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STEADY ARRHENIUS LAMINAR FREE CONVECTIVE MHD FLOW AND HEAT TRANSFER PAST A VERTICAL STRETCHING SHEET WITH VISCOUS DISSIPATION

A. J. OMOWAYE[1] AND O. K. KORIKO

ABSTRACT. An analysis of the effects of Arrhenius kinetics on hydromagnetic free convective flow(set up due to temperature) of an electrically conducting fluid past a vertical stretching sheet kept at constant temperature with viscous dissipation is presented. A similarity transformation is used to reduce the governing partial differential equations into a system of ordinary differential equations, which is solved numerically . The effect of various parameters on the velocity and temperature profiles as well as the skin friction and Nusselt number are presented in graphs and tables. It was shown that the velocity and temperature increases as local Eckert number (or viscous dissipation parameter) increases.

Keywords and phrases: Steady, MHD, Arrhenius kinetics, free convection, boundary layer
2010 Mathematical Subject Classification: A80

1. INTRODUCTION

The study of laminar flow has attracted the interest of many scientists in recent times because of its considerable importance in many practical and technological applications. Hydromagnetic free convection flow has great application in the field of steller and planetary, magnetispheres and aeronautics.However,hydro-magnetic flow and heat transfer problems have become more important industrially in many metallurgical processes involving the cooling of many continuous strips or filaments by drawing them through an electrically conducting fluid subject to magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. Another important application of hydromagnetic flow to metallurgy lies in the purification of molten metals from nonmetallic inclusion by application of magnetic field.

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[1]Corresponding author

Shrama and Singh [17] investigated the effect of temperature dependent electrical conductivity on steady natural convection flow of a viscous incompressible low Prandtl ($Pr << 1$) electrically conducting fluid along an isothermal vertical non-conducting plate in the presence of transverse magnetic field and exponentially decaying heat generation. The governing equations of continuity, momentum and energy are transformed into ordinary differential equations using similarity transformation. The resulting coupled non-linear ordinary differential equations were solved numerically. They showed that increase in Prandtl number decreases the skin friction and velocity profiles.Chamha[1] reported analytical solutions for the problem of heat and mass transfer by steady flow of an electrically conducting and heat generating/absorbing fluid on a uniformly moving vertical permeable surface in the presence of a magnetic field of first order chemical reaction. He showed that the fluid velocity decreased as either of the prandtl number, the schmidt number or the strength of the magnetic field was increased as either of the thermal or concentration buoyancy effects were increased. Mansuor et al[10] considered a steady two dimensional nonlinear MHD boundary layer flow of an incompressible, viscous and electrically conducting fluid in the presence of a uniform magnetic field with heat, mass transfer and chemical reaction in a porous medium. The fluid properties were assumed to be constant. The results showed that the flow field was influenced appreciably by the presence of chemical reaction, viscous dissipation and suction or injection flow. Makinde[9] reported steady flow of a reactive variable viscosity fluid in a cylindrical pipe with an isothermal wall. The steady state thermal critically conditions which does not depend on Frank-Kamentskii and viscous heating parameters were obtained. The result also revealed that rapid convergence of the approximation procedure with gradual increase in the number of series coefficients utilised in the approximants. Recently, Mohyud-Din et al [11] studied free convective boundary layer flow modeled by a system of nonlinear ordinary differential equations. It was observed that combination of modified variational iteration method and the pade approximation improved the accuracy and the convergence domain of the problem.

Crane [4] first introduced the study of steady two-dimensional boundary layer flow caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point in the sheet.(cf. Gupta and Gupta [6]; Rajagopal et al.[14]; Siddapa and Abel [18]; Chen and Char [2]; Laha et al. [8]; Vajravelu and Nayfeh[21] and

Tan et al. [20]) studied various aspects of this problem,such as the heat,mass and momentum transfer in viscous flows with or without suction or blowing. Shit [16] discussed heat and mass transfer over a stretching sheet under the influence of applied uniform magnetic field and the effects of Hall current. The non-linear boundary layer equations together with the boundary conditions were reduced to a system of non-linear ordinary differential equations by using the similarity transformation. The system of non-linear ordinary differential equations were solved numerically by finite difference scheme and Newton's method of linearization. He showed that all the instantaneous flow characteristics are affected by the Hall current parameter.

Shit and Haldar [15]presented the combined effects of thermal radiation and Hall current on momentum, heat and mass transfer in laminar boundary-layer flow over an inclined permeable stretching sheet with variable viscosity. The sheet was linearly stretched in the presence of an external magnetic field and the fluid motion was subjected to a uniform porous medium. The effect of internal heat generation/absorption was also taken into account. The fluid viscosity was assumed to vary as an inverse linear function of temperature. The boundary-layer equations that governed the flow problem were reduced to a system of non-linear ordinary differential equations with a suitable similarity transformation. Then the transformed equations were solved numerically by employing a finite difference scheme. It was noted that the velocity profiles were strongly affected by the inclination of the sheet, whereas the temperature and concentration profiles are weakly affected.

Viscous dissipation which, appear as a source term in the fluid flow generates appreciable temperature, gives the rate at which mechanical energy is converted into heat in a viscous fluid per unit volume. This effect is of particular significance in natural convection in various devices that are subjected to large variation of gravitational force or that operate at high rotational speeds,as pointed out by Gebhart [5] in his study of viscous dissipation on natural convection in fluids. Ostrach [13] presented similarity solution of natural convective flow along vertical isothermal plate. Gebhart[5] studied the effect of dissipation on natural convective flow.Also, Soundalgekar [19] studied natural convective flow along vertical porous plate with suction and viscous dissipation. Joshi and Gebhart[7] observed the effect of pressure stress work and viscous dissipation in some natural convective flow: isothermal, uniform flux and plumes. Watanabe and Pop [22] investigated the heat transfer in thermal boundary layer of MHD flow over a flat plate.

In the present work, we study steady laminar free convective MHD flow and heat transfer past a vertical stretching plate under Arrhenius kinetics . We further assume that the flow is subject to magnetic field and there is heat generated by viscous dissipation. This extends the previous work of Mohyud-Din et al [11] . We then reduced the two dimensional continuity, momentum and energy equations to a system of ordinary differential equations using similarity solutions. The resulting ordinary differential equations are solved numerically.

2. MATHEMATICAL FOMULATION OF THE PROBLEM

Consider steady laminar two-dimensional free convection flow of a viscous incompressible fluid along a vertical stretching sheet kept at constant temperature T_w , under Arrhenius kinetics. The flow is assumed to be in x direction,which is taken along the sheet and y-axis is normal to the sheet. A uniform strong magnetic field β_0 is assumed to be applied. It is assumed that external field is zero. The electrical field owing to polarization of charges and Hall effect are neglected and there is heat generation due to viscous dissipation. The flow configuration and coordinate system is shown in Figure 1.

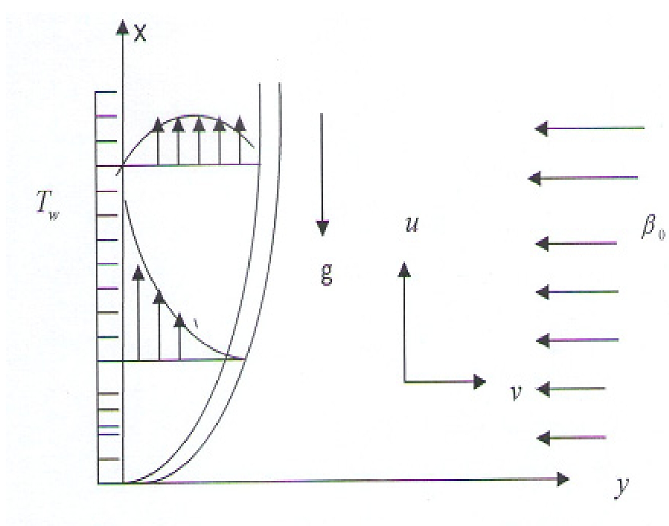

Fig. 1. Physical model and coordinate system

Incorporating the Buossinesq approximation within the boundary layer, the governing equation of continuity, momentum and energy respectively are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma\beta_o^2}{\rho}\ u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho\ c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho\ c_p}(\frac{\partial u}{\partial y})^2 + \frac{AQe^{-(\frac{E}{RT})}}{\rho\ c_p} \tag{3}$$

The boundary conditions are:

$$y = 0, u = u(x) = cx, v = 0, T = T_w \tag{4}$$

$$y \to 0, u = 0, T = T_\infty \tag{5}$$

Introducing the stream function $\psi(x, y)$ as defined by

$$u = \frac{\partial\psi}{\partial y}\ \ and\ \ v = -\frac{\partial\psi}{\partial x} \tag{6}$$

the similarity variable $\eta = y\left(\frac{c}{\nu}\right)^{\frac{1}{2}}$ and

$$\psi(x, y) = (c\nu)^{\frac{1}{2}} x f(\eta) \tag{7}$$

the dimensionless temperature

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

where c is the stretching plate parameter. Substituting equations(6)-(8) into equations (2) and (3) and simplify, we have

$$f''' + f''f - (f')^2 + Gr\theta - Mf' = 0 \tag{9}$$

$$\frac{1}{Pr}\theta'' + f\theta' + Ec(f'')^2 + \delta e^{\frac{\theta}{\alpha+\epsilon\theta}} = 0 \tag{10}$$

where the local grashof number $Gr = \frac{g\beta(T_w - T_\infty)\nu}{c^2 x}$,the activation energy parameter $\epsilon = \frac{RT_\infty}{E}$, the magnetic parameter $M = \frac{\sigma\beta_0^2}{\rho c}$, Prandtl number $Pr = \frac{c_p\mu}{k}$,the Frank-Kamenetskii parameter $\delta = \frac{AQEe^{-\frac{E}{RT_0}}}{\rho c_p c R T_\infty^2}$ and the local Eckert number(or viscous dissipation parameter) $Ec = \frac{c^2 x^2}{c_p T_\infty}$, $\alpha = \frac{\epsilon T_\infty}{(T_w - T_\infty)}$. For this analysis we assume $\alpha = 1$

Using equation(6) in equation(1), the equation is identically satisfied. The corresponding boundary conditions are reduced to:

$$f(0) = 0, f^{'}(0) = 1, f^{'}(\infty) = 0, \theta(0) = 1, \theta(\infty) = 0 \tag{11}$$

Equations(9) and (10) subject to (11) are the local similarity equations governing the flow.

We now move to examining Nusselt number and skin-friction coefficient. We therefore denote and define respectively as in Omowaye and Koriko[12]: $Nu = (\frac{\nu}{c})^{\frac{1}{2}} \frac{q_w}{k^*(T_w - T_\infty)} = -\theta'(0)$, where $q_w = -k^*(\frac{\partial T}{\partial y})_{y=0}$,$C_f = \frac{\tau_w}{\rho c(\nu c)^{\frac{1}{2}}} = xf^{''}(0)$,where $\tau_w = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})_{y=0}$ is the shear stress at the sheet.

3. NUMERICAL COMPUTATION

The set of nonlinear coupled ordinary differential equations (9) and (10) with boundary conditions (11) are solved using Runge-Kutta Fourth order technique along with the shooting method (Conte and Boor [3]). The basic idea is to reduce the higher order non-linear differential equations (9) and (10) to first order linear differential equations and they are further transformed into initial value problem and then apply shooting technique with Pr, Gr, Ec, M, ϵ and δ as prescribed parameters. The results are presented in figures 2 - 8.

4. DISCUSSION AND RESULTS

The local similarity equations governing the flow along with the boundary conditions have been solved in the preceding section in order to give the detail of flow fields, thermal and velocity distributions. The effects of the main controlling parameters as they appear in the governing equations are discussed in the current section. In the entire numerical calculations Pr, ϵ, Gr, Ec, M and δ are varied over ranges which are listed in the figure legends.

Typical variations of the velocity profiles along the span wise coordinate are shown in figures 2 - 5 for different values of Grashof number, magnetic parameter, local Eckert number and Frank-kamenetskii parameter. In figure 2, the numerical result shows that the velocity increases with increase in Gr. This is in agreement with physical fact that buoyancy force assists the flow and this result is in agreement with what Chamkha [1] obtained. While in figure 3, the velocity decreases with increase in magnetic parameter. It is noted that the presence of a magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem. This type of resistive force tends to slow down the flow field(velocity). Since the magnetic field has stability effect, the velocity decreases with increasing M , this yields identical results of Chamkha[1]. The velocity increases with the increase in Frank-Kamenetskii parameter. This is shown in figure 5. Also, the temperature profiles are shown in figures 6 and 7 for various values of local Grashof number and Frank-kamenetskii parameter. It is reported in figure 6 that the temperature decreases as local Grashof number and the temperature increases as Frank-kamenetskii parameter increases. Also, the effect of viscous dissipation on velocity and temperature profiles are shown in figures 4 and 8 respectively. By analysing these figures, it is clearly revealed that the effect of local Eckert number is to increase both the velocity and temperature distributions in the flow region. This is due to the fact that heat energy is stored in liquid due to frictional heating thus, the effect of increasing local eckert number is to enhace the temperature at any point as well as the velocity. Numerical values of heat transfer rate and local skin friction are presented in tables 1 and 2. The effects of Pr, M, Gr, Ec, ϵ and δ on the rate of heat transfer(Nu) are numerically shown in table1. We observed that an increase in Pr, Gr, Ec and δ leads to an incease in the rate of heat transfer expressed in terms of Nusselt number while the reverse effect is noted for M and ϵ . The effects of Pr, M, Gr, Ec, ϵ and δ on the skin friction (cf) are numerically shown in table2. We observed that an increase in Pr, M, Ec and δ leads to a decease in the skin friction while the reverse is the case for Gr and ϵ . Finally,we observed from table 3 that the numerical values of temperature gradient in present paper are in agreement with the result of Mohyud -Din et al [11].

5. CONCLUDING REMARKS

The problem of steady laminar free convective MHD flow and heat transfer past a vertical stretching sheet under Arrhenius Kinetics and viscous dissipation has been considered. Numerical results are presented to illustrate the details of the flow condition and fluid properties.

The main findings of the present study are:

(i) The velocity profiles increase with increase in local grashof number and local eckert number. While temperature increases as local eckert number increases.

(ii) An increase in local eckert number enhance the rate of heat transfer in terms of Nusselt number while the reverse is the case for skin friction cofficient.

Table 1. The rate of heat transfer in terms of Nusselt number (Nu)

Pr	M	Gr	Ec	ϵ	δ	Nu
0.071	0.1	2	1	0.01	0.1	0.2710
1	0.1	2	1	0.01	0.1	1.3414
0.071	0.8	2	1	0.01	0.1	0.2564
0.071	0.1	4	1	0.01	0.1	0.3024
0.071	0.1	2	3	0.01	0.1	0.2849
0.071	0.1	2	1	0.5	0.1	0.2657
0.071	0.1	2	1	0.01	0.3	0.3488

Table 2. Numerical values of shear stress in terms of Skin friction(C_f)

Pr	M	Gr	Ec	ϵ	δ	Cf
0.071	0.1	2	1	0.01	0.1	0.3511
1	0.1	2	1	0.01	0.1	-0.4852
0.071	0.8	2	1	0.01	0.1	-0.1179
0.071	0.1	4	1	0.01	0.1	1.4026
0.071	0.1	2	3	0.01	0.1	0.3351
0.071	0.1	2	1	0.5	0.1	0.3542
0.071	0.1	2	1	0.01	0.3	0.2800

Table 3. Values of $-\theta'(0)$ for different values of Pr are compared with results obtained by Mohyud-Din et al [11]

Pr	$-\theta'(0)$ $Gr = 0.1, Ec = 0.1, \epsilon = 0.09, \delta = 0.0815$	
	Mohyud-Din et al [11]	Present Paper
0.710	0.2133	0.2132
0.711	0.2134	0.2137
0.712	0.2135	0.2142

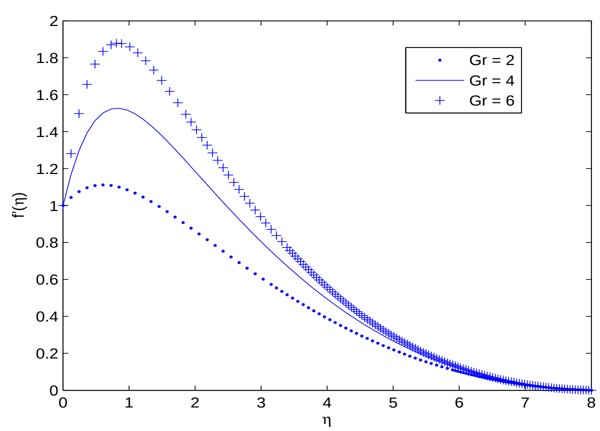

Fig. 2. Velocity Profiles against spanwise coordinate η for various Gr

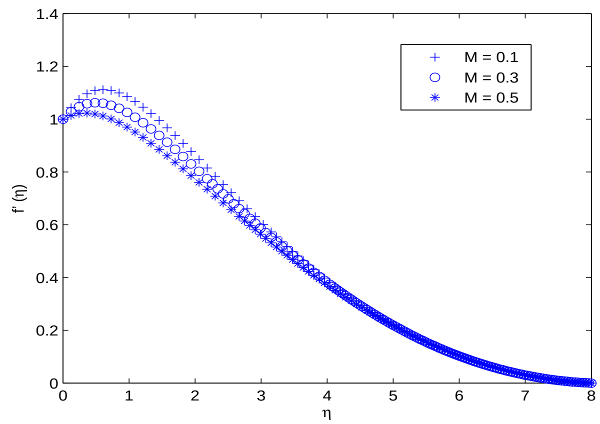

Fig. 3. The variation of f'(η) against η for various M

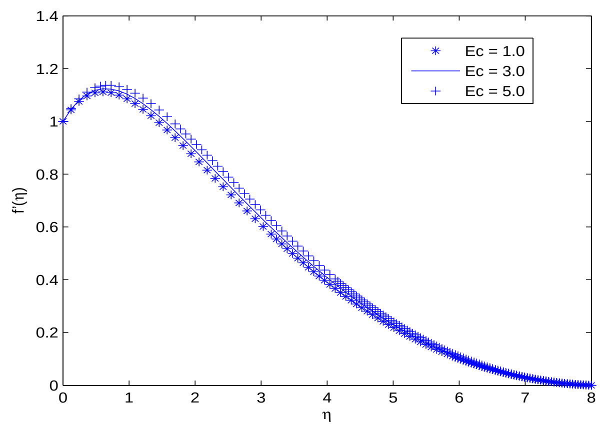

Fig. 4. Variation of Velocity Profiles for different values of Ec

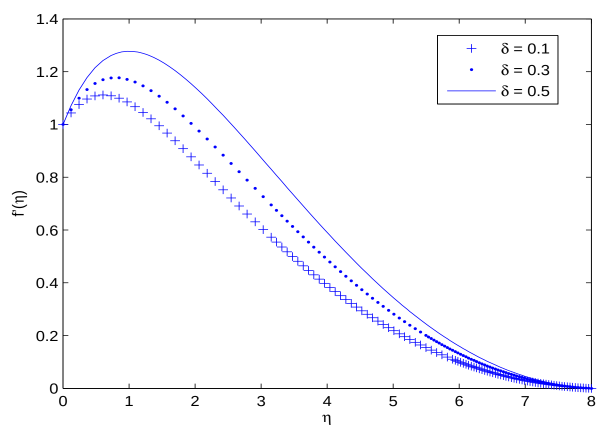

Fig. 5. Velocity distribution versus η for various δ

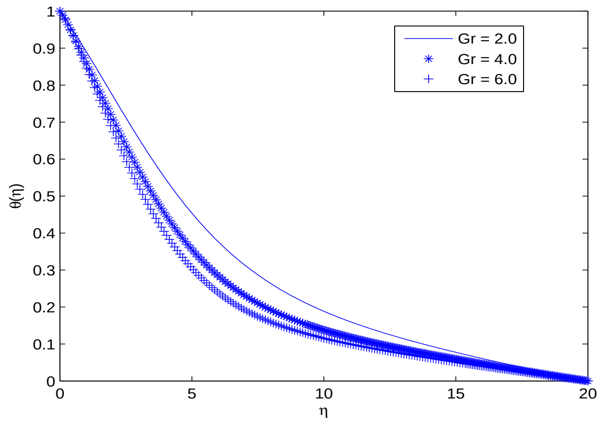

Fig. 6. Temperature profiles for different values of Gr

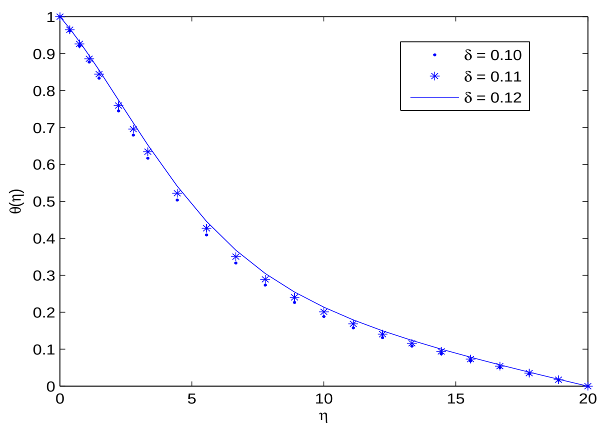

Fig. 7. Temperature Profiles against η for various δ

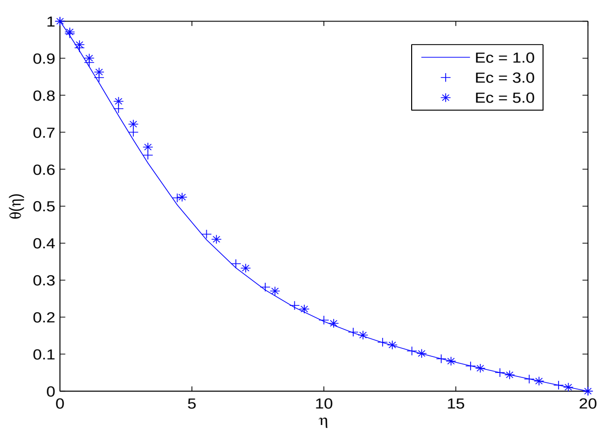

Fig. 8. Temperature Profiles for various Ec

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NOMENCLATURE

- A-Pre-exponent factor
- c-Stretching sheet parameter
- c_p-specific heat capacity at constant pressure
- C_f-Skin friction coefficient
- E - Activation energy
- Ec-Local Eckert number (or viscous dissipation parameter)
- f -Dimensionless velocity
- g -Acceleration due to earth gravity
- Gr- Local Grashof number
- k - Thermal conductivity
- M - Magnetic parameter
- Nu - Nusselt number
- Pr - Prandtl number
- Q - Heat release
- q_w - Local heat flux
- R - Universal gas constant
- T - Fluid temperature
- T_w -Temperature at the wall
- u,v - Velocity components along x-and y-axes respectively
- x,y - Cartesian coordinates along x-and y-axes respectively

Greek Letters

- β - Coefficient of volumetric expansion
- β_o - Magnetic field intensity
- ϵ - Activation energy parameter
- δ - Frank Kamenetskii parameter
- η - Similarity variable
- ψ - Stream function
- θ - Dimensionless temperature
- ρ - Density of fluid
- σ - Electrical conductivity
- ν - Kinematic viscosity
- μ - dynamic viscosity
- τ_w - Shear stress
- T_∞ - Free stream temperature

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DEPARTMENT OF MATHEMATICAL SCIENCES, FEDERAL UNIVERSITY OF TECHNOLOGY, AKURE, NIGERIA

E-mail address: ajomowaye@futa.edu.ng

DEPARTMENT OF MATHEMATICAL SCIENCES, FEDERAL UNIVERSITY OF TECHNOLOGY, AKURE, NIGERIA

E-mail address: okkoriko@futa.edu.ng