [4] are commonly used for semi-explicit index-1 DAEs, however, these methods approximate the solution of (1) at one point. The algorithm presented in this paper is based on block method and approximates the solution at several points (see [5],[6],[7]). Block methods were first introduced by Milne [8] for use only as a means of obtaining starting values for predictor-corrector algorithms and has since then been developed by several researchers (see [[1],[9],[10],[11]), for general use. This paper presents a block method which preserves the Runge-Kutta traditional advantage of being self-starting and efficient.

2. DERIVATION OF THE METHOD

In this section, we construct the main method and additional methods derived from its first derivative and are combined to form the five step Extended Block Backward Differentiation Formula (EBBDF) on the interval from t_n to $t_{n+5} = t_n + 5h$ where h is the chosen step-length and k is the step number. We assume that the exact solution $y(t)$ on the interval $[t_n, t_{n+k}]$ is locally represented by $Y(t)$ given by

$$Y(t) = \sum_{j=0}^{p+q-1} b_j \varphi_j(t), \tag{2}$$

b_j are unknown coefficients to be determined, and $\varphi_j(t)$ are polynomial basis function of degree $p+q-1$ such that the number of interpolation points p and the number of distinct collocation points q are respectively chosen to satisfy $p = k$ and $q > 0$. The proposed class of methods is thus constructed by specifying the following parameters: $\varphi_j(t) = t^j_{n+i}, j = 0, \ldots, k$, $p = k$, $q = 2$, $k = 5$. By imposing the following conditions

$$\sum_{j=0}^{6} b_j t^j_{n+i} = y_{n+i}, \ i = 0, \ldots, 4, \tag{3}$$

$$\sum_{j=0}^{6} j b_j t^j_{n+i} - 1 = f_{n+i}, \ i = 4, 5, \tag{4}$$

assuming that $y_{n+i} = Y(t_n + ih)$, denote the numerical approximation to the exact solution $y(t_{n+i})$, $f_{n+i} = Y'(t_n + ih, y_{n+j})$,denote the approximation to $y'(t_{n+i})$ n is the grid index. It should be noted that equation (3) and (4) lead to a system of seven equations which must be solved to obtain the coefficients b_j, $j = 0, 1, \ldots, 6$. The main method is then obtained by substituting the values of b_j into equation (2). After some algebraic computation, the method yields the expression in the form (5)

$$Y(t) = \sum_{j=0}^{4} \alpha_j(t) y_{n+j} + h(\beta_4(t) f_{n+4} + \beta_5(t) f_{n+5}), \tag{5}$$

where $\alpha_j(t), j = 0, 1, \ldots, 4$, $\beta_4(t)$ and $\beta_5(t)$ are continuous coefficients. The continuous coefficients are also express as a function of$x = \frac{t - t_{n+4}}{h}$ given as

$\alpha_0(x) = -\frac{1}{788}(197x^5 - 185x^6)$, $\alpha_1(x) = \frac{5}{1182}(485x^5 - 455x^6)$
$\alpha_2(x) = -\frac{5}{197}(315x^5 - 295x^6)$, $\alpha_3(x) = \frac{5}{197}(895x^5 - 835x^6)$ $\alpha_4(x) = \frac{5}{2364}(62080x^4 - 124665x^5 + 62525x^6)$
$\beta_4(x) = \frac{5}{197}(745x^5 - 685x^6)$, $\beta_5(x) = \frac{60}{197}x^6)$. The main method is obtained for $k = 5$ by evaluating (5) at $t = t_{n+5}$, which is equivalent to $x = 1$ to obtain the formula

$$y_{n+5} = -\frac{3}{197}y_n + \frac{25}{197}y_{n+1} - \frac{100}{197}y_{n+2} + \frac{300}{197}y_{n+3} - \frac{25}{197}y_{n+4} + \frac{300h}{197}f_{n+4} + \frac{60h}{197}f_{n+5} \tag{6}$$

. To obtain the additional methods, differentiate (5) with respect to t we have

$$Y'(t) = \frac{1}{h}[\sum_{j=0}^{4} \overline{\alpha}_j(t) y_{n+j} + h(\overline{\beta}_4(t) f_{n+4} + \overline{\beta}_5(t) f_{n+5})]. \tag{7}$$

. additional discrete methods are then obtained by evaluating (7) at the points $t = [t_n, t_{n+1}, t_{n+2}, t_{n+3}]$ to give

$$\left.\begin{aligned}
hf_n &= -\frac{1490}{591}y_n + \frac{3880}{591}y_{n+1} - \frac{1890}{197}y_{n+2} + \frac{7160}{591}y_{n+3} - \frac{3880}{591}y_{n+4} + \frac{745h}{197}f_{n+4} - \frac{48h}{197}f_{n+5} \\
hf_{n+1} &= -\frac{90}{197}y_n - \frac{826}{591}y_{n+1} + \frac{576}{197}y_{n+2} - \frac{546}{197}y_{n+3} + \frac{826}{591}y_{n+4} - \frac{152h}{197}f_{n+4} + \frac{9h}{197}f_{n+5} \\
hf_{n+2} &= \frac{41}{1576}y_n - \frac{202}{591}y_{n+1} - \frac{315}{394}y_{n+2} + \frac{374}{197}y_{n+3} - \frac{3703}{4728}y_{n+4} + \frac{157h}{394}f_{n+4} - \frac{4h}{197}f_{n+5} \\
hf_{n+3} &= -\frac{43}{4728}y_n + \frac{53}{591}y_{n+1} - \frac{207}{591}y_{n+2} - \frac{349}{591}y_{n+3} + \frac{4895}{4728}y_{n+4} - \frac{167h}{394}f_{n+4} + \frac{3h}{197}f_{n+5}
\end{aligned}\right\} \tag{8}$$

the methods (6) and (8) are thus combined to give the EBBDF.

3. ORDER OF ACCURACY AND STABILITY OF THE EBBDF

The five step extended block backward differentiation formulae can be represented by a matrix finite difference equation in block form as

$$A^{(1)}Y_\omega = A^{(0)}Y_{\omega-1} + hB^{(1)}F_\omega + hB^{(0)}F_{\omega-1}, \tag{9}$$

where
$Y_\omega = (y_{n+1}\ y_{n+2}\ y_{n+3}\ y_{n+4}\ y_{n+5})^T$,

$$Y_{\omega-1} = (y_{n-4}\ y_{n-3}\ y_{n-2}\ y_{n-1}\ y_n)^T,$$

$$F_\omega = (f_{n+1}\ f_{n+2}\ f_{n+3}\ y_{n+4}\ y_{n+5})^T,$$

$$F_{\omega-1} = (f_{n-4}\ f_{n-3}\ f_{n-2}\ f_{n-1}\ f_n)^T,$$

for $\omega = 1, 2, \ldots$ and $n = 0, 5, \ldots, N-5$.

And the matrices $A^{(1)}$, $A^{(0)}$, $B^{(1)}$ and $B^{(0)}$ are 5 by 5 matrices whose entries are given by the coefficients of (9) given as

$$A^{(1)} = \begin{pmatrix} \frac{826}{591} & \frac{-576}{197} & \frac{546}{197} & \frac{-826}{591} & 0 \\ \frac{202}{591} & \frac{315}{394} & \frac{-374}{197} & \frac{3703}{4728} & 0 \\ \frac{-53}{591} & \frac{207}{394} & \frac{349}{591} & \frac{-4895}{4728} & 0 \\ \frac{-3880}{591} & \frac{1890}{197} & \frac{-7160}{591} & \frac{3880}{591} & 0 \\ \frac{-25}{197} & \frac{100}{197} & \frac{-300}{197} & \frac{25}{197} & 1 \end{pmatrix}$$

$$A^{(0)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{-30}{197} \\ 0 & 0 & 0 & 0 & \frac{41}{1576} \\ 0 & 0 & 0 & 0 & \frac{-43}{4728} \\ 0 & 0 & 0 & 0 & \frac{-1490}{591} \\ 0 & 0 & 0 & 0 & \frac{-3}{197} \end{pmatrix}$$

$$B^{(1)} = \begin{pmatrix} -1 & 0 & 0 & \frac{-152}{37} & \frac{9}{197} \\ 0 & -1 & 0 & \frac{157}{394} & \frac{-4}{197} \\ 0 & 0 & -1 & \frac{-167}{394} & \frac{3}{197} \\ 0 & 0 & 0 & \frac{745}{197} & \frac{-48}{197} \\ 0 & 0 & 0 & \frac{300}{197} & \frac{60}{197} \end{pmatrix}$$

$$B^{(0)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Following Fatunla[12] and Lambert [13] the local truncation error associated with each of the method in the EBBDF can be defined to be the linear difference operator

$$L[y(t); h] = \sum_{j=0}^{k-1} \alpha_j y_{n+j} - h(\beta_{k-1} f_{n+k-1} + \beta_k f_{n+k} - \gamma_l), \qquad (10)$$

where $\gamma_l = 1$, $l = 0, 1, \ldots, k-2$.

Assuming that $y(t)$ is sufficiently differentiable,we can write the terms in (10) as a Taylor series expression of $y(t_{n+j})$ and $f(t_{n+j}) = y'(t_{n+j})$ as

$$y(t_{n+j}) = \sum_{j=0}^{\infty} \frac{(jh)^m}{m!} y^{(m)}(t_n) \text{ and } y'(t_{n+j}) = \sum_{j=0}^{\infty} \frac{(jh)^{m+1}}{(m+1)!} y^{(m+1)}(t_n). \tag{11}$$

Substituting (11) into equations (10) we obtain the expression

$$L[y(t); h] = C_0 y(t) + C_1 h y'(t) + C_2 h^2 y''(t) + \ldots + C_p h^p y^p(t) + \ldots, \tag{12}$$

where the constant coefficients C_m , $m = 0, 1, 2, \ldots$, $l = 1, 2, \ldots, k$ are given as follows:

$C_0 = \sum_{j=0}^{k-1} \alpha_j$

$C_1 = \sum_{j=1}^{k-1} j\alpha_j - \beta_{k-1} - \beta_k + \gamma_l$

$C_2 = \frac{1}{2!}(\sum_{j=1}^{k} j^2 \alpha_j - 2(k-1)\beta_{k-1} - 2k\beta_k + 2l\gamma_l)$

$\vdots$

$C_m = \frac{1}{m!}(\sum_{j=1}^{k-1} j^m \alpha_j - m(k-1)^{m-1}\beta_{k-1} - mk^{m-1}\beta_k + ml^{m-1}\gamma_l)$

where $\gamma_k = 0$ and $\gamma_l = 1, l = 0, 1, \ldots k-2$.

The block method in (9) is said to have a maximal order of accuracy m if

$$L[y(t); h] = \bigcirc(h^{m+1}),\ C_0 = C_1 = \ldots = C_m = 0,\ C_{m+1} \neq 0. \tag{13}$$

Therefore, C_{m+1} is the error constant and $C_{m+1} h^{m+1} y^{(m+1)}(t_n)$ the principal local truncation error at the point t_n.

Therefore the values of the error constant calculated for the five step EBBDF (9) is given as:

$(-\frac{10}{1379}, \frac{418}{4137}, -\frac{106}{6895}, \frac{31}{5910}, -\frac{227}{82740})$ with order p=$(6\ 6\ 6\ 6\ 6)^T$ and T is the transpose.

Zero Stability: The zero stability of the method is concerned with the stability of the difference system in the limit as $h \to 0$ [12]. The difference system (9) becomes

$$A^{(1)} Y_\omega = A^{(0)} Y_{\omega-1}$$

whose first characteristics polynomial $\rho(R)$ given by $|R_j| \leq 1$, $j = 1, 2, \ldots, 5$

Thus from (14)the EBBDF $k = 5$, $\rho(R) = 0$ implies

$$\rho(R) = Det[RA^{(1)} - A^{(0)}] = \frac{1800}{197} R^4 (1 - R). \tag{14}$$

The block method (9) is zero stable since from (15) $\rho(R) = 0$ satisfies $|R_j| \leq 1$, and for those roots with $|R_j| = 1$, the multiplicity does not exceed 1 hence the extended block BDF with continuous coefficients is zero stable.

Consistency and Convergence: We note that the new block method (9) is consistent as it has order $p > 1$. Since the block method (11) is zero stable. According to Henrici [14],
Convergence = zero stability + consistency. Hence the block method (9) converges.

Linear stability: The linear stability properties of the block formula is discussed and determined through the application to the test equation

$$y' = \lambda y, \lambda < 0 \tag{15}$$

applying (9) and (15) yields

$$Y_\omega = Q(z) Y_{\omega-1} \ , \tag{16}$$

where $Q(z)$ is the amplification matrix with $z = \lambda h$ given by

$$Q(z) = (A^{(1)} - zB^{(1)})^{-1}.\ (A^{(0)} + B^{(0)})$$

The matrix $Q(z)$ has eigenvalues $\{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5\} = \{0, 0, 0, 0, \xi_5\}$, where the dominant eigenvalue ξ_5 is a rational function of z given by

$$\xi_5 = \frac{360 + 900z + 1020z^2 + 675z^3 + 274z^4 + 60z^5}{360 - 900z + 1020z^2 - 675z^3 + 274z^4 - 60z^5} \tag{17}$$

which is the stability function of our block method (9). From (17) the usual property of A-stability which requires that for all $z = \lambda h \epsilon\ C^-$ and $\xi_5 < 0$ is obtained. The absolute stability region S associated with the block method (9) is the set $S = \{z = \lambda h$ for that z where the roots of the stability function (17) are moduli $<$ $1\}$.

In the spirit of Hairer and Wanner [15], the stability region S is presented in white colour which corresponds to the 5- step extended block BDF stability function (17). Clearly, from Figure 1 above, it is obvious that the method is A- stable, since it has no pole of the stability function (17) represented by the plus sign in the left half complex plane.

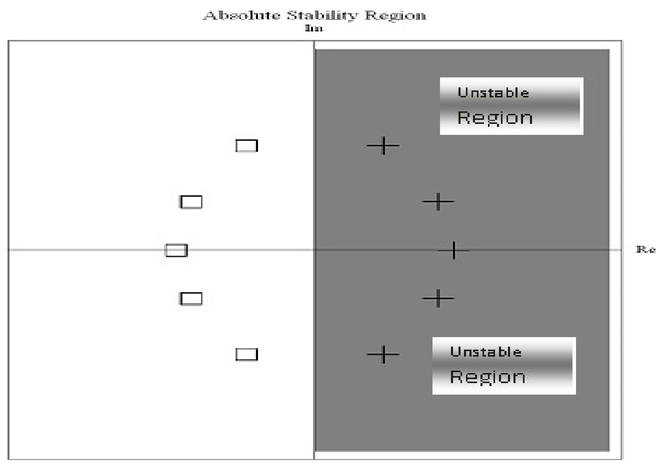

Fig. 1. Absolute stability region.

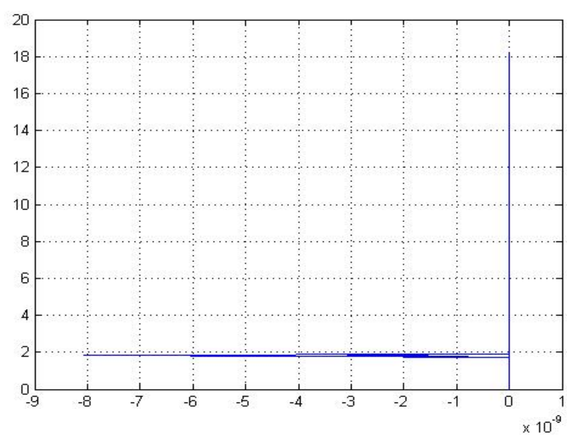

Fig. 2. All points for stability region.

4. COMPUTING WITH THE EBBDF

The method is implemented more efficiently as a 5-step block numerical integrators for (1) to simultaneously obtain the approximations $(y_{n+1}\ y_{n+2}\ y_{n+3}\ y_{n+4}\ y_{n+5})^T$ without requiring back values and predictors,taking $n = 0, 5, \ldots, N-5$, over sub-intervals $[t_0, t_5], \ldots, [t_{N-5}, t_N]$, where N is the total number of points. For example $n = 0, \omega = 1$ $((y_{n+1}\ y_{n+2}\ y_{n+3}\ y_{n+4}\ y_{n+5})^T$, are simultaneously obtained over the sub-interval $[t_0, t_5]$, as y_0 is known from (1).

For $n = 1, \omega = 2$, $(y_6\ y_7\ y_8\ y_9\ y_{10})^T$ are simultaneously obtained over the sub-interval $[t_5, t_{10}]$, as y_5 is known from the previous block. Hence, the sub-intervals do not over-lap. The computations were carried out using our written code in Matlab. It should be noted that for linear problems, the code uses Gaussian elimination and uses the Newton's method for nonlinear problems.

5. Numerical Examples

In this section, we give three examples to illustrate the accuracy of the method. The three problems are standard DAE problems whose solutions are of importance in applied systems. We find absolute errors of the approximate solutions. All computations were carried out using our written Mathematica code in Mathematica 9.0.

Example 5.1:

$$y'(t) = tcost - y + (1-t)z,\ y(0) = 1$$

$$sint - z = 0,\ z(0) = sint$$

The exact solution is $y(t) = e^{-t} + t\sin t,\ z(t) = \sin t$

Example 5.2:

$$\begin{gathered} y_1'(t) = -ty_2 - (1+t)z_1,\ y_1(0) = 5 \\ y_2'(t) = ty_1 - (1+t)z_2,\ y_2(0) = 1 \end{gathered}$$

$$\frac{y_1 - z_2}{5} - cos(\frac{t^2}{2}),\ z_1(0) = -1$$

$$\frac{y_2 - z_1}{5} - sin(\frac{t^2}{2}),\ z_2(0) = 0$$

$0 \leq t \leq 10$

The exact solution is $y_1 = \sin t + 5\cos(\frac{t^2}{2})$, $y_2 = \cos t + 5\sin(\frac{t^2}{2})$, $z_1 = -\cos t,\ z_2 = \sin t$

Example 5.3:

$$y'(t) = z,\ y(0) = 1$$

$$z^3 - y^2 = 0,\ z(0) = 1$$

The exact solution is
$y(t) = (1 + \frac{t}{3})^3$, $z(t) = (1 + \frac{t}{3})^2$

The tables below show the numerical results of extended block BDF method for k=5 applied to semi explicit index-1 DAEs. Tables 1-6 display the results for example 5.1, 5.2, and 5.3. The results obtained show that the method is efficient for semi-explicit index-1 DAEs and can cope with large step size

Table 1. Numerical results for the for Example 5.1 .

t	i	Exact y(t) z(t)	Block method(9) y_i z_i	Error $h = 0.1$ $\lvert y(t) - y_i\rvert$ $\lvert z(t) - z_i\rvert$	i	Error $h = 0.01$ $\lvert y(t) - y_i\rvert$ $\lvert z(t) - z_i\rvert$
2	20	1.95393014	1.95393606	3.34×10^{-8}	200	4.32×10^{-14}
		0.90929743	0.90929861	4.16×10^{-9}		8.54×10^{-15}
4	40	-3.00889434	-3.00889931	1.44×10^{-8}	400	2.89×10^{-14}
		-0.75680249	-0.75680352	3.46×10^{-9}		7.33×10^{-15}
6	60	-1.67401423	-1.67402028	2.81×10^{-8}	600	3.55×10^{-15}
		-0.27941549	-0.27941584	1.28×10^{-9}		3.16×10^{-15}
8	80	7.91520143	7.91521420	5.46×10^{-8}	800	8.52×10^{-14}
		0.98935824	0.98935949	4.53×10^{-9}		8.88×10^{-15}
10	100	-5.44016570	-5.44017306	1.96×10^{-8}	1000	1.78×10^{-13}
		-0.54402111	-0.54402190	2.49×10^{-9}		1.82×10^{-14}

Table 2. comparison of methods for the for Example 5.1 Max Error = $|y(t_i) - y_{i_{approx}}|$,$|z(t_i) - z_{i_{approx}}|$.

h	Abass et.al.[7] Max Error	h	Block Method (9) Max Error
0.01	4.78153×10^{-4}	0.5	5.70843×10^{-4}
0.001	4.91863×10^{-6}	0.1	5.76343×10^{-8}
0.0001	4.94542×10^{-8}	0.05	8.85906×10^{-10}
0.00001	4.94939×10^{-10}	0.01	1.77636×10^{-13}
0.000001	1.21411×10^{-9}	0.005	3.37508×10^{-13}

Table 3. Numerical results for the for Example 5.2.

t	Error $h = 0.1$ $\lvert y(t_1) - y_1\rvert$ $\lvert z(t_1) - z_1\rvert$	 $\lvert y(t_2) - y_2\rvert$ $\lvert z(t_2) - z_2\rvert$	Error $h = 0.01$ $\lvert y(t_1) - y_1\rvert$ $\lvert z(t_1) - z_1\rvert$	 $\lvert y(t_2) - y_2\rvert$ $\lvert z(t_2) - z_2\rvert$
2	8.92×10^{-6}	2.88×10^{-7}	8.70×10^{-12}	2.53×10^{-13}
	8.44×10^{-7}	2.99×10^{-6}	8.49×10^{-13}	2.93×10^{-12}
4	1.26×10^{-4}	1.70×10^{-4}	1.12×10^{-10}	1.63×10^{-10}
	7.88×10^{-5}	2.99×10^{-5}	7.55×10^{-11}	7.64×10^{-11}
6	8.70×10^{-4}	2.03×10^{-3}	7.54×10^{-10}	1.90×10^{-9}
	5.22×10^{-4}	6.03×10^{-4}	8.41×10^{-10}	4.11×10^{-10}
8	7.25×10^{-3}	8.47×10^{-3}	1.06×10^{-8}	4.34×10^{-9}
	6.68×10^{-3}	3.75×10^{-3}	2.25×10^{-9}	4.84×10^{-9}
104	7.24×10^{-2}	1.24×10^{-1}	4.05×10^{-8}	1.66×10^{-8}
	1.80×10^{-1}	2.22×10^{-2}	7.27×10^{-9}	1.93×10^{-8}

Table 4. Comparison of methods for the for Example 5.2 Max Error $= |y(t_i) - y_{i_{approx}}|$, $|z(t_i) - z_{i_{approx}}|$.

	Abass et.al. [7]		Block Method (9)
h	Max Error	h	Max Error
0.01	6.60563×10^{-4}	0.05	2.51154×10^{-4}
0.001	6.60001×10^{-6}	0.01	4.04563×10^{-8}
0.0001	6.60125×10^{-8}	0.005	6.31331×10^{-10}
0.00001	6.87188×10^{-10}	0.002	3.85381×10^{-12}
0.000001	2.02971×10^{-9}	0.001	1.92988×10^{-11}

Table 5. Numerical results for the for Example 5.3.

t	i	Exact y(t) z(t)	Block method(9) y_i z_i	Error $h = 0.1$ $\|y(t) - y_i\|$ $\|z(t) - z_i\|$	Error $h = 0.01$ $\|y(t) - y_i\|$ $\|z(t) - z_i\|$
2	20	4.62962962	4.62962962	6.21×10^{-15}	8.88×10^{-15}
		2.77777777	2.77777777	2.66×10^{-15}	3.99×10^{-15}
4	40	12.70370370	12.70370370	2.49×10^{-14}	4.08×10^{-14}
		5.44444444	5.44444444	7.11×10^{-15}	1.33×10^{-14}
6	60	26.99999999	26.99999999	5.68×10^{-14}	1.08×10^{-13}
		8.99999999	8.99999999	1.24×10^{-14}	2.66×10^{-14}
8	80	49.29629629	49.29629629	1.28×10^{-13}	2.48×10^{-13}
		13.44444444	13.44444444	2.66×10^{-14}	5.15×10^{-14}
10	100	81.37037037	81.37037037	2.55×10^{-13}	4.83×10^{-13}
		18.77777777	18.77777777	4.26×10^{-14}	8.17×10^{-14}

Table 6. comparison of methods for the for Example 5.3 Max Error$=|y(t_i) - y_{i_{approx}}|$,$|z(t_i) - z_{i_{approx}}|$.

	Abass et.al.[7]		Block Method (9)
h	Max Error	h	Max Error
0.01	2.04173×10^{-3}	0.5	1.27898×10^{-13}
0.001	2.06314×10^{-5}	0.1	4.83169×10^{-13}
0.0001	2.06367×10^{-7}	0.05	8.95284×10^{-13}
0.00001	1.01275×10^{-9}	0.01	1.84741×10^{-12}
0.000001	1.04160×10^{-8}	0.005	4.81748×10^{-12}

6. CONCLUDING REMARKS

We have proposed in this paper a five step EBBDF for the solutions of semi-explicit index-1 DAEs. The method is of order 6, it is self-starting and provides good accuracy. Numerical examples using the five step EBBDF showed that the method is accurate and efficient as evident in Tables 1-6. The EBBDF is also found to be convergent and A-stable, making it a suitable method for this class of problems.

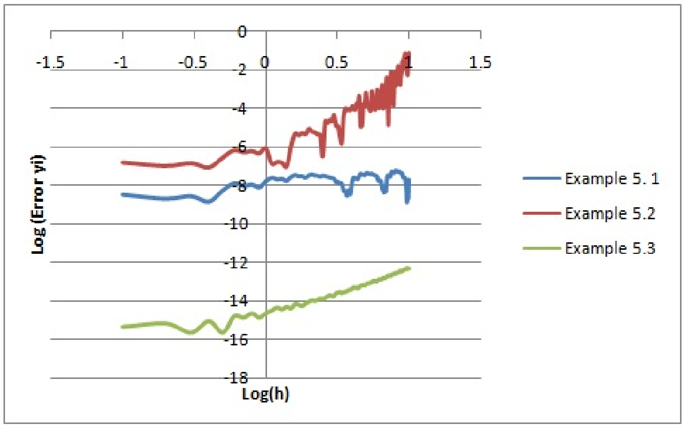

Fig. 3. Error distribution for Example 5.1, 5.2, 5.3 for stepsize h=0.1

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