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ENTROPY GENERATION DUE TO COUETTE FLOW OF A VISCOUS INCOMPRESSIBLE FLUID BETWEEN TWO VERTICAL PARALLEL POROUS PLATES

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ABSTRACT. This work studies entropy generation and irreversibility distribution due to Couettte driven flow of a viscous incompressible fluid in a vertical channel formed by two parallel porous plates. One of the porous plates is stationary while fluid flow in the channel is induced by uniform motion of the other parallel porous plate. Isothermal heating of the moving plate and viscous dissipation cause heat transfer within the channel. The viscous dissipation is combined with natural convection, giving rise to non-linearity of the energy equation which is then coupled with the momentum equation. The coupling of the non-linear energy equation with the momentum equation makes it practically not feasible to obtain a closed form solution to the problem, the homotopy perturbation method is therefore employed to obtain approximate analytical solutions to the formulated mathematical model capturing this physical phenomena. The approximate analytical solutions obtained for velocity and temperature are used to compute entropy distribution and irreversibility distribution. Effects of the governing parameters on velocity, temperature, entropy distribution and irreversibility are presented, studied and discussed with the aid of graphs. Results obtained from the present study reveal that there is higher entropy generation near the stationary cold porous wall than the moving hot wall. Systems with low Prandtl number tend to exhibit lower entropy generation number than those with higher Prandtl number.

Keywords and phrases: Entropy generation; Porous wall; Couette; Homotopy perturbation; Irreversibility distribution.
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1. INTRODUCTION

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Entropy is the measure of molecular disorder or randomness of a system, it accounts for unavailability of energy to do work. It is well known that all forms of energy can be converted to one another, and that all forms of energy can be converted to useful work, but it is practically impossible to convert the entire available energy into work. Efforts to convert heat energy into work give rise to thermodynamics as a field, the energy that is not available to do work is of interest in thermodynamics. The second law of thermodynamics establishes the concept of entropy as a physical property of a thermodynamic system. Entropy determines whether a process is reversible or impossible despite obeying the principle of conservation of energy as stated in the first law of thermodynamics. The entropy of various parts of the system may change, but the sum total change is zero for any given reversible process. The entropy balance for a closed system shows that entropy change of the system is equal to the sum of entropy transfer with heat and entropy generation. Since entropy generation accounts for the available work of the system, a very good knowledge of the major factors influencing entropy generation is vital in optimizing system performances. Entropy generation is associated with thermodynamic irreversibility which is one of the methods used in predicting the performance of engineering processes.

A number of research findings have been carried out to investigate entropy generation between parallel plates. Among the related studies on entropy generation is Bejan [1] which studied second law analysis in heat transfer and thermal design. Another closely related work is entropy generation in the flow system generated between two parallel plates due to bi-vertical motion of the top plate which was studied by Sahin and Yilbas [2]. They concluded that increasing constant velocity and force, lower entropy generation. Ajibade et al. [3] investigated entropy generation in a steady flow of viscous incompressible fluids between two infinite parallel plates. The fluid temperature variation is due to asymmetric heating of the porous plates as well as viscous dissipation. Their investigation further reveals that suction/injection exerts a significant influence on the temperature and velocity distribution, which consequently affects the entropy generation within the channel. Bejan [4] discovered constructal thermodynamics, which he claimed unites the animate, inanimate, and engineered system. Das and Jana [5] studied effects of magnetic field and Navier slip on the entropy generation

in a flow of viscous incompressible electrically conducting fluid between two infinite horizontal parallel plates under constant pressure gradient. In a related work, Das and Jana [6] studied effects of magnetic field and suction/injection on entropy generation in a flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous plates under a constant pressure gradient. Using analytical results, Aksoy [7] studied effects of couple stresses on the heat transfer and entropy generation rates for a flow between parallel plates with constant heat flux. He found that the couple stress parameter alters the parabolic and symmetric velocity profile and that increasing couple stress effects lead to a rise in fluid temperature. He further noted that there is a decrease of entropy generation with increasing couple stress effects while increasing Brinkman number grows the entropy generation in the fluid. Torabi et al. [8] studied the interface entropy generation in micro porous channels with velocity slip and temperature jump. They noted that the total entropy generation rate may increase or decrease in accordance with the temperature jump parameter with respect to the micro porous channel outer boundary conditions.
Makinde and Gbolagade [9] investigated the second law analysis on a laminar flow of incompressible fluid through an inclined channel with isothermal boundaries. From their study, it was shown that the heat transfer irreversibility dominates along the centerline of the channel. Jain et al. [10] studied entropy generation in generalized Couette flow through porous medium with different thermal boundary conditions. They observed that total entropy generation increases throughout the channel by increasing the heat flux. Related studies involving entropy generation due to Couette flow in cylindrical annulus were also carried out by [11, 12].

Other important studies were carried out on fluid flow between parallel plates, in which a plate moves relative to a stationary plate. Among such works include Jha and Ajibade [13] which investigated free convective flow of heat generating/absorbing fluid between vertical parallel porous plates due to periodic heating of the porous plates. Jha et al. [14] presented a numerical solution for transient free convective flow of reactive viscous fluid in a vertical parallel plates channel. Their study found excellent agreement between the analytical solution of the steady state problem and numerical solution of the transient problem at large value of time. Das et al.

[15] considered the MHD flow and heat transfer in a viscous incompressible fluid between two parallel porous plates experiencing a discontinuous change in temperature. Hafeez and Ndikilar [16] studied the problem of steady laminar flow of viscous incompressible fluid between two parallel porous plates with bottom injection and top suction. Sharma [17] investigated the steady plane Couette flow of viscous incompressible fluid between two parallel porous plates through porous medium, while Mandal et al. [18] analyzed effects of thermal radiation and constant mass diffusion on the transient laminar free convective flow in a vertical channel with variable temperature and mass diffusion. One of their results show that velocity and temperature fields decrease with an increase in radiation parameter. Recently, in an analysis of entropy generation for MHD flow of viscous fluid embedded in a vertical porous channel with thermal radiation, Abbas et al,[19] discussed the effects of magnetic field, suction/injection, radiation and entropy generation for a fully developed flow in a vertical porous micro channel, it was concluded that entropy generation rate improve with higher values of the injection parameter. Many authors [20, 21, 22, 23, 24] have also investigated entropy generation in a Couette flow through porous channels for different geometrical configurations and under different thermal conditions.

Because the channel presentation in Ajibade et al. [3] is horizontal, the effect of heat input on the momentum was ignored thereby underestimating the contribution of fluid friction to total entropy generation in the system. To address this issue, the present problem is set to investigate the effect of viscous dissipation on entropy generation due to natural convection flow of viscous incompressible fluids between two vertical parallel porous plates. In this case, the external heat input as well as viscous dissipation is captured to have significant effects on the buoyancy of the fluid and hence the velocity. Combining viscous dissipation effects with natural convection eventually leads to nonlinearity of the energy equation as well as coupling of the momentum and energy equations. Therefore, it become practically not feasible to obtain a closed form solution. Hence, the Homotopy perturbation method (HPM) has been adopted to obtain an approximate solution to the problem. The solutions obtained for velocity and temperature are used to determine the entropy generation as well as the irreversibility distribution within the channel. The solutions are presented graphically and discussed for some carefully selected values of the governing parameters.

2. ENTROPY GENERATION RATE AND IRREVERSIBILITY ANALYSIS

According to Bejan [25], the volumetric rate of entropy generation rate for the flow of a Newtonian incompressible fluid under the effect of Fourier law of heat conduction, is given in Cartesian coordinates as

$$E_G = \frac{k}{T_0^2}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{\mu}{T_0}\left(2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]^2\right) \tag{1}$$

In this form of entropy generation, irreversibility is due to two effects, which are; conduction (k) and viscosity (μ). The presence of temperature and velocity gradients in a medium implies entropy generation rate is finite and positive. Assuming the flow to be hydro-dynamically fully developed $\left(\frac{\partial u}{\partial x} = 0\right)$ and thermally developing $\left(\frac{\partial T}{\partial x} \neq 0\right)$ or thermally fully developed $\left(\frac{\partial T}{\partial x} = 0\right)$, (see [26, 27, 28]), then eq. (1) is reduced to the form:

$$E_G = \frac{k}{T_0^2}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{\mu}{T_0}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2}$$

The dimensionless form of E_G has been presented as entropy generation number (N_s) by Bejan [1] which is given to be equal to the ratio of the actual entropy generation rate to the characteristic entropy transfer rate $E_{G,C}$.

$$E_{G,C} = \frac{q^2}{kT_0^2} = \frac{k(\Delta T)^2}{L_2 T_0^2} \tag{3}$$

q is the heat flux, T_0 is the absolute reference temperature, ΔT is the temperature difference and L is the characteristic length depending on geometry of the channel and problem type.

The expression on the right hand side of eq. (3) is applied for isothermal boundary conditions while that at the middle of the same equation is used for isoflux boundary conditions. Eq. (2) is reduced, in dimensionless form, to:

$$N_S = \frac{1}{Pe^2}\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 + \frac{Br}{\Omega}\left(\frac{\partial u}{\partial y}\right)^2 = N_c + N_y + N_f \quad (4)$$

This is achieved by scaling the velocity u with the reference velocity u_0, the distance y with L, the distance x with $L^2u_0\alpha^{-1}$ and expressing the dimensionless temperature θ as $(T-T_0)\Delta T^{-1}$ where $Pe = L^2u_0\alpha^{-1}$ is the Peclet number, and $\Omega = \Delta T T_0^{-1}$ is the dimensionless temperature difference. The first term (N_c) stands for the entropy generation by heat transfer due to axial conduction, the middle term (N_y) represents entropy generation due to heat transfer across different fluid sections within the channel, and the last term (N_f) gives the contribution of viscous dissipation to entropy generation.

Both heat transfer and fluid friction irreversibility account for the entropy generation rate in many convective problem. Eq. 4 shows the extent of spatial distribution of entropy generation but could not indicate the relative contributions of each irreversibility to the total entropy generation. In order to identify which of the irreversibilities (between fluid friction or heat transfer) that dominate total entropy generation, Bejan [1] showed the irreversibility ratio ϕ as the ratio of entropy generation due to fluid friction N_f to entropy generation due to heat transfer $(N_c + N_y)$. Since flow is assumed to be thermally fully developed in the present work, heat transfer due to axial conduction is neglected $(N_c = 0)$ (see also Aydin and Avci [29]) so that irreversibility distribution ratio becomes.

$$\phi = \frac{N_f}{N_y} \quad (5)$$

Entropy due to heat transfer dominates irreversibility for $0 \leq \phi \leq 1$, entropy due to fluid friction dominates for $\phi > 1$ while $\phi = 1$ implies both heat transfer and fluid friction irreversibilities contribute equally to the total entropy generation. Due to the relevance of contribution from heat transfer irreversibility to entropy generation in many physical situations, Paoletti et al [30] developed an alternative irreversibility distribution parameter in terms of Bejan number (Be), which they defined as the ratio of entropy due to heat transfer $(N_c + N_y)$ to the total entropy generation (N_s). Bejan number could be Mathematically expressed as

$$Be = \frac{N_c + N_y}{N_s} = \frac{1}{1+\theta} \quad (6)$$

One can easily conclude from the expression for Bejan Number (eq. 6) that $0 \leq Be \leq 1$, with the following possibilities; $Be = 1$ implies that entropy is generated only by heat transfer irreversibility, for $Be = 0$ signifies that total entropy generation is only due to fluid friction irreversibility while $Be = \frac{1}{2}$ indicates that both heat transfer and fluid friction contribute equally to total entropy generation.

We investigate the second law analysis for a couette flow in a channel between vertical porous walls.

3. MATHEMATICAL ANALYSIS

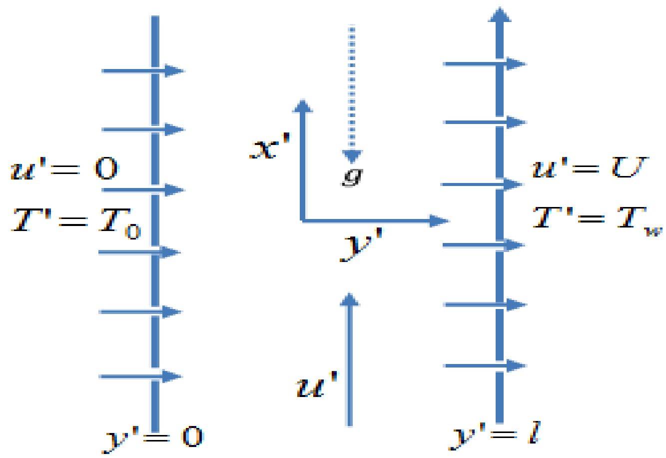

Fig. 1. Diagramatic Representation of the Flow Domain.

We consider a steady flow of viscous incompressible heat generating/absorbing fluid in a vertical channel bounded by two infinite parallel porous walls. The x' axis is taken vertically parallel to one of the channel porous walls and normal to the y' axis. The two porous walls are placed parallel to each other at l distance apart and the flow is induced by the uniform motion of the porous wall placed at $y' = l$, see Figure 1. Heat transfer in the system is due to the isothermal heating of the moving porous wall as well as viscous dissipation within the channel. Also, we assume the flow to be steady and fully developed; hence, the temperature and velocity fields are functions of y' alone.

The governing equations for the steady flow of viscous incompressible fluid between two heated parallel porous walls are the conservation of mass

$$\nabla \bullet \vec{V} = 0 \tag{7}$$

conservation of momentum

$$\left(\vec{V}\bullet\nabla\right)\vec{V}=v\nabla^2\vec{V}-\frac{1}{\rho}\nabla P+g\beta\left(T'-T_0\right) \tag{8}$$

and conservation of energy

$$\left(\vec{V}\bullet\nabla\right)T=\frac{k}{\rho C_p}\nabla^2 T+\frac{\mu}{\rho C_p}\left(\nabla u\right)^2+\frac{Q_0\left(T'-T_0\right)}{\rho C_p} \tag{9}$$

where $\nabla=\frac{\partial}{\partial x'}i+\frac{\partial}{\partial y'}j+\frac{\partial}{\partial z'}k$. Physical quantities $\rho,\vec{V},v,k$ and P have been defined in nomenclature.
Considering a two dimensional flow so that $\vec{V}=(u',v',0)$ where u' and v' are the vertical and horizontal (suction/injection) components of the velocity respectively. Also, we assume the flow is along the x'-axis which is fully developed and depends on y alone so that eq. (7) is reduces to

$$\frac{dv'}{dy'}=0 \tag{10}$$

We integrate (10) to obtain the horizontal velocity, $v'=-v_0$ (constant), which is the velocity of suction/injection.

Adopting the Boussinesqs approximation, the equations of motion for steady Couette flow of viscous incompressible heat generating fluid with viscous dissipation are given as,

$$v\frac{d^2u'}{dy'^2}+v_0\frac{du'}{dy'}+g\beta\left(T'-T_0\right)=0 \tag{11}$$

$$\frac{k}{\rho C_p}\frac{d^2T'}{dy'^2}+v_0\frac{dT'}{dy'}+\frac{\mu}{\rho C_p}\left(\frac{du'}{dy'}\right)^2+Q_0\frac{\left(T'-T_0\right)}{\rho C_p}=0 \tag{12}$$

with the boundary conditions:

$$\left.\begin{array}{l} u'(0)=0,T'(0)=T_0 \\ u'(l)=U,T'(l)=T_w \end{array}\right\} \tag{13}$$

We present eqs. (11) and (12) in dimensionless form as follows

$$\frac{d^2u}{dy^2}+S\frac{du}{dy}+Gr\theta=0 \tag{14}$$

$$\frac{d^2\theta}{dy^2}+SPr\frac{d\theta}{dy}+EcPr\left(\frac{du}{dy}\right)^2+\delta\theta=0 \tag{15}$$

With boundary conditions

$$\left.\begin{array}{l} u(0)=0,\theta(0)=0 \\ u(1)=1,\theta(1)=1 \end{array}\right\} \tag{16}$$

The non-dimensional quantities in equations (14)-(16) are;

$$y=\frac{y'}{l}, u=\frac{u'}{U}, \theta=\frac{T'-T_0}{T_w-T_0}, S=\frac{v_0 l}{v}, Pr=\frac{\mu C_p}{k}, \delta=\frac{Q_0 l^2}{k},$$

$$Gr=\frac{g\beta l^3\left(T_w-T_0\right)}{v^2}, Ec=\frac{U^2}{C_p\left(T_w-T_0\right)}$$

Pr is the Prandtl number which is inversely proportional to the thermal diffusivity of the fluid. S is the dimensionless suction/injection parameter, where positive values denote suction at the porous wall $y'=0$ with a corresponding injection on the wall $y'=l$ while negative values denote injection at the porous wall $y'=0$ with a corresponding suction on the other wall. Ec, the Eckert number is the measure of viscous dissipation in the system. Gr is the Grashof number which is the ratio of the buoyancy to the viscous force acting on the fluid. δ is the temperature dependent heat source/sink parameter, positive values represent heat source and negative values represent heat sink.

In view of the nonlinearity and coupling of the governing equations, obtaining an exact solution becomes elusive; we therefore employ the Homotopy Perturbation Method, (He [31, 32, 33, 34]) to obtain approximate analytical solutions. Solving equations (14) and (15) with the boundary conditions (16) using the Homotopy perturbation technique, He [31, 32, 33, 34] we obtain the following approximate solutions for velocity and temperature

$$u(y)=u_0(y)+u_1(y)+u_2(y)+\ldots \tag{17}$$

$$\theta(y)=\theta_0(y)+\theta_1(y)+\theta_2(y)+\ldots \tag{18}$$

where; $\theta_0(y)=y$

$\theta_1(y)=A_2y^6+A_3y^4+A_4y^3+A_5y^2+k_5y$

$\theta_2(y)=A_{12}y^{11}+A_{13}y^9+A_{14}y^8+A_{15}y^7+A_{16}y^6+A_{17}y^5+A_{18}y^4+A_{19}y^3+A_{20}y^2+k_9y$

$u_0(y)=A_1y^3+k_3y$

$u_1(y)=B_1y^8+B_2y^6+B_3y^5+B_4y^4+B_5y^3+B_6y^2+k_1y$

$u_2(y)=A_{21}y^{13}+A_{22}y^{11}+A_{23}y^{10}+A_{24}y^9+A_{25}y^8+A_{26}y^7+A_{27}y^6+A_{28}y^5+A_{29}y^4+A_{30}y^3+A_{31}y^2+k_{11}y$

Where A_i, k_i, for $i=1,2,\ldots$ are constants of integration and are given as;

$A_1=\frac{-Gr}{6}$, $k_1=1$, $A_2=\frac{3A_1^2EcPr}{10}$, $k_3=\frac{6+6Gr}{6}$, $A_3=\frac{A_1k_3EcPr}{2}$, $A_4=\frac{-\delta}{6}$, $A_5=\frac{SPr+k_3^2EcPr}{2}$ $k_5=-(A_2+A_3+A_4+A_5)$ $A_6=-\frac{GrA_2}{56}$, $A_7=-\frac{GrA_3}{30}$, $A_8=-\frac{GrA_4}{20}$, $A_9=-\frac{GrA_5+3SA_1}{12}$, $A_{10}=-\frac{Grk_5}{6}$,

$A_{11}=-\frac{Sk_3}{2}$, $A_{12}=-\left(\frac{24A_1A_{11}EcPr}{55}\right)$, $A_{13}=-\left(\frac{9A_1A_7EcPr+4A_6k_3EcPr}{18}\right)$, $A_{14}=-\left(\frac{30A_1A_8EcPr+A_2\delta}{56}\right)$, $A_{15}=-\left(\frac{A_2SPr+4A_1A_9EcPr+2A_7k_3EcPr}{7}\right)$, $A_{16}=-\left(\frac{18A_1A_{10}EcPr+10A_8k_3EcPr+A_3\delta}{30}\right)$, $A_{17}=-\left(\frac{4A_3SPr+12A_1A_{11}EcPr+8A_9k_3EcPr+A_4\delta}{20}\right)$, $A_{18}=-\left(\frac{3A_4SPr+6A_1k_7EcPr+6A_{10}k_3EcPr+A_5\delta}{12}\right)$, $A_{19}=-\left(\frac{2A_5SPr+4A_{11}k_3EcPr+k_5\delta}{6}\right)$, $A_{20}=-\left(\frac{k_5SPr+2k_{3k_7EcPr}}{2}\right)$, $k_9=-\left(A_{12}+A_{13}+A_{14}+A_{15}+A_{16}+A_{17}+A_{18}+A_{19}+A_{20}\right)$, $A_{21}=-\frac{A_{12}Gr}{156}$, $A_{22}=-\frac{A_{13}Gr}{110}$, $A_{23}=-\frac{A_{14}Gr}{90}$, $A_{24}=-\frac{A_{15}Gr+8A_6S}{72}$, $A_{25}=-\frac{A_{16}Gr}{56}$, $A_{26}=-\frac{A_{17}Gr+6A_7S}{42}$, $A_{27}=-\frac{A_{18}Gr+5A_8S}{42}$, $A_{28}=-\frac{A_{19}Gr+4A_9S}{20}$, $A_{29}=-\frac{A_{20}Gr+A_{10}S}{12}$, $A_{30}=-\frac{k_9Gr+2A_{11}S}{6}$, $A_{31}=-\frac{k_7S}{2}$, $k_{11}=-(A_{21}+A_{22}+A_{23}+A_{24}+A_{25}+A_{26}+A_{27}+A_{28}+A_{29}+A_{30}+A_{31})$

We then compute the entropy generation in the system by substituting the expressions for velocity and temperature into eq. (4).

$$N_s=\left(\frac{d\theta}{dy}\right)^2+\frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 \tag{19}$$

to obtain;

$$N_s=\tau_1+\frac{Br}{\Omega}\tau_2^2 \tag{20}$$

where, $\left(\frac{d\theta}{dy}\right)^2=\tau_1$, $\left(\frac{du}{dy}\right)^2=\tau_2^2$, $\tau_1=d_1y^{20}+d_2y^{18}+d_3y^{17}+d_4y^{16}+d_5y^{15}+d_6y^{14}+d_7y^{13}+d_8y^{12}+d_9y^{11}+d_{10}y^{10}+d_{11}y^9+d_{12}y^8+d_{13}y^7+d_{14}y^6+d_{15}y^5+d_{16}y^4+d_{17}y^3+d_{18}y^2+d_{19}y+d_{20}$,

$\tau_2^2=13A_{21}y^{12}+11A_{22}y^{10}+10A_{23}y^9+9A_{24}y^8+(8A_6+8A_{25})y^7+7A_{26}y^6+(6A_7+6A_{27})y^5+(5A_8+5A_{28})y^4+(4A_9+4A_{29})y^3+(3A_1+3A_{10}+3A_{30})y^2+(2A_{11}+2A_{31})y+(k_3+k_7+k_{11})$ $d_1=(11A_{12})^2$, $d_2=2\left(99_{12}A_{13}\right)$, $d_3=2\left(88A_{12}A_{14}\right)$, $d_4=154A_{12}A_{15}+(9A_{13})^2$, $d_5=66A_{12}(A_2+A_{16})+144A_{13}A_{14}$, $d_6=110A_{12}A_{17}+126A_{13}A_{14}+(8A_{14})^2$, $d_7=22A_{12}(4A_3+4A_{18})+18A_{13}(6A_2+6A_{16})+112A_{14}A_{15}$, $d_8=22A_{12}(3A_4+3A_{19})+90A_{13}A_{17}+16A_{14}(6A_2+6A_{16})+(7A_{15})^2$, $d_9=22A_{12}(4A_3+4A_{18})+18A_{13}(6A_2+6A_{16})+112A_{14}A_{15}$, $d_{10}=22A_{12}(1+k_5+k_9)+18A_{13}(3A_4+3A_{19})+16A_{14}(4A_1+4A_{18})+70A_{15}A_{17}+(6A_{12}+6A_{16})^2$, $d_{11}=18A_{13}(2A_5+2A_{20})+16A_{14}(A_4+3A_{19})+14A_{15}(4A_3+4A_{18})+10A_{17}(6A_2+6A_{16})$, $d_{12}=18A_{13}(1+k_5+k_9)+16A_{14}(2A_2+2A_{20})+14A_{15}(3A_4+3A_{19})+2(6A_2+6A_{16})(4A_3+4A_{18})+(5A_{17})^2$, $d_{13}=16A_{14}(1+k_5+k_9)+14A_{15}(2A_5+2A_{20})+2(6A_2+6A_{16})(3A_4+3A_{19})+10A_{17}(4A_3+4A_{18})$, $d_{14}=14A_{15}(1+k_5+k_9)+2(6A_2+6A_{17})(2A_5+2A_{20})+10A_{17}(3A_4+3A_{19})+(4A_3+4A_{18})^2$, $d_{15}=2(6A_2+$

$6A_{16})(1+k_5+k_9)+10A_{17}(2A_5+2A_{20})+2(4A_3+4A_{18})(3A_4+3A_{19})$, $d_{16} = 10A_{17}(1+k_5+k_9)+2(4A_3+4A_{18})(2A_5+2A_{20})+(3A_4+3A_{19})^2$, $d_{17} = 2(4A_3 + 4A_{18})(1 + k_5 + k_9) + 2(3A_4 + 3A_{19})(2A_5 + 2A_{20})$, $d_{18} = 2(3A_4 + 3A_{19})(1 + k_5 + k_9) + (2A_5 + 2A_{20})^2$, $d_{19} = 2(2A_5 + 2A_{20})(1 + k_5 + k_9)$, $d_{20} = (1 + k_5 + k_9)^2$

In order to analyze the irreversibility distribution within the system, we compute the Bejan number by substituting the expressions for velocity and temperature into eq.(6) to get:

$$Be = \frac{\tau_1}{\tau_1 + \frac{Br}{\Omega}\tau_2^2} \tag{21}$$

Results of these computations are plotted on line graphs generated by MATLAB.

VALIDATION OF RESULTS

In order to validate the results obtained in the present study, we set the buoyancy term, Gr and the heat generation/absorption term, δ to zero in the present work and compare with the exact solutions obtained in [3]. The result of this comparison shows that with Gr and δ set to zero in the present study, a very good approximation of [3] is recovered as shown in table 1. The slight variation is as a result the semi-analytical method (homotopy perturbation method) used in obtaining the solution of the governing equations of the present problem due to its non linearity, which was truncated at the second order of the homotopy parameter (P^2). A better approximation is expected with a higher order of the homotopy parameter, P.

TABLE 1:*Numerical comparison between the present problem and* [3]

	[3]; $y = 0.5$	Present work; $Gr = \delta = 0$, $y = 0.5$
S	θ	θ
-1.0	0.437758611397176	0.437875000000000
-0.5	0.482101262450140	0.482250000000000
0.5	0.570618979285886	0.571000000000000
1.5	0.654438795486174	0.659750000000000
S	u	u
-1.0	0.377540668798145	0.377604166666667
-0.5	0.437823499114202	0.437825520833333
0.5	0.562176500885798	0.562174479166667
1.5	0.679178699175393	0.678710937500000

4. RESULTS AND DISCUSSION

The present study presented entropy generation due to flow of viscous incompressible heat generating/absorbing fluids between two vertical parallel porous walls. The considered problem is a Couette flow between two vertical porous walls with asymmetric lateral heating and viscous dissipation. The study investigates the roles of Grashof number (Gr), temperature dependent heat source/sink (δ), Prandtl number (Pr), suction/injection parameter S and the Eckert number (Ec) on entropy generation and irreversibility distribution within the channel. In this discussion, the values of Pr are chosen between the non-dimensional values of 0.04 to 0.99 to accommodate some known fluids such as mercury (0.008-0.041), water vapour (0.882-0.994), oxygen (0.729-0.759) and air (0.703-0.784) (see Lienhard and Lienhard [35]). S is chosen from -1.0 to 1.0 to account for injection and suction, δ is chosen between -2.2 and 2.2 to accommodate both heat source $\delta > 0$ and heat sink $\delta < 0$ while the values of Gr are selected between 5 and 25.

Figs. 2 and 3 show the velocity profiles for different values of Gr and δ respectively. Fig 2 reveals that increasing Gr leads to increase in the velocity of fluid within the channel. This is true because increasing Gr in the channel leads to increase in the buoyancy force which tends to overcome the restraining viscous force in the fluid and consequently cause rise in fluid velocity within the channel. Fig 3 shows that fluid velocity increases with heat source ($\delta > 0$) and decreases with heat sink ($\delta < 0$).

Effects of Gr and δ on the temperature distribution within the system are displayed in figures 4 and 5. Fig. 4 shows that temperature increases with increase in Gr. Physically speaking, the significance of the Grashof number is that it represents the ratio of the buoyancy force as a result of spatial variation in fluid density as a result of temperature differences, to the restraining force caused by viscous force on the fluid. Fig. 5 shows the temperature profiles for different values of δ. It is observed that the temperature decreases with heat sink ($+\delta$) but increases with heat source ($-\delta$).

Figures 6-10 describe the response of entropy generation number with respect to the different flow parameters. Fig. 6 shows that entropy generation number increases with increase in Gr. The effect is higher close to the porous walls but decreases towards the centerline of the channel, which is in excellent agreement with [23]. This effect is stronger near the stationary/cold wall ($y = 0$) as compared to the heated/moving wall, ($y = 1$). Entropy generation number increases with increase in heat source, $\delta > 0$; this is shown in fig. 7. This change is stronger near the stationary wall, $y = 0$ than the moving wall, $y = 1$. Fig.8 shows entropy generation number for different values of the Prandtl number. It shows that entropy generation number grows with increasing Pr. This implies that systems designed with fluids having low Prandtl number performs better than those with high Prandtl number. Fig. 9 describes entropy generation number for different values of Ec. The figure shows that entropy generation increases with growing Ec near the cold wall. In addition, the influence of Ec on the total entropy generation diminishes near the center line while it is higher near the cold stationary wall, $y = 0$. Effect of the group parameters $(Br\Omega^{-1})$ on entropy generation number is shown in fig.10.

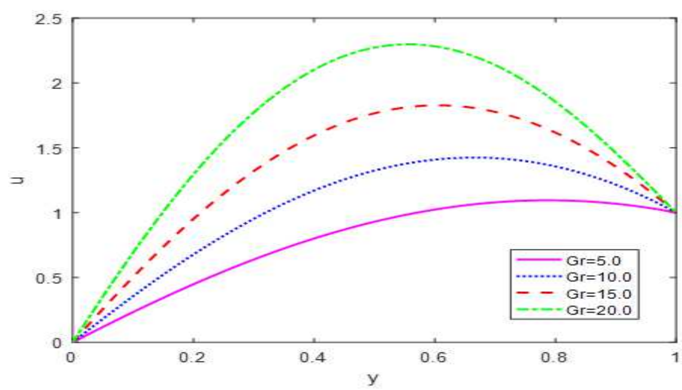

Fig. 2. Velocity for different Gr
$(Pr = 0.71, Ec = 0.3, S = 0.5, \delta = 0.2)$.

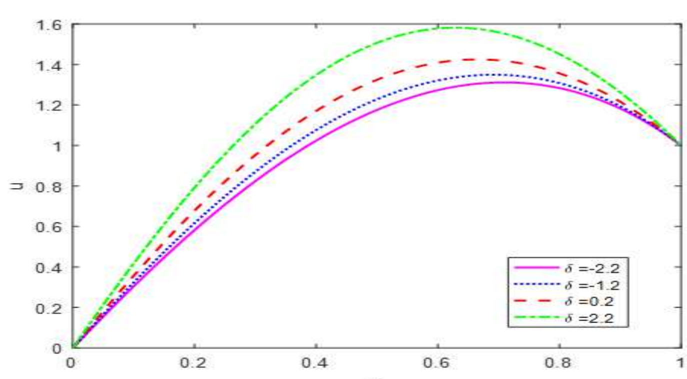

Fig. 3. Velocity for different δ
$(Pr = 0.71, Ec = 0.3, S = 0.5, Gr = 10)$.

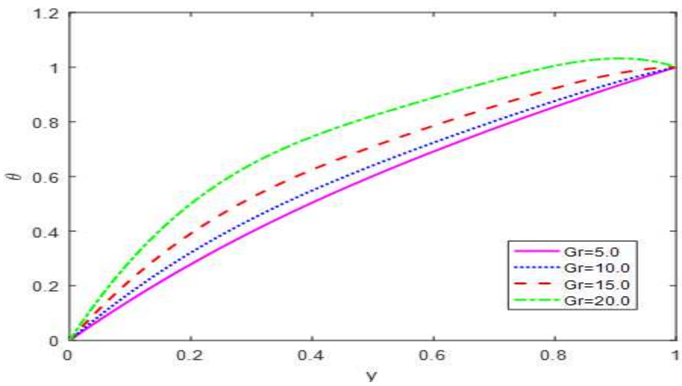

Fig. 4. Temperature for different Gr
$(Pr = 0.71, Ec = 0.3, S = 0.5, \delta = 0.2)$.

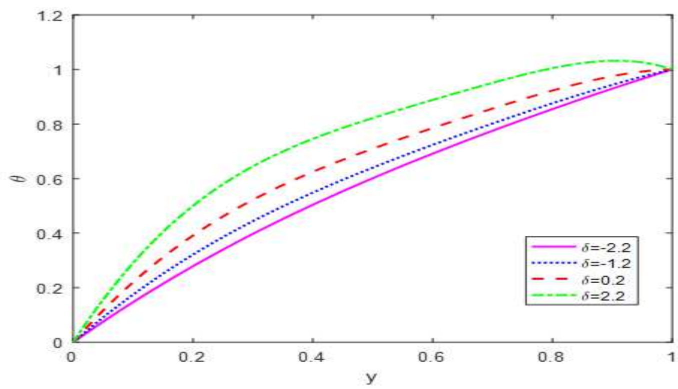

Fig. 5. Temperature for different
$\delta(Pr = 0.71, Ec = 0.3, S = 0.5, Gr = 10)$.

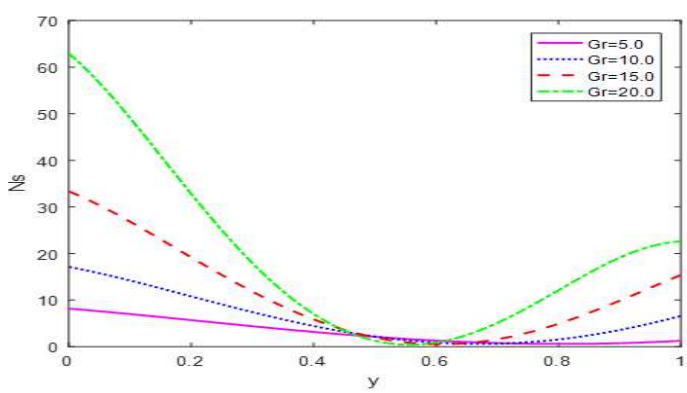

Fig. 6. Entropy Generation number for different Gr
$(Pr = 0.71, Ec = 0.3, S = 0.5, \delta = 0.2, Br\Omega^{-1} = 1)$.

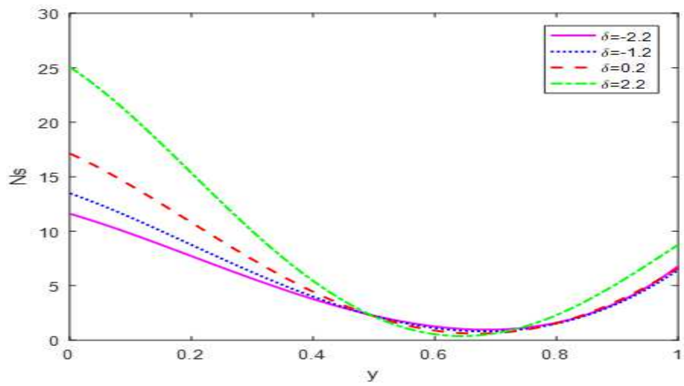

Fig. 7. Entropy Generation number for different $\delta(Pr = 0.71, Ec = 0.3, S = 0.5, Gr = 10, Br\Omega^{-1} = 1)$.

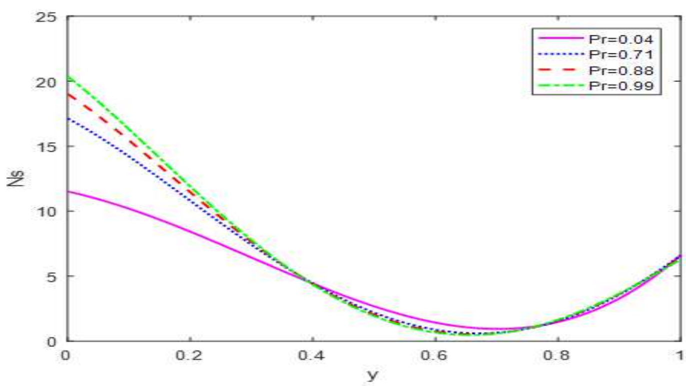

Fig. 8. Entropy Generation number for different $\mathrm{Pr}(\delta = 0.2, Ec = 0.3, S = 0.5, Gr = 10, Br\Omega^{-1} = 1)$.

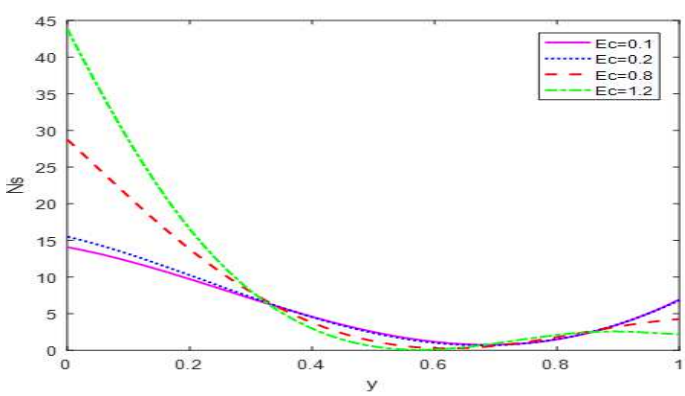

Fig. 9. Entropy Generation number for different $\mathrm{Ec}(\delta = 0.2, Pr = 0.71, S = 0.5, Gr = 10, Br\Omega^{-1} = 1)$.

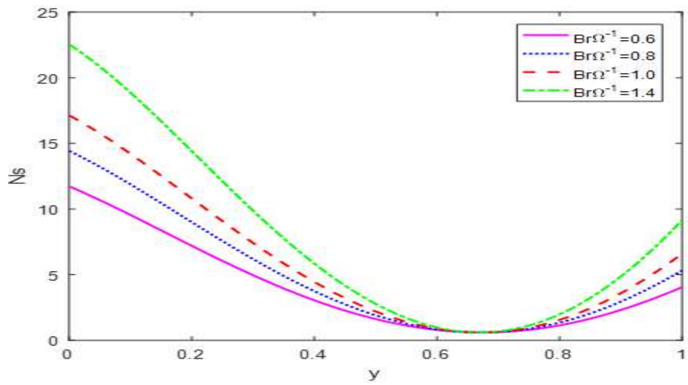

Fig. 10. Entropy Generation number for different $Br\Omega^{-1}$
$(\delta = 0.2, Ec = 0.3, S = 0.5, Gr = 10, Pr = 0.71)$.

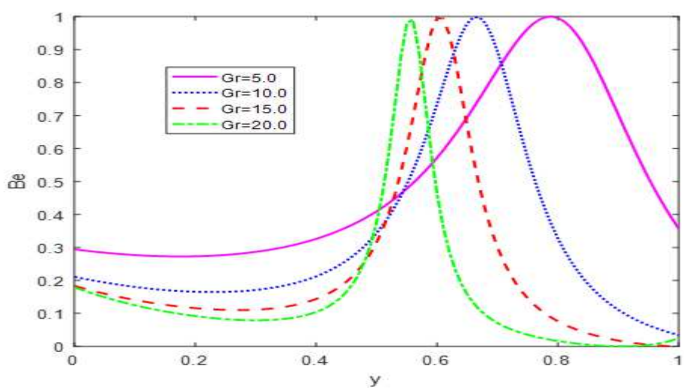

Fig. 11. Bejan number for different Gr
$(\delta = 0.2, Ec = 0.3, S = 0.5, Pr = 0.71, Br\Omega^{-1} = 1)$.

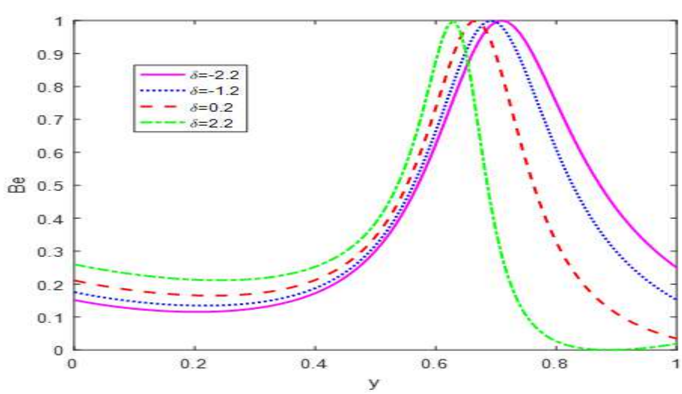

Fig. 12. Bejan number for different δ
$(Ec = 0.3, S = 0.5, Gr = 10, Pr = 0.71, Br\Omega^{-1} = 1)$.

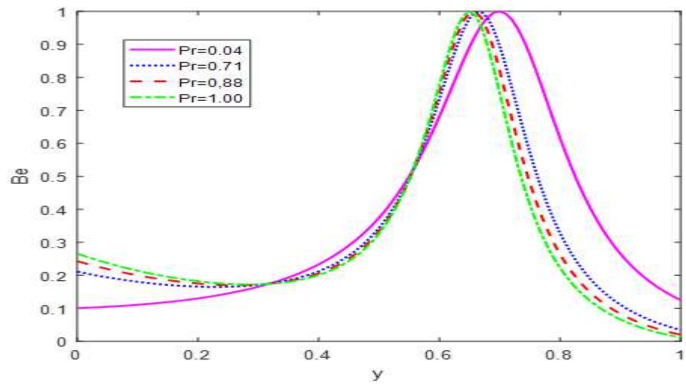

Fig. 13. Bejan number for different Pr
$(Ec = 0.3, S = 0.5, Gr = 10, \delta = 0.2, Br\Omega^{-1} = 1)$.

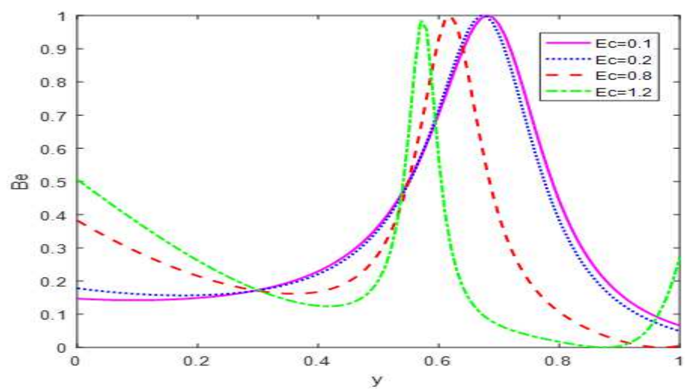

Fig. 14. Bejan number for different
Ec$(\delta = 0.2, Pr = 0.71, S = 0.5, Gr = 10, Br\Omega^{-1} = 1)$.

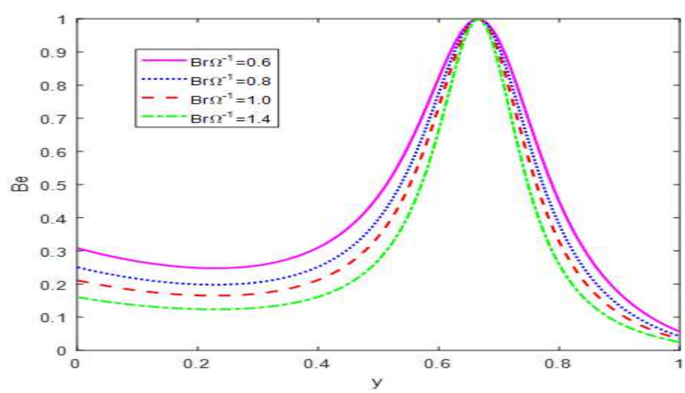

Fig. 15. Bejan number for different $Br\Omega^{-1}$
$(\delta = 0.2, Ec = 0.3, S = 0.5, Gr = 10, Pr = 0.71)$.

The figure shows that entropy generation increases with growing $(Br\Omega^{-1})$. However, there is a fluid section at which entropy generation number is insensitive to changes in magnitude of the group parameter. This fluid section where viscous dissipation is negligible for each value of the parameter is closer to the hot moving wall $(y = 1)$ than the stationary wall, $(y = 0)$. A critical look at this figure shows that the magnitude of entropy generation is higher at the interface of the cold/stationary wall than at the moving/hot wall. This is physically true because effect of viscous dissipation is lowered due to higher temperature and fluid movement near the moving/hot wall which eventually lowered the entropy generation compared to the cold/stationary wall.

Effects of the governing parameters on the irreversibility distribution ratio (Bejan number) are presented in figures 11-15. Figure 11 shows that irreversibility due to heat transfer took dominance over total entropy generation near the center of the channel while fluid friction irreversibility took the dominance near the channel porous walls. However, as (Gr) increases, fluid temperature as well as velocity increases and there is a decrease in the level of dominance of heat transfer irreversibility close to the porous walls. Increasing heat source is observed to increase contribution of heat transfer irreversibility near the cold stationary wall while it decreases that dominance near the moving heated wall. However, heat sink has influenced the entropy generation in a way that is directly opposite to that of heat source in which growing the heat sink act in support of the fluid friction irreversibility dominance of the total entropy generation as shown in figure 12. From fig. 13, it is clearly shown that fluid friction irreversibility is the major cause of entropy generation near the cold wall when the working fluid has a low Prandtl number while the trend is reversed towards the heated wall. A particular fluid section is observed in the neighborhood of $y = 0.3$ in which irreversibility distribution does not depend on fluid type. At this section, minimum entropy generation is achieved for the different fluid types considered.

Fig. 15 presents the response of Be to different values of the group parameter $(Br\Omega^{-1})$. It is shown that fluid friction irreversibility dominates near the porous walls while heat transfer irreversibility dominates towards the centerline of the channel. Increasing group parameter is observed to increase the dominance of fluid friction irreversibility near both porous walls. However, influence of the group parameter ($Br\Omega^{-1}$) diminishes as we approach that fluid section with absolute heat transfer dominance.

5. CONCLUSION

We have investigated entropy generation and irreversibility distribution in a steady fully developed Couette flow and heat transfer with viscous dissipation in a vertical channel formed by two infinite parallel porous walls. The governing momentum and energy equations were solved using the Homotopy perturbation method. Impacts of the operating parameters on the flow have been discussed with the aid of graphs. The results of the validation of the present work agree significantly with Ajibade et al. [3]. It has been discovered that heat source/sink and the Grashof number exert significant influence on the temperature, velocity, as well as entropy generation rate and irreversibility distribution within the channel. The following major conclusions have also been drawn from the present study:
i. Minimum entropy generation is obtained towards the channel's center-line.
ii. Entropy generation is higher at the surface of the cold/stationary wall than the hot/moving wall.
iii. Fluid friction irreversibility dominates near the channel walls while heat transfer irreversibility is dominant towards the centerline of the channel.
iv. Dominance of fluid friction irreversibility is higher near the hot/moving wall than the cold /stationary wall.
v. For optimal performance, engineering systems should be designed with fluids having low Prandtl number.

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NOMENCLATURE

Be	Bejan number	
Br	Brinkman number	
C_p	Specific heat at constant preasure	$[m^2 s^{-2} K^{-1}]$
Ec	Eckert number	
E_G	volumetric rate of entropy generation	
$E_{G,C}$	characteristic entropy transfer rate	
g	acceleration due to gravity	$[ms^{-2}]$

Gr	Grashof number	
l	channel width	
N_c	entropy generation by heat transfer due to axial conduction	
N_f	entropy generation by viscous dissipation	
N_s	dimensionless entropy generation number	
N_y	entropy generation due to heat transfer across fluid sections	
Pe	Peclet number	
Pr	Prandtl number	
Q_0	dimensional heat generating parameter $[Kgm^{-1}s^{-3}K^{-1}]$	
Re	Reynolds number	
S	dimensionless suction/injection parameter	
T_0	dimensional temperature at $y' = 0$	$[K]$
T'	dimensional fluid temperature	$[K]$
T_w	temperature of plate at $y' = l$	$[K]$
u'	dimensional velocity of fluid	$[ms^{-1}]$
u	dimensionless velocity of fluid	
U	dimensional velocity of the moving plate	$[ms^{-1}]$
$\vec{V}$	velocity vector having the components u', v', w' in the direction $\underline{i}, \underline{j}, \underline{k}$	$[ms^{-1}]$
v_0	velocity of suction/injection	$[ms^{-1}]$
x'	vertical axis	$[m]$
y'	co-ordinate perpendicular to the plate	$[m]$
y	dimensionless horizontal co-ordinate	
Greek Alphabets		
α	thermal diffusivity	$[m^2s^{-1}]$
β	coefficient of thermal expansion	K^{-1}
δ	dimensionless heat generating parameter	
κ	thermal conductivity $[Kgms^{-3}K^{-1}]$	
θ	dimensionless temperature of fluid	
ϕ	irreversibility distribution ratio	
ν	kinematic viscosity	$[m^2s^{-1}]$
μ	coefficient of viscosity $[Kgm^{-1}s^{-1}]$	
ρ	density of the fluid	$[Kgm^{-3}]$
Ω	dimensionless temperature difference	

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