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EXISTENCE AND UNIQUENESS THEOREMS FOR SOME COMMON FIXED POINTS IN HAUSDORFF UNIFORM SPACES

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ABSTRACT. In this paper, we prove some existence and uniqueness theorems for some common fixed point theorems in Hausdorff uniform spaces for selfmappings using the concepts of $A-$distance and $E-$distance. A class of $\psi-$contractive condition more general than those of Aamri and El Moutawakil [1], Olatinwo [13] and Bosede [6] was employed to establish our results. Our generalizations can be viewed as an improvement to some of the known results in literature.

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1. INTRODUCTION

Several authors such as Berinde [2], Jachymski [10], Kada et al [11], Rhoades [14], Rus [16], Wang et al [18] and Zeidler [19] studied the theory of fixed point or common fixed point for contractive self-mappings in complete metric spaces or Banach spaces in general.
Within the last two decades, Kang [12], Rodríguez-Montes and Charris [15] established some results on fixed and coincidence points of maps by means of appropriate W-contractive or W-expansive assumptions in uniform space.
In the sequel we shall define a uniform space as follows: Let X be a nonempty set and let Φ be a nonempty family of subsets of $X\times X$. The pair (X,Φ) is called a uniform space if it satisfies the following properties:
(i) if G is in Φ, then G contains the diagonal $\{(x,x)|x\in X\}$;
(ii) if G is in Φ and H is a subset of $X\times X$ which contains G, then H is in Φ;
(iii) if G and H are in Φ, then $G\bigcap H$ is in Φ;
(iv) if G is in Φ, then there exists H in Φ, such that, whenever (x,y) and (y,z)

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are in H, then (x, z) is in H;
(v) if G is in Φ, then $\{(y, x)|(x, y) \in G\}$ is also in Φ.
Φ is called the uniform structure of X and its elements are called entourages or neighbourhoods or surroundings.
If property (v) is omitted, then (X, Φ) is called a quasiuniform space. [For examples, see Bourbaki [9] and Zeidler [19]].
In 2004, Aamri and El Moutawakil [1] proved some common fixed point theorems for some new contractive or expansive maps in uniform spaces by introducing the notions of an A-distance and an E-distance.
Aamri and El Moutawakil [1] introduced and employed the following contractive definition: Let $f, g : X \longrightarrow X$ be selfmappings of X. Then, we have

$$p(f(x), f(y)) \leq \psi(p(g(x), g(y))), \ \forall x, y \in X \tag{1}$$

where $\psi : \Re^+ \longrightarrow \Re^+$ is a nondecreasing function satisfying
(i) for each $t \in (0, +\infty), \ 0 < \psi(t)$,
(ii) $\lim_{n \longrightarrow \infty} \psi^n(t) = 0, \forall \ t \in (0, +\infty)$.
ψ satisfies also the condition $\psi(t) < t$, for each $t > 0$.
In 2007, Olatinwo [13] established some common fixed point theorems by employing the following contractive definition: Let $f, g : X \longrightarrow X$ be selfmappings of X. There exist $L \geq 0$ and a comparison function $\psi : \Re^+ \longrightarrow \Re^+$ such that $\forall \ x, y \in X$, we have

$$p(f(x), f(y)) \leq Lp(x, g(x)) + \psi(p(g(x), g(y))), \ \forall x, y \in X \tag{2}$$

Recently, the author [6] proved some common fixed point theorems by employing the following contractive definition: Let $f, g : X \longrightarrow X$ be selfmappings of X. There exist comparison functions $\psi_1 : \Re^+ \longrightarrow \Re^+$ and $\psi_2 : \Re^+ \longrightarrow \Re^+$ with $\psi_1(0) = 0$ such that $\forall \ x, y \in X$, we have

$$p(f(x), f(y)) \leq \psi_1(p(x, g(x))) + \psi_2(p(g(x), g(y))), \ \forall x, y \in X \tag{3}$$

In this paper, we shall establish some common fixed point theorems by using a contractive condition more general than (1), (2) and (3).
We shall also employ the concepts of an A-distance, an E-distance as well as the notion of comparison function in this paper.

2. PRELIMINARY

The following definitions contained in Aamri and El Moutawakil [1] shall be required in the sequel: Let (X, Φ) be a uniform space and $(X, \tau(\Phi))$ a topological space whenever topological concepts are mentioned in the context of a uniform space (X, Φ).

Definition 1. If $H \in \Phi$ and $(x, y) \in H, (y, x) \in H$, x and y are said to be *H-close*. A sequence $\{x_n\}_{n=0}^{\infty} \subset X$ is said to be a $Cauchy$ *sequence* for Φ if for any $H \in \Phi$, there exists $N \geq 1$ such that x_n and x_m are H-close for $n, m \geq N$.

Definition 2. A function p :X×X$\longrightarrow \Re^+$ is said to be an *A-distance* if for any $H \in \Phi$, there exists $\delta > 0$ such that if $p(z, x) \leq \delta$ and $p(z, y) \leq \delta$ for some $z \in X$, then $(x, y) \in H$.

Definition 3. A function p :X×X$\longrightarrow \Re^+$ is said to be an *E-distance* if
(p_1) p is an A-distance,
(p_2) $p(x, y) \leq p(x, z) + p(z, y), \ \forall \ x, y \in X$.

Definition 4. A uniform space (X, Φ) is said to be $Hausdorff$ if and only if the intersection of all $H \in \Phi$ reduces to the diagonal $\{(x, x) | x \in X\}$, i.e. if $(x, y) \in H$ for all $H \in \Phi$ implies $x = y$. This guarantees the uniqueness of limits of sequences. $H \in \Phi$ is said to be *symmetrical* if $H = H^{-1} = \{(y, x) | (x, y) \in H\}$.

Definition 5. Let (X, Φ) be a uniform space and p be an A-distance on X.
(i) Sequence $\{x_n\}_{n=0}^{\infty}$ is p-Cauchy if given $\epsilon > 0$, there exists N such that if $m, n > N$, then $p(x_m, x_n) < \epsilon$.
(ii) X is said to be *S-complete* if for every p-Cauchy sequence $\{x_n\}_{n=0}^{\infty}$, there exists $x \in X$ with $\lim_{n \longrightarrow \infty} p(x_n, x) = 0$.
(iii) X is said to be *p-Cauchy complete* if for every p-Cauchy sequence $\{x_n\}_{n=0}^{\infty}$, there exists $x \in X$ with $\lim_{n \longrightarrow \infty} x_n = x$ with respect to $\tau(\Phi)$.
(iv) $f : X \longrightarrow X$ is said to be *p-continuous* if $\lim_{n \longrightarrow \infty} p(x_n, x) = 0$ implies that $\lim_{n \longrightarrow \infty} p(f(x_n), f(x)) = 0$.
(v) $f : X \longrightarrow X$ is $\tau(\Phi)$-*continuous* if $\lim_{n \longrightarrow \infty} x_n = x$ with respect to $\tau(\Phi)$ implies $\lim_{n \longrightarrow \infty} f(x_n) = f(x)$ with respect to $\tau(\Phi)$.
(vi) X is said to be *p-bounded* if $\delta_p = \sup\{p(x, y) | x, y \in X\} < \infty$.

Definition 6. Let (X, Φ) be a Hausdorff uniform space and p an A-distance on X. Two selfmappings f and g on X are said to be *p-compatible* if, for each sequence $\{x_n\}_{n=0}^{\infty}$ of X such that $\lim_{n \longrightarrow \infty} p(f(x_n), u) = \lim_{n \longrightarrow \infty} p(g(x_n), u) = 0$ for some $u \in X$, then we have $\lim_{n \longrightarrow \infty} p(f(g(x_n)), g(f(x_n))) = 0$.

The following definition which is also required in the sequel to establish some common fixed point results is contained in Berinde [2], Rus [16] and Rus et al [17].

Definition 7. A function $\psi : \Re^+ \longrightarrow \Re^+$ is called a *comparison function* if
(i) ψ is monotone increasing;
(ii) $\lim_{n\longrightarrow\infty} \psi^n(t) = 0,\ \forall\ t \geq 0$.

Remark 1. Every comparison function satisfies the condition $\psi(0) = 0$. Both conditions (i) and (ii) imply that $\psi(t) < t,\ \forall\ t > 0$.
Our aim in this paper is to establish some common fixed point theorems by using the following contractive condition more general than (1), (2) and (3): Let $f, g : X \longrightarrow X$ be selfmappings of X. There exist $M \geq 0$ and comparison functions $\psi_1 : \Re^+ \longrightarrow \Re^+$ and $\psi_2 : \Re^+ \longrightarrow \Re^+$ with $\psi_1(0) = 0$ such that $\forall\ x, y \in X$, we have

$$p(f(x), f(y)) \leq \Big(\psi_1(p(x, g(x))) + \psi_2(p(g(x), g(y)))\Big)X \Big(1 + Mp(x, g(x))\Big), \tag{4}$$

for all $x, y \in X$.

Remark 2. The contractive condition (4) is more general than (1), (2) and (3) in the sense that if $M = 0$ in (4), then we obtain

$$p(f(x), f(y)) \leq \psi_1(p(x, g(x))) + \psi_2(p(g(x), g(y))),\ \forall x, y \in X$$

which is the contractive condition employed by Bosede [6] in (3).
Also, if $M = 0$ and $\psi_1(u) = Lu$ in (4), for $L \geq 0$, $u \in \Re^+$, then we obtain

$$p(f(x), f(y)) \leq Lp(x, g(x)) + \psi_2(p(g(x), g(y))),\ \forall x, y \in X$$

which is the contractive condition employed by Olatinwo [13] in (2).
Moreover, if $L = 0$ in the above inequality, then we obtain (1), which was employed by Aamri and El Moutawakil [1].
Thus, our contractive condition (4) is a generalization of the contractive definitions (1), (2) and (3) of Aamri and El Moutawakil [1], Olatinwo [13] and Bosede [6] respectively.

In the sequel, we shall require the following Lemma which is contained in Kang [12], Rodríguez-Montes and Charris [15] and Aamri and El Moutawakil [1].

Lemma 1. Let (X, Φ) be a Hausdorff uniform space and p an A-distance on X. Let $\{x_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$ be arbitrary sequences in X

and $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ be sequences in $\Re^+$ converging to 0. Then, for $x, y, z \in X$, the following hold:
(a) If $p(x_n, y) \leq \alpha_n$ and $p(x_n, z) \leq \beta_n$, $\forall\ n \in N$, then $y = z$. In particular, if $p(x, y) = 0$ and $p(x, z) = 0$, then $y = z$.
(b) If $p(x_n, y_n) \leq \alpha_n$ and $p(x_n, z) \leq \beta_n$, $\forall\ n \in N$, then $\{y_n\}_{n=0}^{\infty}$ converges to z.
(c) If $p(x_n, x_m) \leq \alpha_n$, $\forall\ m > n$, then $\{x_n\}_{n=0}^{\infty}$ is a Cauchy sequence in (X, Φ).

Remark 3. A sequence in X is p-Cauchy if it satisfies the usual metric property. [For Example, See Aamri and El Moutawakil [1]].

3. THE MAIN RESULTS

The following is the existence result for the common fixed point of f and g:

Theorem 1. Let (X, Φ) be a Hausdorff uniform space and p an A-distance on X such that X is p-bounded and S-complete. For arbitrary $x_0 \in X$, define a sequence $\{x_n\}_{n=0}^{\infty}$ iteratively by
$x_n = f(x_{n-1}),\ n = 1, 2, ...$
Suppose that f and g are commuting p-continuous or $\tau(\Phi)$ - continuous selfmappings of X such that
(i) $f(X) \subseteq g(X)$,
(ii) $p(f(x_i), f(x_i)) = 0,\ \forall\ x_i \in X,\ i = 0, 1, 2, ...,$
(iii) $f, g : X \longrightarrow X$ satisfy the contractive condition (4) with $M \geq 0$.
Suppose also that $\psi_1 : \Re^+ \longrightarrow \Re^+$ and $\psi_2 : \Re^+ \longrightarrow \Re^+$ are comparison functions with $\psi_1(0) = 0$.
Then, f and g have a common fixed point.

Proof. For arbitrary $x_0 \in X$, select $x_1 \in X$ such that $f(x_0) = g(x_1)$. Similarly, for $x_1 \in X$, select $x_2 \in X$ such that $f(x_1) = g(x_2)$. Continuing this process, we select $x_n \in X$ such that $f(x_{n-1}) = g(x_n)$.
We shall show that the sequence $\{f(x_n)\}_{n=0}^{\infty}$ so generated is a p-Cauchy sequence.
Indeed, since $x_n = f(x_{n-1}),\ n = 1, 2, ...$, then by using conditions (ii) and (iii) of the Theorem, we get

$$p(f(x_n), f(x_{n+m})) \leq \Big(\psi_1(p(x_n, g(x_n))) + \psi_2(p(g(x_n), g(x_{n+m})))\Big)\Big(1 + Mp(x_n, g(x_n))\Big)$$

$= \Big(\psi_1(p(f(x_{n-1}), f(x_{n-1}))) + \psi_2(p(f(x_{n-1}), f(x_{n+m-1})))\Big)\Big(1 +$
$Mp(f(x_{n-1}), f(x_{n-1}))\Big)$
$= \Big(\psi_1(0) + \psi_2(p(f(x_{n-1}), f(x_{n+m-1})))\Big)\Big(1 + M(0)\Big)$
$= \Big(0 + \psi_2(p(f(x_{n-1}), f(x_{n+m-1})))\Big)\Big(1 + 0\Big)$
$= \psi_2\Big(p(f(x_{n-1}), f(x_{n+m-1}))\Big)$
$\leq \psi_2\Big(\psi_1(p(x_{n-1}, g(x_{n-1}))) + \psi_2(p(g(x_{n-1}), g(x_{n+m-1})))\Big)\Big(1 +$
$Mp(x_{n-1}, g(x_{n-1}))\Big)$
$= \psi_2\Big(\psi_1(p(f(x_{n-2}), f(x_{n-2}))) + \psi_2(p(f(x_{n-2}), f(x_{n+m-2})))\Big)\Big(1 +$
$Mp(f(x_{n-2}, f(x_{n-2})))\Big)$
$= \psi_2\Big(\psi_1(0) + \psi_2(p(f(x_{n-2}), f(x_{n+m-2})))\Big)\Big(1 + M(0)\Big)$
$= \psi_2\Big(0 + \psi_2(p(f(x_{n-2}), f(x_{n+m-2})))\Big)\Big(1 + 0\Big)$
$= \psi_2^2\Big(p(f(x_{n-2}), f(x_{n+m-2}))\Big)$
$\leq ... \leq \psi_2^n(p(f(x_0), f(x_m))) \leq \psi_2^n(\delta_p(X)),$
which implies that

$$p(f(x_n), f(x_{n+m})) \leq \psi_2^n(\delta_p(X), \tag{5}$$

where $p(f(x_0), f(x_m))) \leq \delta_p(X)$ and
$\delta_p(X) = \sup\{p(x, y)|x, y \in X\} < \infty.$
Using the definition of comparison function in (5) gives

$$\lim_{n\longrightarrow\infty} \psi_2^n(\delta_p(X)) = 0$$

and hence,
$p(f(x_n), f(x_{n+m})) \longrightarrow 0$ as $n \longrightarrow \infty$.
Therefore, by using Lemma 1(c), we have that $\{f(x_n)\}_{n=0}^{\infty}$ is a p-Cauchy sequence.
But X is S-complete. Hence, $\lim_{n\longrightarrow\infty} p(f(x_n), u)) = 0$, for some $u \in X$.
Since $x_n \in X$ implies that $f(x_{n-1}) = g(x_n)$, therefore, we have $\lim_{n\longrightarrow\infty} p(g(x_n), u)) = 0$.
Also, since f and g are p-continuous, then

$$\lim_{n\longrightarrow\infty} p(f(g(x_n)), f(u)) = \lim_{n\longrightarrow\infty} p(g(f(x_n)), g(u)) = 0.$$

But f and g are commuting, therefore $fg = gf$. Hence,

$$\lim_{n\longrightarrow\infty} p(f(g(x_n)), f(u)) = \lim_{n\longrightarrow\infty} p(f(g(x_n)), g(u)) = 0.$$

By applying Lemma 1(a), we have that $f(u) = g(u)$.
Since $f(u) = g(u)$ and $fg = gf$, then we have $f(f(u)) = f(g(u)) = g(f(u)) = g(g(u))$.

We need to show that $p(f(u), f(f(u))) = 0$. Suppose on the contrary that $p(f(u), f(f(u))) \neq 0$. By using the contractive definition (4) and the condition that $\psi(t) < t$, $\forall\ t > 0$ in the Remark 1, we obtain

$$p(f(u), f(f(u))) \leq \Big(\psi_1(p(u, g(u))) + \psi_2(p(g(u), g(f(u))))\Big)\Big(1 + Mp(u, g(u))\Big)$$
$$= \Big(\psi_1(p(f(u), f(u))) + \psi_2(p(f(u), f(f(u))))\Big)\Big(1 + Mp(f(u), f(u))\Big)$$
$$= \Big(\psi_1(0) + \psi_2(p(f(u), f(f(u))))\Big)\Big(1 + M(0)\Big)$$
$$= \Big(0 + \psi_2(p(f(u), f(f(u))))\Big)\Big(1 + 0\Big)$$
$$= \psi_2(p(f(u), f(f(u))))$$
$$< p(f(u), f(f(u))),$$

which is a contradiction. Hence, $p(f(u), f(f(u))) = 0$.

By using condition (ii) of the Theorem, we have $p(f(u), f(u)) = 0$. Therefore, since $p(f(u), f(f(u))) = 0$ and $p(f(u), f(u)) = 0$, by using Lemma 1(a), we get $f(f(u)) = f(u)$, which implies that $f(u)$ is a fixed point of f.
But, $f(u) = f(f(u)) = f(g(u)) = g(f(u))$, which shows that $f(u)$ is also a fixed point of g. Thus, $f(u)$ is a common fixed point of f and g.
The proof of when f and g are $\tau(\Phi)$-continuous is similar since S-completeness implies p-Cauchy completeness.
This completes the proof.

Remark 4. The existence result in Theorem 1 is a generalization of Theorem 3.1 of Aamri and El Moutawakil [1], Theorem 3.1 of Olatinwo [13] as well as Theorem 3.1 of Bosede [6].
The next two Theorems establish the uniqueness of the common fixed point of f and g.

Theorem 2. Let $(X, \Phi), f, g, \psi_1, \psi_2, \{x_n\}_{n=0}^{\infty}$ be as defined in Theorem 1 above and p an E-distance on X. Then, f and g have a unique common fixed point.

Proof. Since an E-distance function p is also an A-distance, then by Theorem 1 above, we know that f and g have a common fixed point. Suppose that there exist $u, v \in X$ such that $f(u) = g(u) = u$ and $f(v) = g(v) = v$.

We need to show that $u = v$. Suppose on the contrary that $u \neq v$, i.e. let $p(u, v) \neq 0$.
Then, by using the contractive definition (4) and the condition that $\psi(t) < t,\ \forall\ t > 0$ in the Remark 1, we obtain
$p(u, v) = p(f(u), f(v))$
$\leq \Big(\psi_1(p(u, g(u))) + \psi_2(p(g(u), g(v)))\Big)\Big(1 + Mp(u, g(u))\Big)$
$= \Big(\psi_1(p(u, u)) + \psi_2(p(u, v))\Big)\Big(1 + Mp(u, u)\Big)$
$= \Big(\psi_1(0) + \psi_2(p(u, v))\Big)\Big(1 + M(0)\Big)$
$= \Big(0 + \psi_2(p(u, v))\Big)\Big(1 + 0\Big)$
$= \psi_2(p(u, v))$
$< p(u, v)$,
which is a contradiction. Hence, we have $p(u, v) = 0$.
Similarly, we have $p(v, u) = 0$. By applying condition (p_2) of Definition 3, we obtain $p(u, u) \leq p(u, v) + p(v, u)$, and hence $p(u, u) = 0$. Since $p(u, u) = 0$ and $p(u, v) = 0$, then by using Lemma 1(a), we get $u = v$.
This completes the proof.

Remark 5. The uniqueness result in Theorem 2 is a generalization of Theorem 3.2 as well as Corollaries 3.1 and 3.2 of Aamri and El Moutawakil [1].
Also, the uniqueness result in Theorem 2 is a generalization of Theorem 3.3 of Olatinwo [13] as well as Theorem 3.3 of Bosede [6].

Theorem 3. Let $(X, \Phi), p, \psi_1, \psi_2$ and $\{x_n\}_{n=0}^{\infty}$ be as defined in Theorem 1 above. Suppose that f and g are p-compatible, p-continuous or $\tau(\Phi)$-continuous selfmappings of X satisfying conditions (i), (ii) and (iii) of Theorem 1 above. Then, f and g have a unique common fixed point.

Proof. By Theorem 1 above, we know that f and g have a common fixed point. Hence, for some $u \in X$, we have $\lim_{n \longrightarrow \infty} p(f(x_n, u)) = \lim_{n \longrightarrow \infty} p(g(x_n, u)) = 0$.
Since f and g are p-continuous, then

$$\lim_{n \longrightarrow \infty} p(f(g(x_n)), f(u)) = \lim_{n \longrightarrow \infty} p(g(f(x_n)), g(u)) = 0.$$

Also, since f and g are p-compatible, then

$$\lim_{n \longrightarrow \infty} p(f(g(x_n)), g(f(x_n))) = 0.$$

By applying condition (p_2) of Definition 3, we obtain

$$p(f(g(x_n)), g(u)) \leq p(f(g(x_n)), g(f(x_n))) + p(g(f(x_n)), g(u)).$$

Letting $n \longrightarrow \infty$ and using Lemma 1(a) yields

$$\lim_{n \longrightarrow \infty} p(f(g(x_n)), g(u)) = 0.$$

Since $\lim_{n \longrightarrow \infty} p(f(g(x_n)), f(u)) = 0$ and $\lim_{n \longrightarrow \infty} p(f(g(x_n)), g(u)) = 0$, then by Lemma 1(a), we obtain $f(u) = g(u)$.

The rest of the proof follows the same standard argument as in Theorem 1 and therefore it is omitted.
This completes the proof.

Remark 6. The uniqueness result in Theorem 3 is a generalization of Theorem 3.3 of Aamri and El Moutawakil [1]. Also, the uniqueness result in Theorem 3 is a generalization of Theorem 3.5 of Olatinwo [13] as well as Theorem 3.5 of Bosede [6].

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