Journal of the
Nigerian Mathematical Society

Vol. 41, Issue 1, pp. 27-48, 2022

INSIGHT INTO THE EFFECTS OF THERMAL RADIATION AND OHMIC HEATING ON CHEMICALLY REACTIVE MAXWELL FLUID SUBJECT TO LORENTZ FORCE AND BUOYANCY FORCE

D. J. SAMUEL[1] AND B. I. OLAJUWON

ABSTRACT. The objective of the present study is to investigate the effects of thermal radiation and Ohmic heating on magnetohydrodynamic free convective flow of a chemically reactive Maxwell fluid driven by exothermal plate. The temperature dependent fluid viscosity and temperature dependent thermal conductivity are considered. Also, Arrhenius kinetics is used to model the exothermal surface reaction. The boundary layer approach is engaged to analyze the model equations. The governing partial differential equations are transformed into system of nonlinear ordinary differential equations by similarity transformations. The Runge-Kutta Fehlberg method of order four is employed to obtain numerical solutions of the resulting dimensionless non-linear equations. The impacts of dimensionless parameters on the fluid velocity, temperature distributions, skin friction coefficients and Nusselt number are demonstrated through plots and tables. Also, the results elucidate that both the skin friction coefficient and heat transfer rate are increasing functions of Deborah number. The obtained results are compared with published literature and the results are found to be in excellent agreement.

Keywords and phrases: Maxwell fluid, Ohmic heating, natural convection, Runge-Kutta Fehlberg.
2010 Mathematical Subject Classification: A80

1. INTRODUCTION

Ohmic heating is a form of heating technology that employs electrical current to generate heat within fluid products. It provides microbiologically safe and high quality foods in food processing. The study of fluid flow problems associated with Ohmic heating and heat transfer has been widely investigated by many researchers.

Received by the editors April 24, 2021; Revised: January 09, 2022; Accepted: January 25, 2022

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[1]Corresponding author

The process of Ohmic heating is one of the numerous electromagnetic based methods such as capacitive dielectric, radiative dielectric, inductive and radiative magnetic heating. Applications existing for Ohmic heating include blanching, thawing, on-line detection of starch gelatinization, fermentation, peeling, evaporation, dehydration, fermentation and extraction. Alwis and Fryer [1] carried out a thorough review on different processes involved in the development of direct resistance heating method and their advantages. Marcotte et al. [2] investigated impact of salt and citric acid on viscous fluid in the presence of electrical conductivities and Ohmic heating. The Ohmic heating rates of fluids with same electrical conductivity and identical amounts of solid particles in the presence of variable fluid viscosity was investigated by Khalaf and Sastry [3]. It was reported that heating rate for fluid mixture with higher viscosity is faster than that of the mixture with lower viscosity fluid in the continuous flow Ohmic heater. Osalusi et al. [4] examined slip effects on MHD flow of Newtonian fluid in the presence of variable fluid properties. Study of heat transfer and boundary layer flow of optically thick radiating nanofluid past an exponentially permeable stretching sheet with Ohmic heating and chemical reaction in the presence of applied magnetic field was presented by Rao et al. [5]. It was noticed that the temperature and concentration distributions of nanofluid increase with increasing values of Joule heating parameter. Prakash et al. [6] analyzed the effects of variable magnetic field on mixed convective flow of an electrically conducting nanofluid in a porous medium past an inclined nonlinear stretching permeable surface. Tsai et al. [7] numerically examined the combined effects of Ohmic heating and heat transfer on hydromagnetic boundary layer fluid flow with variable viscosity and thermal conductivity past a continuously moving permeable sheet. Awasthi [8] employed perturbation technique to analyze the importance of Ohmic heating and thermal radiation effects on MHD convective flow embedded in a porous medium along a vertical plate in the presence of viscous dissipation. Muhammad et al. [9] presented an analysis to study the impacts of chemical reaction and viscous dissipation on hydromagnetic flow of Newtonian fluid past an exponentially stretching sheet with Joule heating effect. Adegbie et al. [10] analyzed natural convection flow over a moving porous surface under the influence of Ohmic heating and magnetic field. Ohmic heating and variable viscosity effects on MHD flow of chemically reacting fluid subject to convective boundary conditions was modeled by Samuel and Fayemi [11].

All the above authors based their investigations on Newtonian fluids in their areas of study. But in recent times, the investigations of non-Newtonian fluid flow are very popular subject among researchers [12, 13, 14, 15, 16, 17, 18]. Studies of non-Newtonian fluid flow are of great importance in the industrial and engineering processes like petroleum drilling, food processing, manufacturing of plastic sheets e.t.c. There are several proposed non-Newtonian fluid models. One of such model is the Maxwell fluid. At high Deborah number, Maxwell fluid exhibits solid-like behavior and fluid-like behavior at low Deborah numbers [19]. A considerable interest has been shown by different researchers in studying Maxwell fluid with Ohmic heating effects due to its wide range of applications. A numerical study on optically thick radiating Maxwell fluid flow past a stretching sheet under the influence of chemical reaction and Joule heating was presented by Shah et al. [20]. Hayat and Qasim [21] examined viscous dissipation and chemical reaction effects on Maxwell fluid free convective flow through a porous medium in the presence of Ohmic heating. Zhao et al. [22] have investigated the influence of Ohmic heating and viscous dissipation on unsteady two-dimensional Maxwell fluid flow over a vertical plate. Noor [23] discussed chemical reaction effect on boundary layer flow of an incompressible Maxwell fluid in a porous medium with Joule heating. Sohail et al. [24] examined the impact of thermal radiation on the flow of steady incompressible Maxwell nanofluid in the presence of heat source/sink and magnetic field. Numerical investigations of heat transfer with convective boundary condition at the wall for Maxwell hybrid nanofluid flow over a porous surface in the presence of velocity slip and Joule heating have been carried out by Aziz et al. [25]. Their findings revealed that greater concentration of nanoparticles raised the heat transfer rate. Hsiao [26] numerically examined the effects of Ohmic heating and buoyancy on stagnation point flow of Maxwell fluid with chemical reaction. Zaidi and Mohyud-Din [27] studied the influence of Joule heating on the flow of two-dimensional steady incompressible upper convected Maxwell fluid flow through a vertical slit. They have also considered the effect of convective heat transfer and obtained numerical solution of the proposed model. Chemical reaction and Ohmic heating effects on natural convective MHD flow of optically thick radiating

Maxwell fluid through porous medium with heat source was studied by Arifuzzaman et al. [28]. Shateyi [29] numerically examined the flow and heat transfer of MHD Maxwell fluid flow over a linear stretching sheet considering Ohmic heating effect. Khan et al. [30] studied the impacts of chemical reaction on boundary layer flow and heat transfer of Maxwell fluid between two stretching disks under the influence of Joule heating.

The objective of the present study is to investigate the impacts of Ohmic heating and thermal radiation on MHD flow of a chemically reactive Maxwell fluid past a catalytic surface in the presence of thermo-physical parameters. Using similarity transformations, the governing partial differential equations are transformed into system of non-linear ordinary differential equations which are solved numerically. Graphical and tabular results for various values of pertinent parameters are discussed. It is worth noting that the study provides logical answers to the following research questions:

(1) What is the increasing effects of activation energy parameter and reactant consumption parameter on the Maxwell fluid flow?
(2) How does large values of variable viscosity parameter, Deborah number and radiation parameter affect the local skin friction coefcients, heat transfer rate, and mass transfer rate? and
(3) What is the significance of thermal radiation.

2. THE MATHEMATICAL FORMULATION

Consider a two-dimensional, laminar and incompressible flow of Maxwell fluid over an exothermic surface as illustrated in Fig. 1. The flow is taken to be steady and the magnetic Reynolds number is taken to be very small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. In the present study, $u_w(x) = ax$ denotes the velocity of the stretching sheet. Also, Ohmic heating and radiation effects are put into consideration. Following Merkin and Mahmood [31], it was assumed that the exothermic surface reaction generated heat to the Maxwell fluid and it is described by the following first order non-isothermal reaction given below

$$A \to B + Heat, rate = A_0 C e^{-E/RT}, \tag{1}$$

the expressions above are known as Arrhenius kinetics, where B is the product species, C is the concentration of reactant A, R is the universal gas constant, A_0 is the rate constant, E is the activation energy and T is the temperature. The surface reaction produces heat which give rise to a convective flow in the neighborhood of the surface. The boundary layer equations for the considered problem are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left[u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right]= \\ \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)+g\beta_T(T-T_\infty)-\frac{\sigma {B_0}^2}{\rho}\left(u+\lambda_1 v\frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\sigma {B_0}^2}{\rho c_p}u^2+ \\ \frac{QC_0A_0}{\rho c_p}e^{-\frac{E}{RT}}, \tag{4}$$

along with the boundary conditions

$$u=u_w(x),\ v=0,\ -k_\infty\frac{\partial T}{\partial y}=QC_0A_0e^{-\frac{E}{RT}},\ at\ y=0, \tag{5}$$

$$u\to 0,\ T\to T_\infty,\ as\ y\to\infty. \tag{6}$$

In the above expressions, u and v represent the velocity components in x and y directions, respectively, ρ is the fluid density, λ_1 is the relaxation time parameter of the fluid, T is the fluid temperature, B_0 is the constant magnetic field, c_p is the specific heat capacity of the fluid at constant pressure, q_r is the relative heat flux, T_∞ is the ambient temperature, ν is the kinematic viscosity, β_T is the coefficient of thermal expansion, g is the acceleration due to gravity, σ is the fluid electrical conductivity, $u_w(x)$ is the surface velocity, Q is the heat of reaction, C_0 is the initial concentration of the reactant species, $K(T)$ is the variable thermal conductivity of the fluid and it is expressed as [10]

$$K(T)=k_\infty[1+\xi(T-T_\infty)], \tag{7}$$

here, $K(T)$ is the temperature dependent thermal conductivity, k_∞ is the fluid free stream thermal conductivity and ξ is a constant.
Eq. (7) can be further expressed as

$$K(T) = k_\infty(1 + \Lambda\theta). \tag{8}$$

In Eq. (8) above, $\Lambda = \frac{\xi {T_\infty}^2}{E}$ is called the thermal conductivity parameter and θ is the dimensionless temperature.
The mathematical model for temperature dependent viscosity is defined as [32]

$$\mu(T) = \mu_\infty e^{-b(T-T_\infty)} = \mu_\infty e^{-\beta\theta}, \tag{9}$$

where β denotes variable viscosity parameter.
The Rosseland approximation term for optimally thick fluid is written as [33]

$$q_r = -\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y}, \tag{10}$$

where σ^* is the Stefan-Boltzman constant and κ^* is the mean absorption coefficient. Following Koriko et al. [34], it is assumed that the temperature difference within the flow regime is sufficiently small such that T^4 can be expressed as a linear function of the free stream temperature of the flow. By invoking Taylor's series, Eq. (10) is linearized by expanding T^4 about the free stream temperature T_∞. Neglecting higher order terms, the following approximation is obtained

$$T^4 \approx 4{T_\infty}^3 T - 3{T_\infty}^4, \tag{11}$$

Eq. (10) and Eq. (11) yield

$$\frac{\partial q_r}{\partial y} = -\frac{4\sigma^*}{3\kappa^*}\frac{\partial^2 T^4}{\partial y^2} \approx -\frac{16\sigma^* {T_\infty}^3}{3\kappa^*}\frac{\partial^2 T}{\partial y^2}, \tag{12}$$

using the following dimensionless variables for the heat and flow equations

$$\begin{aligned} \psi = (\vartheta_\infty a)^{\frac{1}{2}} x f(\eta), \eta = (\frac{a}{\vartheta_\infty})^{\frac{1}{2}} y, \theta = \frac{E(T - T_\infty)}{R{T_\infty}^2}, \\ u = \frac{\partial \psi}{\partial y} = axf^{'}(\eta),\ v = \frac{\partial \psi}{\partial x} = -(\vartheta_\infty a)^{\frac{1}{2}} f(\eta), \end{aligned} \tag{13}$$

the continuity Eq. (2) is identically satisfied and the governing Eqs. (3)-(4) become

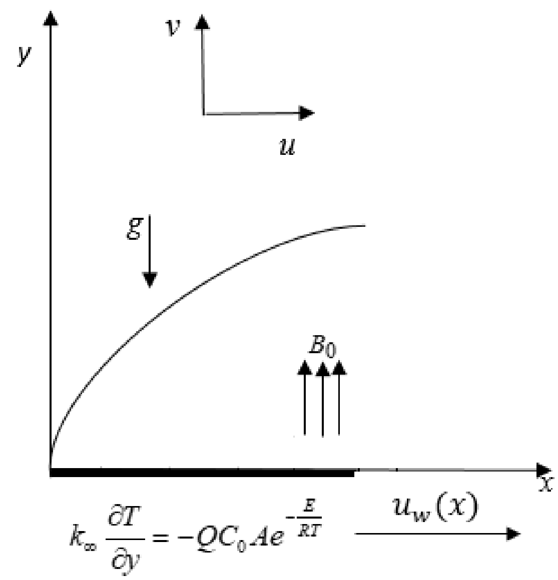

FIGURE 1. Flow geometry of the problem.

$$f''' - \beta\theta' f'' + e^{\beta\theta}\Big[\Omega\big(2ff'f'' - f^2 f'''\big) - M\big(f' - \Omega f f''\big) - (f')^2 + ff'' + G_r\theta\Big] = 0 \quad (14)$$

$$\left(1 + \Lambda\theta + \frac{4}{3}R_a\right)\theta'' + \Lambda(\theta')^2 + P_r M E_c (f')^2 + P_r\theta' f + \delta e^{\theta/1+\epsilon\theta} = 0. \quad (15)$$

The corresponding boundary conditions reduced to:

$$\eta = 0: \ f(0) = 0, \ f'(0) = 1, \ \theta'(0) = -\gamma e^{\theta/1+\epsilon\theta}, \quad (16)$$

$$\eta \to \infty: \ f^{'}(\infty) \to 0, \ \theta(\infty) \to 0, \quad (17)$$

here f' represents an ordinary derivative with respect to η, $G_r = \frac{g\beta R T_\infty{}^2}{Ea^2 x}$ is the Grashof number, $M = \frac{\sigma B_0{}^2}{\rho_\infty a}$ is the magnetic parameter, $R_a = \frac{4\sigma^* T_\infty{}^3}{\kappa^* k_\infty}$ is the radiation parameter, $\Omega = \lambda_1 a$ is the Deborah number, $P_r = \frac{\mu_\infty c_p}{\kappa_\infty}$ is the Prandtl number, $E_c = \frac{Eu_w{}^2}{Rc_p T_\infty{}^2}$ the Eckert number, $\Lambda = \frac{\xi R T_\infty{}^2}{E}$ is the thermal conductivity parameter, $\epsilon = \frac{RT_\infty}{E}$ the activation energy parameter, $\gamma = \frac{QC_0AE}{k_\infty RT_\infty{}^2}\sqrt{\frac{\vartheta_\infty}{a}}e^{-\frac{E}{RT_\infty}}$ the reactant consumption parameter, $\delta = \frac{QC_0A_0E\vartheta_\infty}{k_\infty RT_\infty{}^2 a}e^{-\frac{E}{RT_\infty}}$ the Frank-Kamenetskii parameter and $\beta = \frac{bRT_\infty{}^2}{E}$ is the variable viscosity parameter.

The important physical quantities of interest in this problem are the skin friction coefficient and the local Nusselt number, defined as follows:

$$C_f = \frac{2\tau_w}{\rho u_w^2}, \ Nu_x = \frac{Exq_w}{k_\infty RT_\infty{}^2}, \tag{18}$$

where τ_w is the wall shear stress and q_w represents the heat flux, given by:

$$\tau_w = \mu(1+\Omega)\left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{19}$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} + (q_r)_{y=0}, \tag{20}$$

using Eq. (13) in Eqs. (18)-(20) gives

$$\begin{aligned} Re_x^{\frac{1}{2}} C_f &= 2(1+\Omega)e^{-\beta\theta(0)}f''(0), \\ Re_x^{\frac{-1}{2}} Nu_x &= -(1+\Lambda\theta(0)+\frac{4}{3}R_a)\theta'(0), \end{aligned} \tag{21}$$

where $Re_x = \frac{xu_w}{\nu_\infty}$ denotes the local Reynolds number.

3. NUMERICAL COMPUTATIONS

The coupled non-linear ordinary differential Eqs. (14)-(15) subject to the boundary conditions (16)-(17) are solved in the symbolic computation software MATLAB numerically using Runge-Kutta fourth order scheme with shooting techniques. The strongly coupled boundary value problem in Eqs. (14)-(15) are reduced to a system of first order differential equations as follows:

$$f = f_1, \ f' = f_2, \ f'' = f_3, \ \theta = f_4, \ \theta' = f_5 \tag{22}$$

where,

$$
\begin{aligned}
f_1' &= f_2 \\
f_2' &= f_3 \\
f_3' &= \left(\frac{1}{1-\Omega {f_1}^2 e^{\beta f_4}}\right)\Bigg[\beta f_5 f_3 - e^{\beta f_4}\Bigg(2\Omega f_1 f_2 f_3 - \\
&\quad M\Bigg(f_2 - \Omega f_1 f_3\Bigg) - {f_2}^2 + f_1 f_3 + G_r f_4\Bigg)\Bigg] \\
f_4' &= f_5 \\
f_5' &= \left(\frac{-1}{1+\Lambda f_4 + \frac{4}{3}R_a}\right)\Bigg[\Lambda (f_5)^2 + P_r M E_c (f_2)^2 + \\
&\quad P_r f_5 f_1 + \delta e^{f_4/1+\epsilon f_4}\Bigg].
\end{aligned}
$$

In the present study, eleven parameters emerged and the effects of these parameters are investigated to compute the numerical values of the skin friction coefficient $f''(0)$ and the Nusselt number $\theta'(0)$ as shown in Table 2. Also, graphical solutions are presented in Figs. 2-14.

4. RESULT AND DISCUSSION

Our aim in this section is to describe the behavior of skin friction, Nusselt number, velocity and temperature distributions under the influence of pertinent physical parameters such as Frank-Kamenetskii parameter δ, Grashof number G_r, magnetic parameter M, radiation parameter R_a, Deborah number Ω, Prandtl number P_r, Eckert number E_c, thermal conductivity parameter Λ, activation energy parameter ϵ, reactant consumption parameter γ and variable viscosity parameter β. The values of the parameters are fixed as $G_r = 0.25$, $M = 1.0$, $R_a = 2.0$, $\Omega = 0.5$, $P_r = 0.71$, $E_c = 0.1$, $\Lambda = 1.0$, $\epsilon = 0.2$, $\gamma = 0.01$, $\delta = 0.1$, $\beta = 0.5$ unless otherwise specified. In order to verify the accuracy of our numerical computations, comparison of the present results with those obtained by Prasad et al. [18] is given in Table 1. It is clear from the values obtained that the present results is in excellent agreement the existing results. In Table 2, the numerical values of $-f^{''}(0)$ and $-\theta^{'}(0)$ are given for different values of physical parameters. Results showed that the values of $-f^{''}(0)$ decrease and $-\theta^{'}(0)$ increase as E_c increases. Also, the higher value of β, R_a, P_r and Ω enhances the skin friction coefficient $-f^{''}(0)$. In addition, it is elucidated that

TABLE 1. Comparison of the present results of $-f''(0)$ with those obtained in [18] as the magnetic parameter is varied

	Prasad et al. [18]	Present study
M	$-f''(0)$	$-f''(0)$
0.0	1.0002	1.0014
0.5	1.2248	1.2249
1.0	1.4142	1.4142
1.5	1.5811	1.5811
2.0	1.7321	1.7321

TABLE 2. Computed values of Skin Friction Coefficient $f''(0)$ and local Nusselt Number $\theta'(0)$ for Several Values of E_c, R_a, P_r, β and Ω when $G_r = 1.0,\ M = 0.2,\ \Lambda = 1.0,\ \epsilon = 0.2,\ \gamma = 0.1$ and $\delta = 0.1$.

E_c	R_a	P_r	β	Ω	$-f''(0)$	$-\theta'(0)$
0.5	0.75	0.71	0.5	0.5	1.1502	1.4569
1.0					1.1400	1.4926
2.0					1.1181	1.5667
	0.5				1.1319	1.5154
	1.0				1.1502	1.4569
	2.0				1.1685	1.3967
		0.71			1.1502	1.4569
		1.0			1.1588	1.4356
		2.0			1.1823	1.3780
			0.1		1.0606	1.4561
			0.3		1.1047	1.4565
			0.5		1.1502	1.4569
				0.2	1.0741	1.4558
				0.3	1.0995	1.4562
				0.4	1.1249	1.4565

$-\theta'(0)$ increases for increasing β and Ω but decreases for increasing R_a and P_r.

Figs. 2 present the effects of Eckert number on temperature profile. According to Fig. 2, increasing values of Eckert number significantly enhance the temperature profile. Physically, this is true owing to the fact heat energy is being generated within the fluid due to the presence of frictional heating. The impact of Prandtl number P_r on the fluid temperature is presented in Fig. 3. It is noticed that for the larger values of Prandtl number, the temperature profile decreases. Physically, larger values of P_r lowers the thermal diffusivity which results in the decrease of energy transfer ability. This finding is in agreement with the result obtained by Hsiao [26].

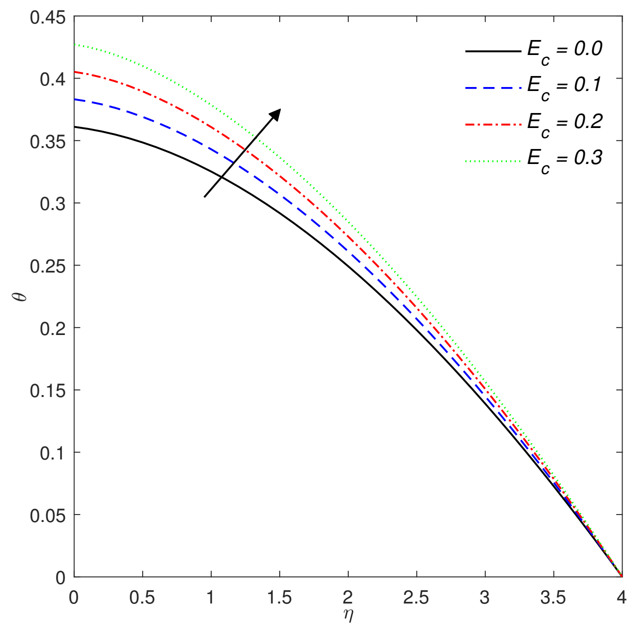

FIGURE 2. Temperature profile for various values of Eckert number E_c.

Fig. 4 describes the impact of Frank-Kamenetskii parameter δ on the fluid flow. From the graph, it is seen that increase in Frank-Kamenetskii parameter raises the fluid temperature. In Fig. 5, the influence of Grashof number G_r on the velocity profile is presented. It is observed that an increase in the Grashof number leads to increase in the velocity field. Physically, $G_r > 0$ represents heating of the fluid or cooling of the boundary surface, while $G_r < 0$ means cooling of the fluid or heating of the boundary surface and $G_r = 0$ corresponds to the absence of natural convection current. Hence, a large value of G_r indicates small viscous effects in the momentum equation and thus, leads to increase in the fluid velocity. The behavior of the temperature profile for various values of reactant consumption parameter γ is plotted in Fig. 6. It is obvious that the temperature profile increases as the values of reactant consumption parameter increases.

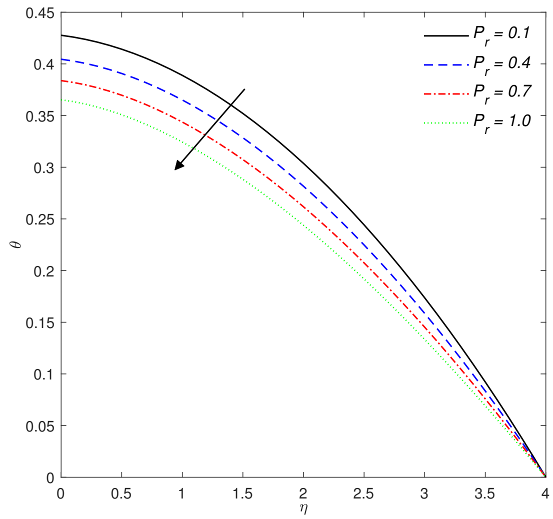

FIGURE 3. Temperature profile for various values of Prandtl number P_r.

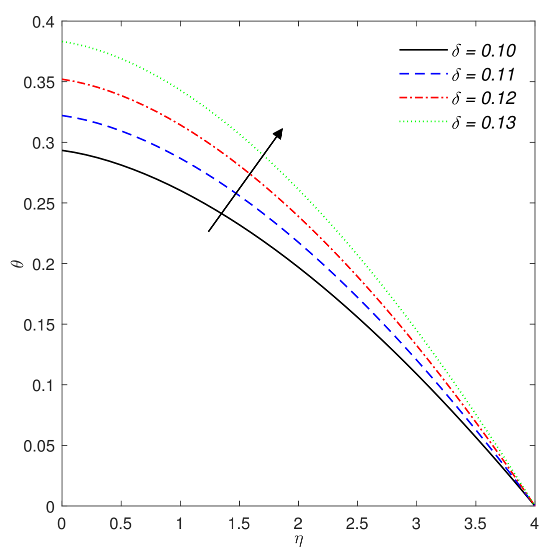

FIGURE 4. Temperature profile for various values of Frank-Kamenetskii parameter δ.

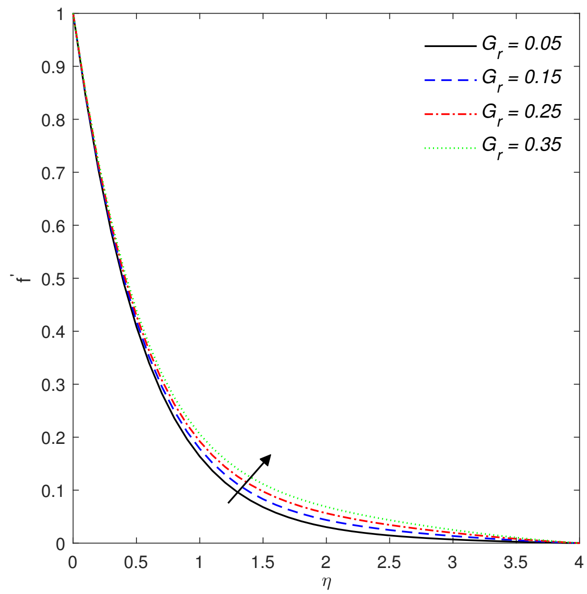

FIGURE 5. Velocity profile for various values of Grashof number G_r.

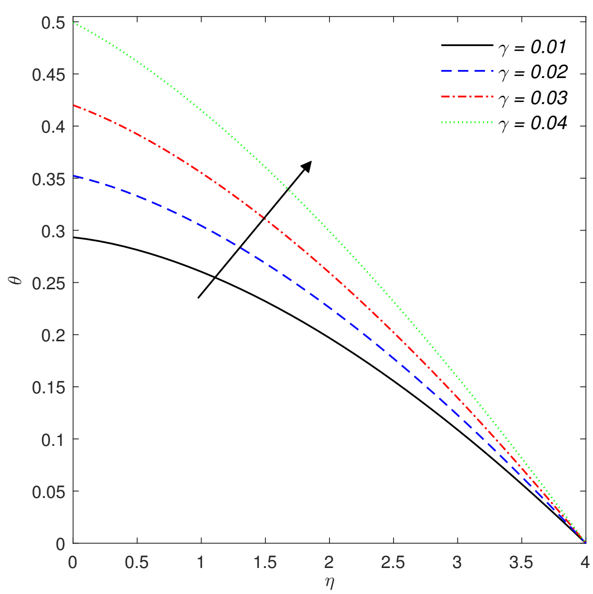

FIGURE 6. Temperature profile for various values of reactant consumption parameter γ.

The impacts of magnetic parameter M on velocity and temperature distributions are depicted in Figs. 7 and 8. It is observed in Fig. 7 that growing values of magnetic parameter decreases the velocity profiles. Physically, the magnetic parameter produces Lorentz force which slows down the flow of the fluid. Fig. 8 exhibits the impact of magnetic parameter on temperature profile. Obviously, growing values of magnetic parameter M increases the temperature profile. In Fig. 9, the influence of Deborah number Ω on the velocity profile is shown. It is noticed that the velocity of the fluid decreases with increasing values of Deborah number. This observation corroborates the report of Zaidi and Mohyud-Din [27].

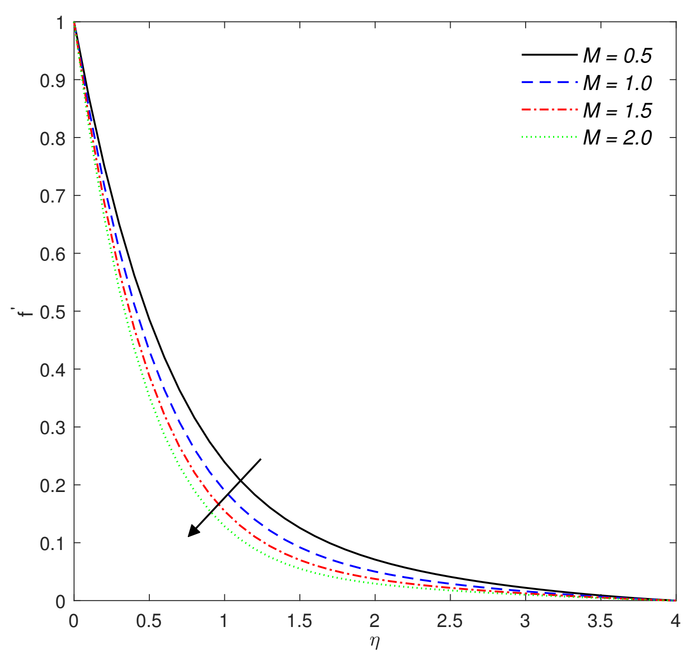

FIGURE 7. Velocity profile for various values of magnetic parameter M.

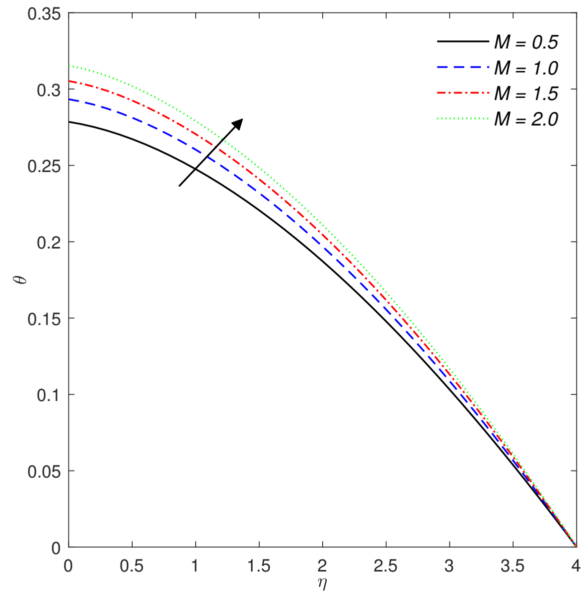

FIGURE 8. Temperature profile for various values of magnetic parameter M.

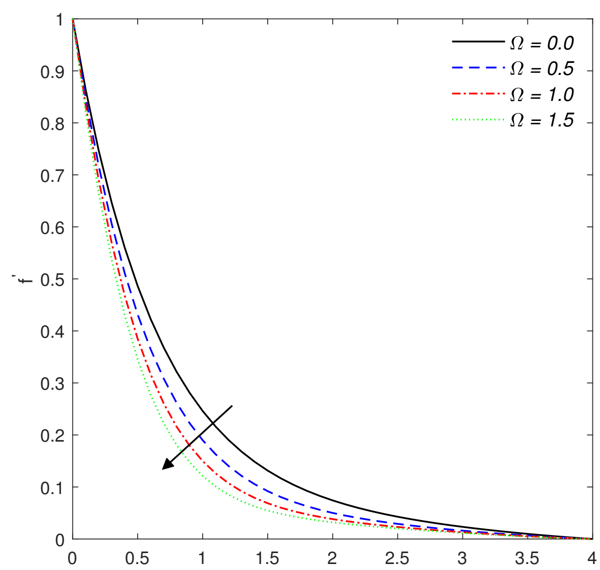

FIGURE 9. Velocity profile for various values of Deborah number Ω.

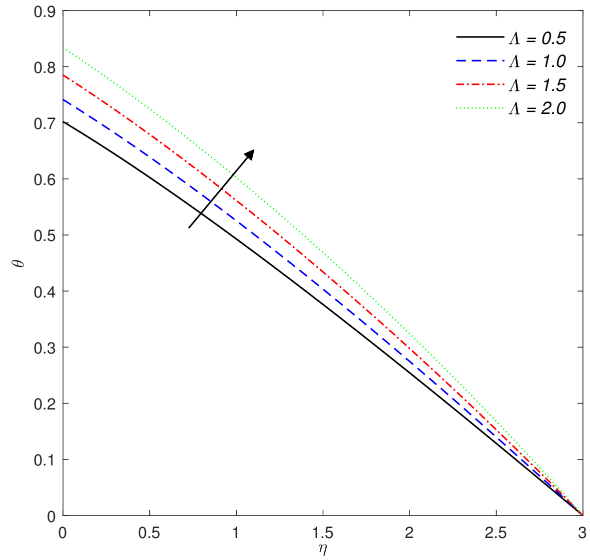

FIGURE 10. Temperature profile for various values of variable thermal conductivity parameter Λ.

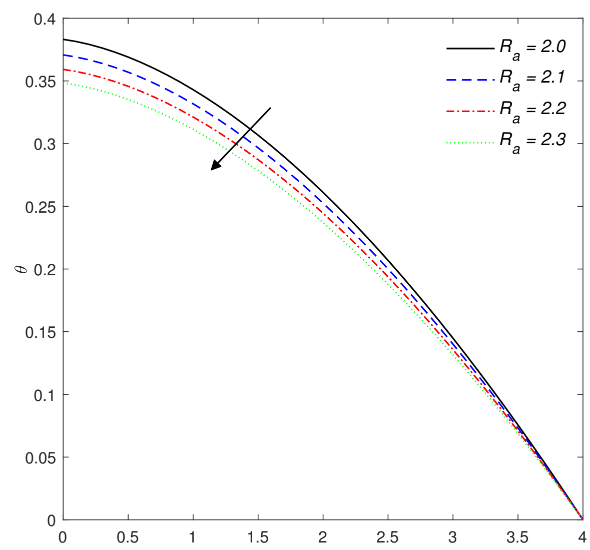

FIGURE 11. Temperature profile for various values of radiation parameter R_a.

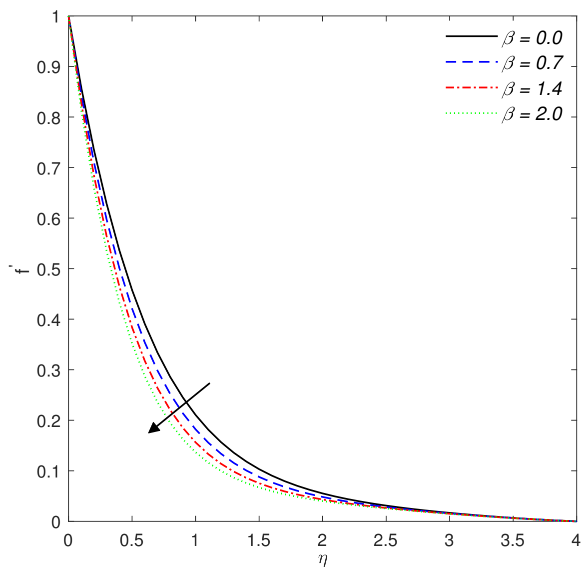

FIGURE 12. Velocity profile for various values of variable viscosity parameter β.

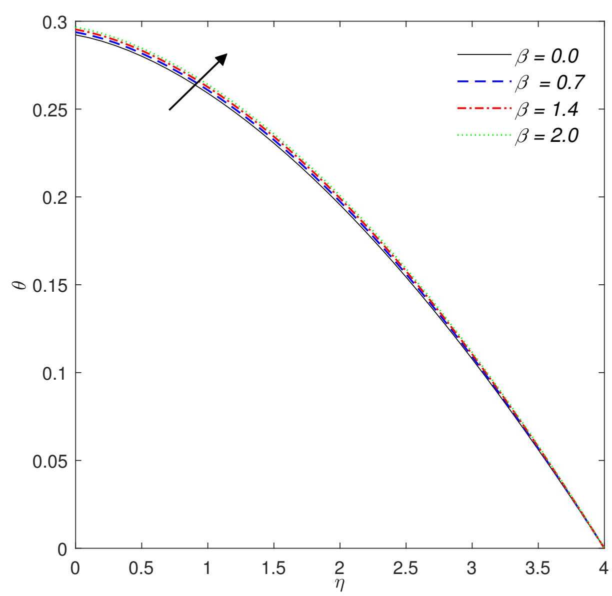

FIGURE 13. Temperature profile for various values of variable viscosity parameter β.

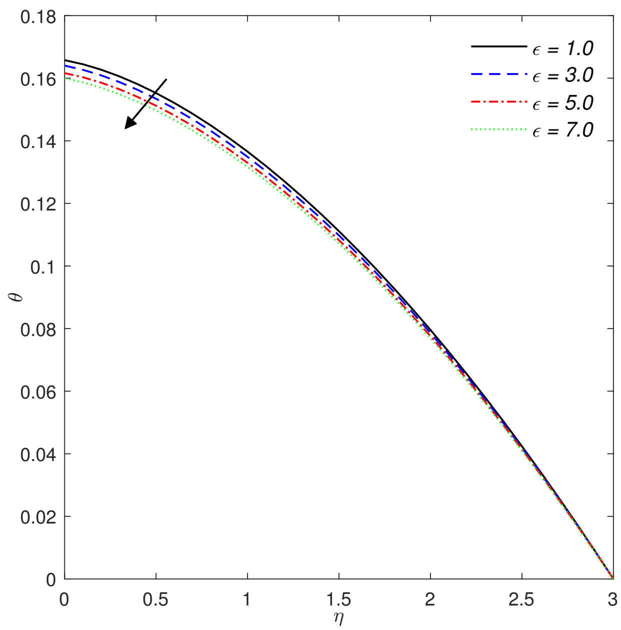

FIGURE 14. Temperature profile for various values of activation energy parameter ϵ.

Fig. 10 discusses the variation of thermal field versus variable thermal conductivity parameter when $\gamma = 0.1$. The fluid temperature enhances when variable thermal conductivity parameter is incremented. Obviously this is anticipated since thermal conductivity is a direct linear function of temperature.
Fig. 11 depicts the impact of radiation parameter R_a on temperature profile. it is observed that the enhancement of R_a reduces the fluid temperature. This is due to the emission of heat to the plate by the fluid.

The influence of variable viscosity parameter β on the flow of Maxwell fluid has been demonstrated in Fig. 12. Here, a decrease in the fluid flow is noticed due to larger values of variable viscosity parameter. Also, Fig. 13 demonstrates a decrease in temperature distribution for various values of variable viscosity parameter. Fig. 14 illustrates the behavior of temperature profile for different values of activation energy parameter ϵ. It is observed that the temperature of the fluid decreases as the activation energy parameter increases.

5. CONCLUSION

In this study, numerical solutions for steady boundary layer flow and heat transfer along a stretching sheet are examined for chemically reactive Maxwell fluid in the presence of Ohmic heating and thermal radiation. The governing non-linear problem is solved numerically via Runge-Kutta Fehlberg method. The findings of this study are as follows::

(1) Fluid velocity is a decreasing function of magnetic parameter, Deborah number and variable viscosity parameter whereas larger values of Grashof number increases the fluid velocity;
(2) the temperature profiles increase in the presence of Frank-Kamenetskii parameter, reactant consumption parameter, Eckert number, thermal conductivity parameter and magnetic parameter whereas a reverse effect is noticed in the presence of thermal radiation parameter and activation energy parameter;
(3) the Skin friction coefficient enhances with larger values of variable viscosity parameter, radiation parameter, Deborah number and Prandtl number but decreases with increase in Eckert number; and
(4) the numerical values of local Nusselt number are larger when values of physical parameters Eckert number, Deborah number and variable viscosity increases whereas reverse trend is observed with larger values of Prandtl number and radiation parameter.

ACKNOWLEDGEMENTS

The authors deeply appreciate the anonymous reviewers for their constructive suggestions which improved the quality of this article.

REFERENCES

[1] D. A. Alwis and P. J. Fryer, *The use of direct resistance heating in the food industry*, J. Food Eng. **11**(1) 3-27, 1990. doi.org/10.1016/0260-8774(90)90036-8

[2] M. Marcotte, M. Trigui and H. S. Ramaswamy, *Effect of salt and citric acid on electrical conductivities and ohmic heating of viscous liquids*, J. Food Process. Preserv. **24**(5) 389-406, 2000. doi:10.1111/j.1745-4549.2000.tb00427.x

[3] W. G. Khalaf and S. K. Sastry, *Effect of fluid viscosity on the Ohmic heating rate of solid-liquid mixtures*, J. Food Eng. **27**(2) 145-158, 1996. doi:10.1016/0260-8774(94)00090-5

[4] E. Osalusi, J. Side and R. Harris, *The effects of Ohmic heating and viscous dissipation on unsteady MHD and slip flow over a porous rotating disk with variable properties in the presence of Hall and ion-slip currents*, Int. Commun. Heat Mass Transf. **34**(9-10) 10171029, 2007. doi:10.1016/j.icheatmasstransfer.2007.05.009

[5] J. A. Rao, G. Vasumathi and J. Mounica, *Joule heating and thermal radiation effects on MHD boundary layer flow of a nanofluid over an exponentially stretching sheet in a porous medium*, World J. Mech. **5**(9) 151164, 2015. doi:10.4236/wjm.2015 .59016

[6] D. Prakash, P. Suriyakumar and Narsu Sivakumar and B. Rushi Kumar, *Influence of viscous and Ohmic heating on MHD flow of nanofluid over an inclined nonlinear stretching sheet embedded in a porous medium*, Int. J. Mech. Eng. Technol. **9**(8) 9921001, 2018.

[7] R. Tsai, K. H. Huang and J. S. Huang, *Effects of variable viscosity and thermal conductivity on heat transfer for hydromagnetic flow over a continuous moving porous plate with Ohmic heating*, Appl. Therm. Eng. **29**(10) 19211926, 2009. doi:10.1016/j.applthermaleng.2008.09.005

[8] B. Awasthi, *Joule heating effect in presence of thermal radiation on MHD convective flow past over a vertical surface in a porous medium*, Int. J. Eng. Sci. Invent. **6**(9) 3040, 2017.

[9] T. Muhammad, T. Hayat, S. A. Shehzad and A. Alsaedi, *Viscous dissipation and Joule heating effects in MHD 3D flow with heat and mass fluxes*, Results Phys. **8** 365371, 2018. doi:10.1016/j.rinp.2017.12.047

[10] K. S. Adegbie, D. J. Samuel and B. O. Ajayi, *Ohmic heating of magnetohydrodynamic viscous flow over a continuous moving plate with viscous dissipation buoyancy and thermal radiation*, Defect Diffus. Forum **392** 7391, 2019. doi:10.4028/www.scientific.net/ddf.392.73

[11] D. J. Samuel and I. A. Fayemi, *Variable viscosity and chemical reaction effects on MHD flow of radiative fluid past a stretching sheet in the presence of Joule heating.* Asian Res. J. Math. **17**(12), 28-42, 2021. doi:10.9734/arjom/2021/v17i1230346

[12] S. S. Saxena and G. K. Dubey, *MHD free convection heat and mass transfer flow of viscoelastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime*, Adv. Appl. Sci. Res. **2**(5) 115129, 2011.

[13] M. S. Abel, J. V. Tawade and M. M. Nandeppanavar, *MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet*, Meccanica **47** 385393, 2012. doi:10.1007/s11012-011-9448-7

[14] D. J. Samuel and B. O. Ajayi, *The effects of thermo-physical parameters on free convective flow of a chemically reactive power law fluid driven by exothermal plate*, Chemical and Biomolecular Engineering **3**(3) 2234, 2018. doi:10.11648/j.cbe.2018 0303.12

[15] T. Sarpkaya, *Flow of non-Newtonian fluids in a magnetic field*, Am. Inst. Chem. Eng. J. **7**(2) 324328, 1961. doi:10.1002/aic.690070231

[16] R. Ellahi and A. Riaz, *Analytical solutions for MHD flow in a third-grade fluid with variable viscosity*, Math. Comput. Model. **52**(9-10) 17831793, 2010. doi:10.1016/ j.mcm.2010.07.005

[17] O. D. Makinde, M. T. Omojola, B. Mahanthesh, F. I. Alao, K. S. Adegbie, I. L. Animasaun, A. Wakif, R. Sivaraj and M. S. Tshehla, *Significance of buoyancy, velocity index and thickness of an upper horizontal surface of a paraboloid of revolution: The case of nonnewtonian Carreau fluid*, Defect Diffus. Forum **387** 550561, 2018. doi:10.4028/www.scientific.net/DDF.387.550

[18] K. Prasad, S. Santhi and P. Datti, *Non-Newtonian power-law fluid flow and heat transfer over a non-linearly stretching surface*, Applied Mathematics **3**(5) 425-435, 2012. doi.org/10.4236/am.2012.35065

[19] A. J. Omowaye and I.L.Animasaun, *Upper-convected Maxwell fluid flow with variable thermo-physical properties over a melting surface situated in hot environment subject to thermal stratification*, Journal of Applied Fluid Mechanics **9**(4) 1777-1790, 2016. doi.org/10.18869/acadpub.jafm.68.235.24939

[20] S. Shah, S. Hussain and M. Sagheer, *MHD effects and heat transfer for the UCM fluid along with Joule heating and thermal radiation using Cattaneo-Christov heat flux model*, AIP Adv. **6** 085103, 2016. doi:10.1063/1.4960830

[21] T. Hayat and M. Qasim, *Influence of thermal radiation and Joule heating on MHD flow of a Maxwell fluid in the presence of thermophoresis*, Int. J. Heat Mass Transf. **53**(21-22) 47804788, 2010. doi:10.1016/j.ijheatmasstransfer.2010.06.014

[22] J. Zhao, L. Zheng, X. Zhang and F. Liu, *Convection heat and mass transfer of fractional MHD Maxwell fluid in a porous medium with Soret and Dufour effects*, Int. J. Heat Mass Transf. **103** 203210, 2016. doi:10.1016/j.ijheatmasstransfer.2016.07.057

[23] N. F. M. Noor, *Analysis for MHD flow of a Maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction, International Journal of Engineering*, Mathematical and Physical Sciences **5**(4) 10191023, 2012. doi:10.5281/zenodo.1332468

[24] M. Sohail, R. Naz and R. Raza, *Application of double diffusion theories to Maxwell nanofluid under the appliance of thermal radiation and gyrotactic microorganism*, Multidiscip. Model. Mater. Struct. **16**(2) 256280, 2019. doi:10.1108/MMMS-05-2019-0101

[25] A. Aziz, W. Jamshed, Y. Ali and M. Shams, *Heat transfer and entropy analysis of Maxwell hybrid nanofluid including effects of inclined magnetic field, Joule heating and thermal radiation*, Discrete Cont. Dyn. S **13**(10) 26672690, 2020. doi:10.3934/dcdss.2020142

[26] K-L. Hsiao, *Combined electrical MHD heat transfer thermal extrusion system using Maxwell fluid with radiative and viscous dissipation effects*, Appl. Therm. Eng. **112** 12811288, 2017. doi:10.1016/j.applthermaleng.2016.08.208

[27] Z. A. Zaidi and S. T. Mohyud-Din, *Effect of Joule heating and MHD in the presence of convective boundary condition for upper convected Maxwell fluid through wall jet*, J. Mol. Liq. **230** 230234, 2017. doi:10.1016/j.molliq.2016.11.092

[28] S. M. Arifuzzaman, Md. S. Khan, M. S. Islam, M. M. Islam, B. M. J. Rana, P. Biswas and S. F. Ahmmed, *MHD Maxwell fluid flow in presence of nano-particle through a vertical porous-plate with heat-generation, radiation absorption and chemical reaction*, Front. Heat Mass Transf. **9**(25) 114, 2017. doi:10.5098/hmt.9.25

[29] S. Shateyi, *A new numerical approach to MHD flow of a Maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction*, Bound. Value Probl. **2013**(196) 114, 2013. doi:10.1186/1687-2770-2013-196

[30] N. Khan, T. Mahmood, M. Sajid and M. S. Hashmi, *Heat and mass transfer on MHD mixed convection axisymmetric chemically reactive flow of Maxwell fluid driven by exothermal and isothermal stretching disks*, Int. J. Heat Mass Transf. **92** 10901105, 2016. doi:10.1016/j.ijheatmasstransfer.2015.09.001

[31] J. Merkin and T. Mahmood, *Convective flows on reactive surfaces in porous media*, Transport Porous Med. **33** 279293, 1998. doi:10.1023/A:1006541819777

[32] D. J. Samuel, *Chemical reaction and melting heat effects on MHD free convective radiative fluid flow past a continuous moving plate in the presence of thermo-physical parameters*, Defect Diffus. Forum **384** 8098, 2018. doi:10.4028/www.scientific.net/DDF.384.80

[33] M. A. Hossain, M. A. Alim, D. A. S. Rees, *The effect of radiation on free convection from a porous vertical plate*, Int. J. Heat Mass Transf. **42**(1) 181191, 1999. doi:10.1016/S0017-9310(98)00097-0

[34] O. K. Koriko, I. L. Animasaun, A. J. Omowaye and T. Oreyeni, *The combined influence of nonlinear thermal radiation and thermal stratification on the dynamics of micropolar fluid along a vertical surface*, Multi. Mod. in Mat. Struct. **15**(1) 2018. doi:10.1108/mmms-12-2017-0155

DEPARTMENT OF MATHEMATICS, FEDERAL UNIVERSITY OF AGRICULTURE, ABEOKUTA, OGUN STATE, NIGERIA
E-mail address: samueldj009@yahoo.com

DEPARTMENT OF MATHEMATICS, FEDERAL UNIVERSITY OF AGRICULTURE, ABEOKUTA, OGUN STATE, NIGERIA
E-mail address: olajuwonishola@yahoo.com