Journal of the Nigerian Mathematical Society
Vol. 40, Issue 3, pp. 321-337, 2021
©Nigerian Mathematical Society

COMBINED EFFECT OF FLUID'S VISCOUS DISSIPATION, VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY ON THE FREE CONVECTIVE HEAT TRANSFER PAST A CIRCULAR CYLINDER

S. I. OPADIRAN[1] AND S. S. OKOYA

ABSTRACT. This research work examines the combined effect of fluid's variable thermal conductivity, viscous dissipation and variable viscosity on the free convection flow over a circular cylinder. The flow is well developed and of an incompressible fluid about a uniformly heated cylinder. The governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by employing regular perturbation technique. While, the resulting nonlinear ordinary differential equations are solved numerically using the Mid-point method with Richardson extrapolation technique. Graphs of the temperature fields and the velocity profiles, the skin friction coefficient and heat transfer reveal that the viscous dissipation effect marginally increases the velocity and temperature of the fluid, while the combined effect of the embedded fluid parameters significantly influences the velocity.

Keywords and phrases: Viscous dissipation, Temperature-dependent material property
2010 Mathematical Subject Classification: 80A25, 76A05

1. INTRODUCTION

Fluid flow and heat transfer attributes in engineering processes are usually outcomes of a collective or accumulated effects of the intrinsic fluid material properties and the dynamic system behaviors. This has attracted a great deal of researchers interest in recent years. Various studies in the literature investigate the effects of both fluid and system properties under different scenarios.

Many researchers have assumed a constant thermo-physical properties of fluid while studying the flow and heat transfer attributes around a

Received by the editors September 29, 2021; Revised: December 05, 2021; Accepted: December 14, 2021
www.nigerianmathematicalsociety.org; Journal available online at https://ojs.ictp.it/jnms/

[1]Corresponding author

cylinder undergoing natural convection heat transfer (Zokri et al. [1]; Rath and Dah [2]; Mohamed et al. [3] and Molla et al. [4]).

Numerous studies have also been conducted to investigate the effect of fluids variable thermo-physical properties. Particularly, the impact of viscosity changing with temperature has been studied by (Cheng [5]; Uddin and Kumar [6] and Ahmad et al. [7]). They revealed that fluids variable viscosity significantly affects the flow and heat transfer performance. Also, appreciable research has considered the contributions of thermal conductivity that depends on temperature (Molla et al. [8], Uddin and Kumar [9], Uddin and Kumar [6]).

Another important fluid material property is the presence of viscous dissipation, whose effect has been neglected by the above-mentioned authors. However, the effect of viscous dissipation is significant in free convective mechanisms working under huge deceleration or at high speed of rotation as well as in processes involving substantial gravitational field and in various processes of geology (Chowdhury et al. [10]). Furthermore, the applications are seen in many industrial mechanisms viz. heat-sink in radiators and cooling systems of engines. These devices operating at high velocity poses a significant effect of dissipation on the heat transfer behavior of systems. There is relatively few studies in the literature investigating the influence of dissipation on free convective flows. Prasad et al. [11] considered viscous dissipation and joule heating effect around a porous cylinder. Mohammed et. al. [3], numerically researched the impact of dissipation on free convective fluid flow around a circular cylinder with constant wall temperature. The investigations conducted by Mohammed et. al. [3], and Prasad et al. [11] examined the impact of viscous dissipation around a horizontal cylinder, but considered the fluid's viscosity and thermal conductivity to be constant. Zokri et al. [1] researched the impact of viscous dissipation on the flow Jeffrey fluid. Nabil et al. [12] considered effect of variable viscosity and viscous dissipation. This investigation considered the dissipative impact around a cylinder, employing a temperature dependent viscosity but a constant thermal conductivity.

In the preceding paragraph, the most common technique for determining the effect of parameter sensitivity is to vary one material property while others are kept constant. To establish a clearer understanding of the combined effect of viscous dissipation, temperature dependent viscosity and thermal conductivity on the free convective flow distributions as well as temperature fields alongside its relationship with the skin friction and the heat transfer coefficients, we designed this study. This is because, the observed graphical behavior of varying just one material property may change significantly, when other material properties are changing as well.

The important aim of this research work is to examine the combined effect of viscous dissipation, temperature dependent viscosity and thermal conductivity on the flow attributes as well as the heat transfer characteristics at different instances, by introducing stream function, regular perturbation technique and Mid-point method (the numerical technique implemented on Maple 17 alongside Richardson extrapolation method). The numerical results obtained were validated through comparison with three relevant works in the literature. The skin friction and the velocity distributions together with the heat transfer coefficients and the temperature profiles are examined for the effects of the embedded fluid parameters. The results of this study were presented and discussed via graphical illustrations.

2. FORMULATION OF PROBLEM'S GOVERNING EQUATIONS

In formulating the problem's governing equations, the following assumptions were considered. A free convective steady, laminar and two-dimensi onal flow of an incompressible fluid is considered. Flow is around a circular cylinder with radius R. Viscous dissipation influence is significant, while the viscosity and thermal conductivity of fluid vary with temperature. T_∞ is ambient temperature T_w is the cylinder's constant surface temperature, where $T_w > T_\infty$. The basic laws of continuity, linear momentum and energy are:

$$\nabla\cdot \vec{v} = 0,$$

$$\rho[\vec{v}\cdot \nabla]\vec{v} = -\nabla p + \nabla\cdot [\mu(T)\nabla\vec{v}] + \vec{F}.$$

$$\rho C_p[\vec{v}\cdot \nabla]T = \nabla\cdot [\kappa(T)\nabla T] + \mu(T)\Phi.$$

Where C_p is the specific heat at constant pressure, ρ is the density, $\vec{v} = (\bar{u}, \bar{v}, 0)$ is the velocity, the body force which is due to gravity is $\vec{F} = \rho\vec{g}$ (where $\vec{g}$ is the acceleration due to gravity), μ(T) is the viscosity varying with temperature, p is the pressure, ∇ is the gradient operator, T is the fluid temperature, $\Phi = \left(\frac{\partial\bar{u}}{\partial\bar{y}} + \frac{\partial\bar{v}}{\partial\bar{x}}\right)^2 + 2\left[\left(\frac{\partial\bar{u}}{\partial\bar{x}}\right)^2 + \left(\frac{\partial\bar{v}}{\partial\bar{y}}\right)^2\right]$ is the viscous dissipation function (see Schlichting and Gersten [13]) and the fluid's thermal conductivity varying with temperature is κ(T).

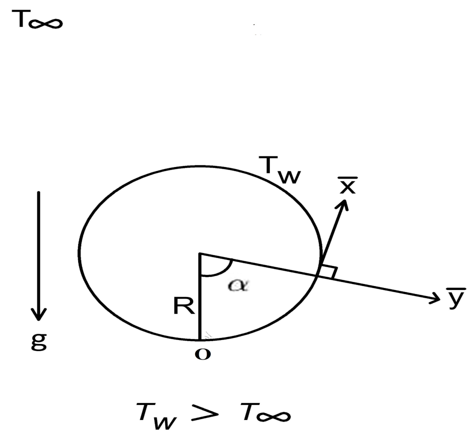


Figure 1: Cross-section and coordinate system of Problem.

Under the boundary layer and Boussinesq approximation we have the resulting continuity, momentum and energy equation with the incorporated viscous dissipation effect, as

$$\left[\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}}\right] = 0, \tag{1}$$

$$\rho\left[\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right] = \rho g \beta_0 (T - T_\infty)\sin\left(\frac{\bar{x}}{R}\right) + \frac{\partial}{\partial \bar{y}}\left[\mu(T)\frac{\partial \bar{u}}{\partial \bar{y}}\right], \tag{2}$$

$$\rho C_p\left[\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right] = \frac{\partial}{\partial \bar{y}}\left[\kappa(T)\frac{\partial T}{\partial \bar{y}}\right] + \mu(T)\left[\frac{\partial \bar{u}}{\partial \bar{y}}\right]^2, \tag{3}$$

$$\bar{u}(x,y) = 0,\ \bar{v}(x,y) = 0,\ T(x,y) = T_w \text{ at } \bar{y} = 0, \tag{4}$$

$$\bar{u}(x,y) \to 0,\ T(x,y) \to T_\infty \text{ as } \bar{y} \to \infty. \tag{5}$$

According to Akinbobola and Okoya [14]; Opadiran and Okoya [15]; Fatunmbi et al. [16]; the thermal conductivity variation and viscosity variation with temperature are given as,

$$\kappa(T) = \kappa_\infty\left(1 + \epsilon\left[\frac{T - T_\infty}{T_w - T_\infty}\right]\right) \tag{6}$$

and

$$\mu(T) = \frac{\mu_0}{1 + \alpha_0(T - T_\infty)}. \tag{7}$$

Where the tangential and normal coordinate axes to the cylinder's surface are $\bar{x}$ and $\bar{y}$, while the parameter for variable thermal conductivity is ϵ, μ_0 and and κ_∞ are the mainstream viscosity and thermal conductivity, respectively and α_0 is a constant reference viscosity.

The following dimensionless variables were considered

$$x = \frac{\bar{x}}{R},\ \bar{u} = \frac{uGr^{1/2}\nu}{R},\ y = \frac{\bar{y}}{RGr^{-1/4}},\ \bar{v} = \frac{vGr^{1/4}\nu}{R},\ Ec = \frac{\nu^2 Gr}{R^2 C_p(T_w - T_\infty)},$$

$$Gr = \frac{g\beta(T_w - T_\infty)R^3}{\nu^2},\ \theta = \left[\frac{T - T_\infty}{T_w - T_\infty}\right], \tag{8}$$

and plugging these into Equations (1) - (3) and Equations (6) - (7), we have

$$\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] = 0, \tag{9}$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{1}{[1+\omega\theta]}\left[\frac{\partial^2 u}{\partial y^2}\right] - \frac{\omega}{[1+\omega\theta]^2}\left[\frac{\partial \theta}{\partial y}\right]\left[\frac{\partial u}{\partial y}\right] + \theta\sin x, \tag{10}$$

$$\left[u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right] = \frac{1}{P_r}(1+\epsilon\theta)\left(\frac{\partial^2 \theta}{\partial y^2}\right) + \frac{1}{P_r}\epsilon\left(\frac{\partial \theta}{\partial y}\right)^2 + \frac{Ec}{1+\omega\theta}\left(\frac{\partial u}{\partial y}\right)^2, \tag{11}$$

where Gr is the Grashof number,Prandtl number is $P_r(=\rho c_p \nu/\kappa)$, Ec is the Eckert number and the viscosity variation parameter $\omega = \alpha_0(T_w - T_\infty)$.

$$u(x,y) = 0,\ v(x,y) = 0,\ \theta(x,y) = 0 \text{ at } y = 0, \tag{12}$$

$$u(x,y) \to 0,\ \theta(x,y) \to 1 \text{ as } y \to \infty. \tag{13}$$

The stream function $\xi(x,y)$ written as

$$u = \partial\xi/\partial y,\ v = -\partial\xi/\partial x, \tag{14}$$

satisfies the continuity equation identically. From the relations defined as

$$\xi = xf(x,y) \text{ and } \theta = \theta(x,y), \tag{15}$$

we have

$$\frac{1}{(1+\omega\theta)}\left(\frac{\partial^3 f}{\partial y^3}\right) - \frac{\omega}{(1+\omega\theta)^2}\left(\frac{\partial \theta}{\partial y}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial f}{\partial y}\right)^2 + f\left(\frac{\partial^2 f}{\partial y^2}\right) + \theta\frac{\sin x}{x}$$
$$= x\left(\frac{\partial f}{\partial y}\left[\frac{\partial^2 f}{\partial x \partial y}\right] - \frac{\partial f}{\partial x}\left[\frac{\partial^2 f}{\partial y^2}\right]\right), \tag{16}$$

$$\frac{1}{P_r}[1+\epsilon\theta]\left[\frac{\partial^2 \theta}{\partial y^2}\right] + \frac{1}{P_r}\epsilon\left[\frac{\partial \theta}{\partial y}\right]^2 + f\left[\frac{\partial \theta}{\partial y}\right] + \frac{x^2 Ec}{(1+\omega\theta)}\left[\frac{\partial^2 f}{\partial y^2}\right]^2$$
$$= x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial y}\frac{\partial f}{\partial x}\right), \tag{17}$$

$$f(x,y) = \partial f/\partial y = 0,\ \theta(x,y) = 1,\ \text{at } y = 0, \tag{18}$$

$$\partial f/\partial y \to 0,\ \theta(x,y) \to 0 \text{ as } y \to \infty. \tag{19}$$

2.1. Limiting cases of the model

Limiting cases of the problem's model are herein considered. Considerations of constant fluid properties ($\omega = \epsilon = 0$) in the absence of viscous dissipation ($Ec = 0$) was studied by Azim and Chowdhury [17] as well as Molla et al. [18]. Also, the case of constant fluid's material properties ($\omega = \epsilon = 0$) in the presence of viscous dissipation (that is, $Ec \neq 0$), has been studied by Mohammed et al. [3]. While, the case of constant thermal conductivity in the absence of Ec (i.e, $\epsilon = Ec = 0$) was considered by Azim [19] and Molla et al. [20]. Furthermore, Uddin and Kumar [6] has considered the case of variable ω and ϵ in the absence of Ec (i.e, ω and $\epsilon \neq 0$ and $Ec = 0$) as well as Uddin and Kumar [9].

3. PERTURBATION AND NUMERICAL SOLUTIONS

This section employed the regular perturbation method to transform the governing partial differential equations of the problem (i.e Equations (16) - (19)) to ordinary differential equations which are solved using the embedded numerical technique (Mid-point method) on Maple 17 platform. Here the normal distance y, that is $y \longrightarrow \infty$ is truncated at 5.5, 6.5 or 8 as the case may be. The flow and temperature functions are expressed as

$$f(x,y) = \sum_{n=0}^{\infty} x^n f_n(y),\ \theta(x,y) = \sum_{n=0}^{\infty} x^n \theta_n(y). \tag{20}$$

From Equation (20) we have the following perturbation expansions,

$$\mathcal{O}(x^0) : f_0''' + \omega f_0'''\theta_0 - \omega f_0''\theta_0' + (1 + 2\omega\theta_0 + \omega^2\theta_0^2)(\theta_0 + f_0 f_0'' - (f_0')^2) = 0, \tag{21}$$

$$(1+\omega\theta_0)\frac{1}{P_r}(\theta_0'' + \epsilon\theta_0\theta_0'' + \epsilon(\theta_0')^2) + f_0\theta_0' = 0, \tag{22}$$

$$f_0(0) = f_0'(0) = 0,\ \theta_0(0) = 1,\ f_0'(\infty) = \theta_0(\infty) = 0, \tag{23}$$

$$\begin{aligned}\mathcal{O}(x^1) : f_1''' + \omega(f_0'''\theta_1 - \theta_1' f_0'' + f_1'''\theta_0 - f_1''\theta_0') + 2\omega\theta_1(1+\omega\theta_0)(\theta_0 + f_0 f_0'' - (f_0')^2) +\\ (1 + 2\omega\theta_0 + (\omega\theta_0)^2)\,(\theta_1 + f_0 f_1'' - 3f_0' f_1') = 0,\end{aligned} \tag{24}$$

$$\begin{aligned}(1+\omega\theta_0)\left(\frac{1}{P_r}(\theta_1'' + \epsilon\theta_0''\theta_1 + \epsilon\theta_0\theta_1'' + 2\epsilon\theta_0'\theta_1') + f_0\theta_1' + 2f_1\theta_0' - f_0'\theta_1\right) +\\ \omega\theta_1\left(\frac{(1+\epsilon\theta_0)\theta_0''}{P_r} + f_0\theta_0' + \frac{\epsilon\theta_0'}{P_r}\right) = 0,\end{aligned} \tag{25}$$

$$f_1(0) = f_1'(0) = 0,\ \theta_1(0) = 0,\ f_1'(\infty) = \theta_1(\infty) = 0, \tag{26}$$

$\mathcal{O}(x^2) : ...$
$\mathcal{O}(x^3) : ...$

$\mathcal{O}(x^4)$: ...
$\mathcal{O}(x^5)$: ...
The skin-fiction together with the heat transfer (specified as the Nusselt number) are:

$$C_f = \frac{\tau_w}{\rho U_\infty^2}, \text{ and } Nu = \frac{Rq_w}{k(T_w - T_\infty)}, \tag{30}$$

where

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \text{ and } \tau_w = \mu\left[\frac{\partial u}{\partial y}\right]_{y=0}, \tag{31}$$

q_w and τ_w are the heat flux and shear stress respectively.

4. RESULTS AND DISCUSSION

In evaluating accuracy of the method used, results of the heat transfer distributions as well as the generated results of the skin friction coefficients are compared with those obtained by Merkin [21], Nazar et al. [22] and Kasim et al. [23] and are presented in Tables 1 and 2, respectively. And the results were found to strongly agree with previous results published in literature.
Figures 2 - 13 show graphs of the dimensionless velocity profiles, temperature fields, skin friction profiles as well as the heat transfer distributions ($f'(y)$, $\theta(y)$, $C_fGr^{1/4}$ and $NuGr^{-1/4}$). Parameters $\omega = \epsilon = Ec = 0$, $\omega = \epsilon = Ec$=1, $\omega = \epsilon = Ec$=3 and $P_r = 0.7$ unless where considered as varying parameters.
Figures 2 - 5 display the variations of $f'(y)$, $\theta(y)$, $C_fGr^{1/4}$ and $NuGr^{-1/4}$ with ω, ϵ and Ec, respectively. Figure 2 shows the velocity profiles for varying ω and the gradual increase in ϵ and Ec.

Table 1. Numerical results for the coefficients of heat transfer when $Ec = \omega = \epsilon = 0$ and $P_r = 1.0$.

x	**Heat transfer coefficient**			
	Merkin [21] Finite difference solution	Nazar et al. [22] Finite difference solution	Kasim et al. [23] Perturbation solution	Present Perturbation solution
0	0.4214	0.4214	0.4214	0.4220
$\pi/6$	0.4161	0.4161	0.4164	0.4169
$\pi/3$	0.4007	0.4005	0.4011	0.4013
$\pi/2$	0.3745	0.3741	0.3752	0.3749
$2\pi/3$	0.3364	0.3355	0.3375	0.3373
$5\pi/6$	0.2825	0.2811	0.2832	0.2875

Table 2. Numerical results for the coefficients of Skin friction when $Ec = \omega = \epsilon = 0$ and $P_r = 1.0$.

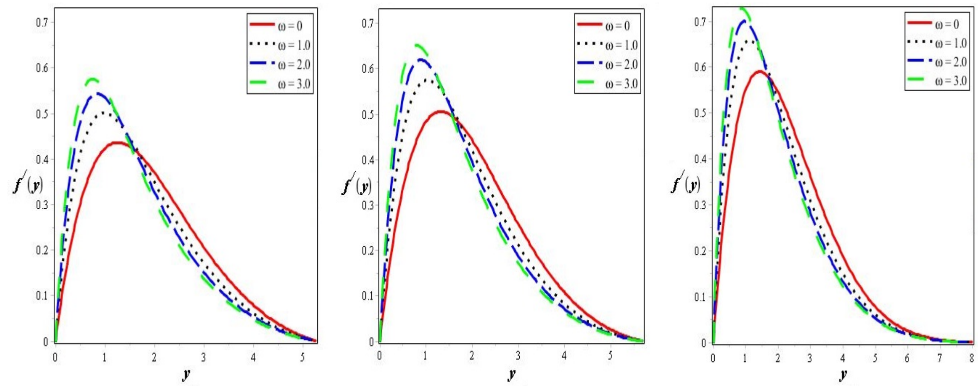
x	**Skin friction coefficient**			
	Merkin [21] Finite difference solution	Nazar et al. [22] Finite difference solution	Kasim et al. [23] Perturbation solution	Present Perturbation solution
0	0.0000	0.0000	0.0000	0.0000
$\pi/6$	0.4151	0.4148	0.4124	0.4119
$\pi/3$	0.7558	0.7542	0.7517	0.7494
$\pi/2$	0.9579	0.9545	0.9552	0.9495
$2\pi/3$	0.9756	0.9698	0.9774	0.9711
$5\pi/6$	0.7822	0.7740	0.7862	0.8066

The velocity is seen to increase to a maximum value before decreasing to zero. Furthermore, the velocity increases as ω increases with the curves in Figure 2c having the highest velocity. The combined effect of increasing ω, ϵ and Ec is also observed to heighten the velocity profiles as well as improving the asymptotic attributes of the curves as they approach infinity.

Figure 3 represents the temperature distributions for varying ω, ϵ and Ec. From Figure 3, it is seen that variation in ω marginally decreases the temperature profiles. While the curves in Figure 3c portray a better asymptotic behavior while approaching infinity. It can also be seen that, though varying ω marginally decreases the fluid temperature, however, the combined effect of increasing ϵ and Ec changes the nature of the curves.

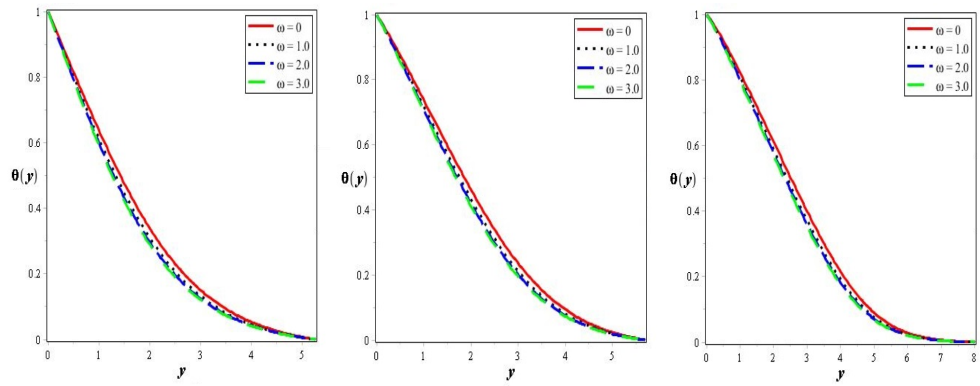
Figure 4 shows that the skin friction coefficient attains a peak value at the point where $x = 2$ and decreases uniformly in the neighborhood of $x > 2$, as seen in Figures 4a, 4b and 4c respectively. The combined effect of increasing ω, ϵ and Ec is observed to heighten the skin friction coefficients. While increase in ω decreases the skin friction.

Figure 5 demonstrates the outcome of increasing ω, ϵ and Ec for the plot of $NuGr^{-1/4}$ against x. Figure 5a has the lowest heat transfer. The outcome of increasing ω as well as the combined effect of increase in ω, ϵ and Ec are observed to increase $NuGr^{-1/4}$.



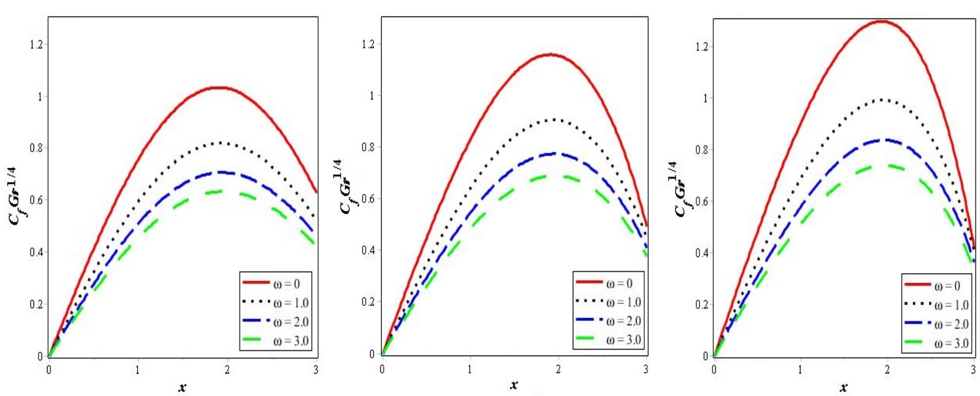
(a) (b) (c)

Figure 2: Velocity graphs when (a) $\epsilon = Ec = 0$, (b) $\epsilon = Ec = 1$ and (c) $\epsilon = Ec = 3$.



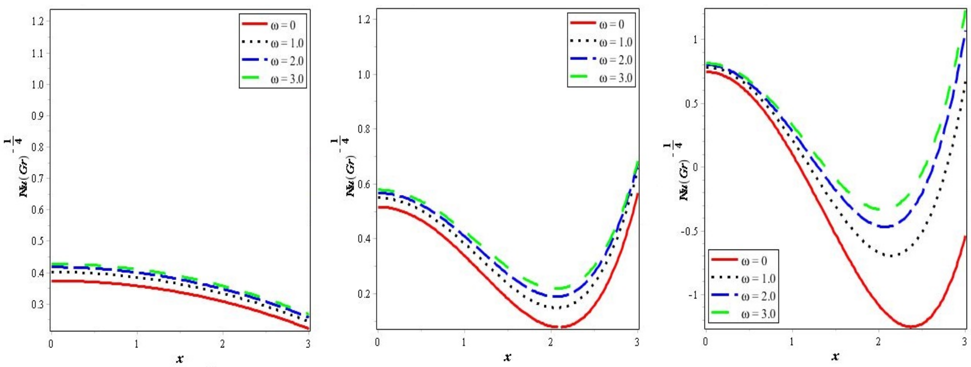
(a) (b) (c)

Figure 3: Graphs of temperature field when (a) $\epsilon = Ec = 0$, (b) $\epsilon = Ec = 1$ and (c) $\epsilon = Ec = 3$.



(a) (b) (c)

Figure 4: Plots of skin friction when (a) $\epsilon = Ec = 0$, (b) $\epsilon = Ec = 1$ and (c) $\epsilon = Ec = 3$.



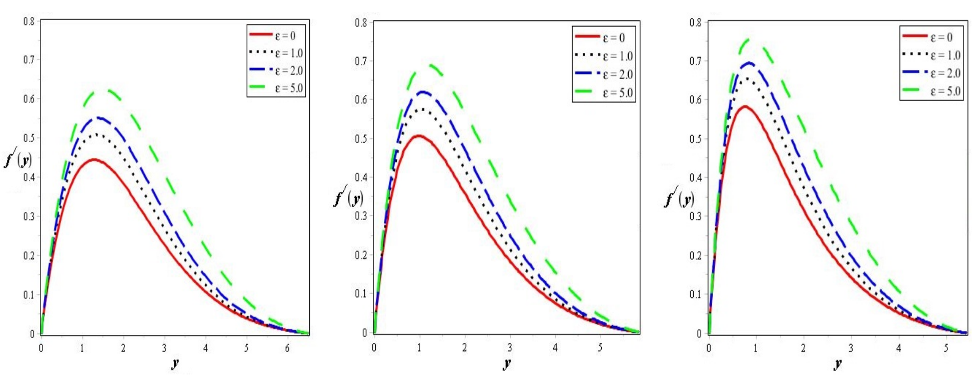
(a) (b) (c)

Figure 5: Heat transfer variations when (a) $\epsilon = Ec = 0$, (b) $\epsilon = Ec = 1$ and (c) $\epsilon = Ec = 3$.

The combined influence of increasing ω, ϵ as well as Ec are displayed in Figures 6 - 9. Figure 6 reveals that the velocity profiles rises as ϵ is increased. Also, the combined effect shown in Figures 6a, 6b and 6c depict that the velocity distributions rises as the fluid parameters increases. And the curves are bounded below by the constant thermal conductivity case. Increase in ϵ leads to increase in the temperature, as shown in Figure 7. Also, the combined effect of increasing ω and Ec is seen to slightly alter the nature of the curves.

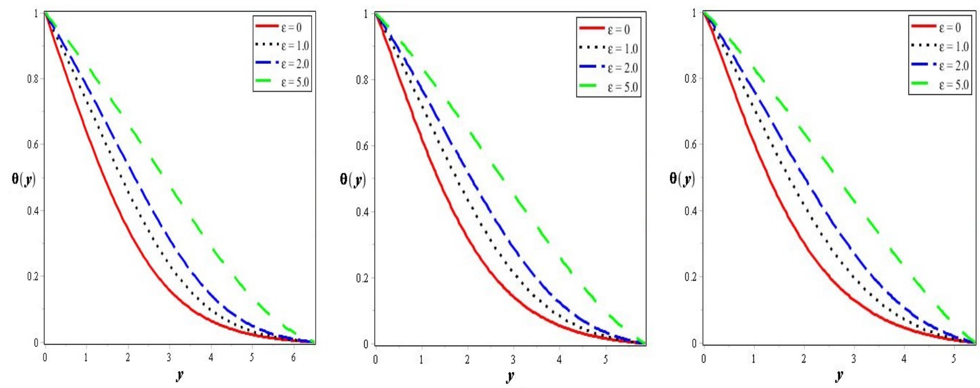
Figure 8 presents the graphs of $C_f Gr^{1/4}$ against x for various values of ϵ, ω and Ec. As ϵ rises, the peaks of $C_f Gr^{1/4}$ rises as well. While a combined increase in the variable viscosity parameter, ω and the Eckert number Ec decreases the skin friction coefficients.

The effect of increasing ϵ shown in Figure 9 increases the $NuGr^{-1/4}$. Also, the combined effects shown in Figures 9a, 9b and 9c increases the heat transfer, with the constant case serving as the lower bound.



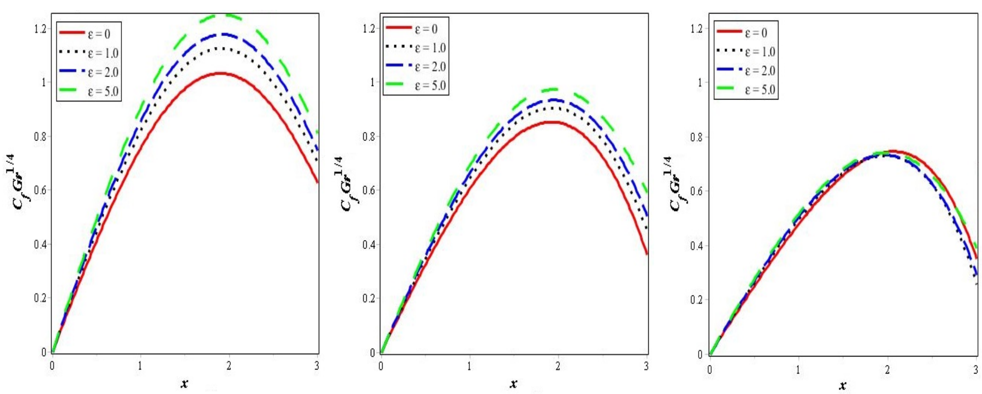
(a) (b) (c)

Figure 6: Velocity distributions when (a) $\omega = Ec = 0$, (b) $\omega = Ec = 1$ and (c) $\omega = Ec = 3$.



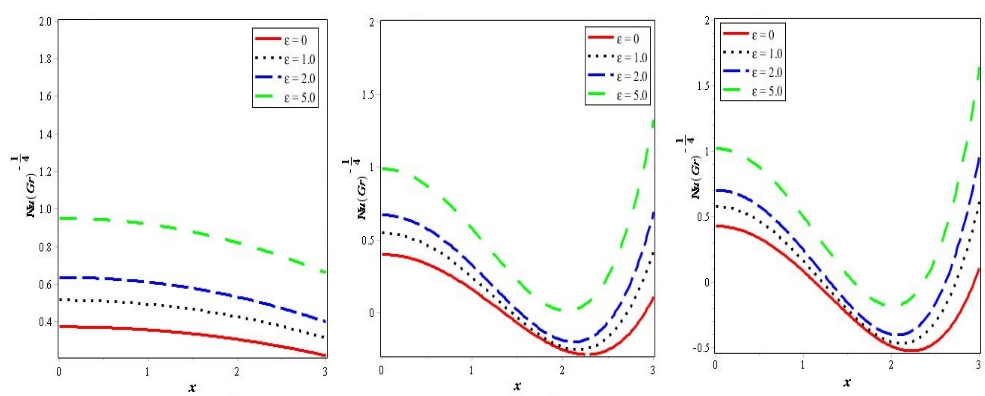
(a) (b) (c)

Figure 7: Temperature fields when (a) $\omega = Ec = 0$, (b) $\omega = Ec = 1$ and (c) $\omega = Ec = 3$.



(a) (b) (c)

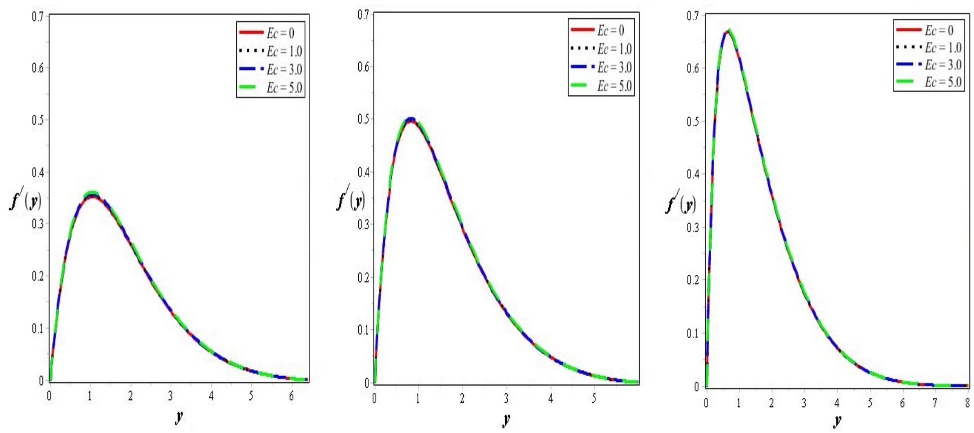
Figure 8: Profiles of skin friction when (a) $\omega = Ec = 0$, (b) $\omega = Ec = 1$ and (c) $\omega = Ec = 3$.



(a) (b) (c)

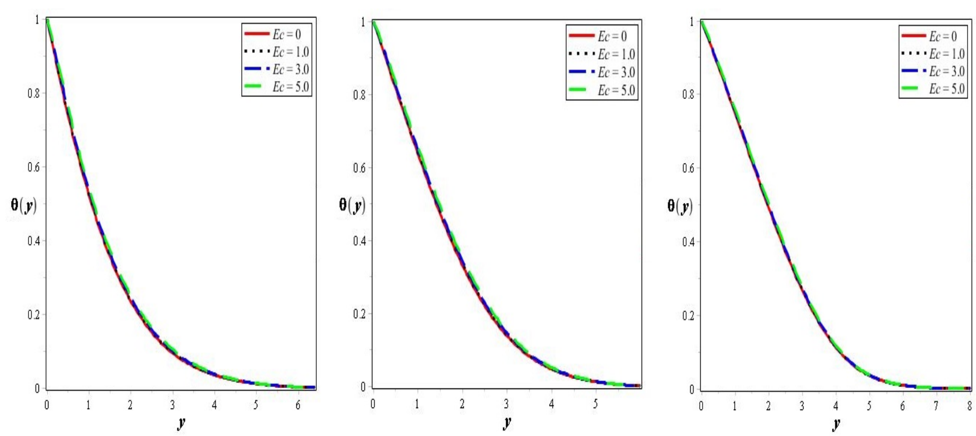
Figure 9: Plots of heat transfer when (a) $\omega = Ec = 0$, (b) $\omega = Ec = 1$ and (c) $\omega = Ec = 3$.

Figure 10 represents the combined effect of increase in the Eckert number together with increase in ω and ϵ. Figures 10a, 10b and 10c respectively show that increase in the Eckert number Ec, do not significantly influence the velocity profiles. However, the combined effect of increasing ω and ϵ appreciably increases the velocity profiles. Figure 11 also depict that Ec do not have effect on the temperature profiles. While the combined effect of increasing ω and ϵ in Figure 11c shows a better asymptotic behavior of the curves.



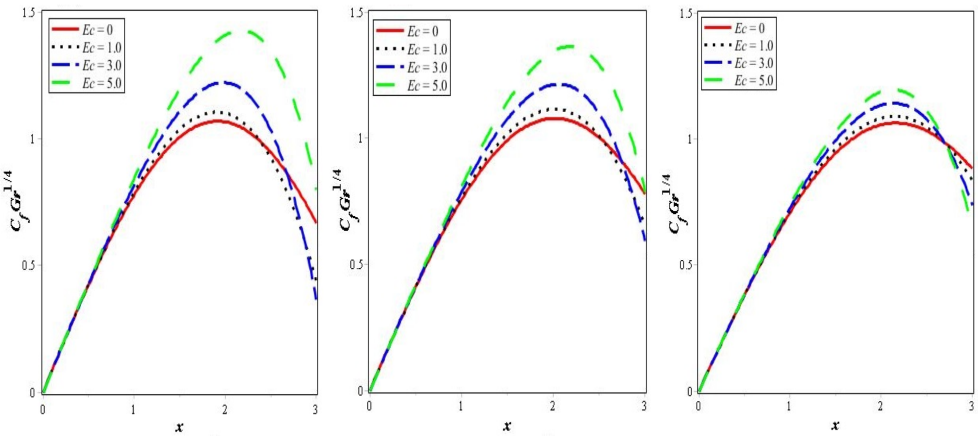
(a) (b) (c)

Figure 10: Behaviour of the velocity plots when (a) $\omega = \epsilon = 0$, (b) $\omega = \epsilon = 1$ and (c) $\omega = \epsilon = 3$.



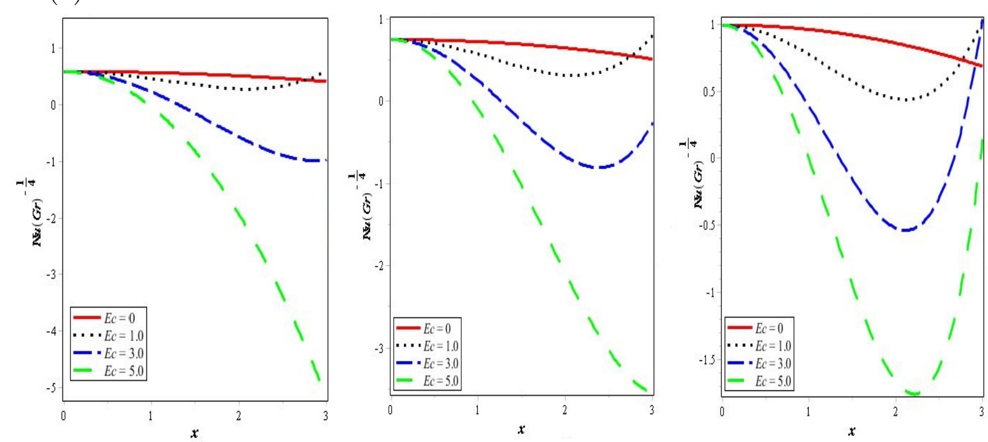
(a) (b) (c)

Figure 11: Observed temperature profiles for (a) $\omega = \epsilon = 0$, (b) $\omega = \epsilon = 1$ and (c) $\omega = \epsilon = 3$.



(a) (b) (c)

Figure 12: Profiles of the skin-friction when (a) $\omega = \epsilon = 0$, (b) $\omega = \epsilon = 1$ and (c) $\omega = \epsilon = 3$.



(a) (b) (c)

Figure 13: Heat transfer coefficients when (a) $\omega = \epsilon = 0$, (b) $\omega = \epsilon = 1$ and (c) $\omega = \epsilon = 3$.

Figure 12 shows the combined effect of Ec, ω and ϵ on the skin friction coefficients. The fluid with constant thermophysical property is observed to serve as the lower bound of the curves. Also, increase in Ec increases the the $C_f Gr^{1/4}$. However, the combined effect of increasing ω and ϵ suppresses the fluid's skin friction coefficients. Figure 13 illustrates the combined effect of Ec, ω and ϵ on the $NuGr^{-1/4}$. The constant fluid property serves as the upper bound of the curves. Increase in Ec suppresses the heat transfer coefficients while the combined effect of increasing ω and ϵ heightens the heat transfer coefficients.

Numerical computations of the velocity and temperature profiles indicate that, for small parameter values the asymptotic behavior is not well portrayed compared to large parameter values as shown in the Figures above. To ensure this asymptotic behavior for small parameter range, if possible, other numerical techniques will be investigated in later works.

5. CONCLUSSION

This study investigated the combined effect of variable ω, viscous dissipation and variable ϵ on the free convective flow across a cylinder. The study employed perturbation and numerical methods. Plots obtained for the limiting cases of the research compared excellently with those in literature. The results obtained in this study show that each fluid property contribute significantly to the flow attributes as well as the heat transfer features. The following important findings are made:
(1) The combined effects of increasing ω, ϵ and Ec significantly heighten the fluid's velocity in all the cases considered.
(2) The combined effects of increasing ω, ϵ and Ec is observed to pose a marginal increase on the fluid's temperature for the different scenarios studied.
(3) The combined effects of increasing ω, ϵ and Ec on the skin friction was discovered to heighten the skin friction only in the variation of ω.
(4) The combined effects of increasing ω, ϵ and Ec was found to spike the heat transfer rates of the fluids in all considered cases.
(5) Plots displaying the variation of one fluid material property (while other properties are kept constant) compares favorably and confirms existing works in literature.
(6) Increasing the viscous dissipation effect marginally increases the velocity distributions together with the temperature fields, whereas it significantly heightens the $C_f Gr^{1/4}$ and suppresses the $NuGr^{-1/4}$.

ACKNOWLEDGEMENTS

The authors are indeed grateful to the referees for their invaluable comments and suggestions which has helped in revising this research work.

REFERENCES

[1] S. M. Zokri, N. S. Arifin, M. K. A. Mohamed, A. R. M. Kasim, N. F. Mohammad and M. Z. Salleh, *Influence of viscous dissipation on the flow and heat transfer of a Jeffrey fluid towards horizontal circular cylinder with free convection: A numerical study*, Malaysian Journal of Fundamental and Applied Sciences **14** (1) 40-47, 2018.

[2] S. Rath and S. K. Dah, *Numerical study of laminar and turbulent natural convection from a stack of solid horizontal cylinders*, Int. J. Thermal Sciences **148**, 106-147, 2020

[3] M. K. A. Mohamed, N. M. Sarif, A. R. M. Kasim, N. A. Z. M. Noar, M. Z. Salleh and A. Ishak, *Effects of viscous dissipation on free convection boundary layer flow towards a horizontal circular cylinder*, ARPN Journal of Engineering and Applied Sciences **11** (11), 258-7263, 2016.

[4] M. M. Molla, S. C. Paul and M. A. Hossain, *Natural convection flow from a horizontal circular cylinder with uniform heat flux in presence of heat generation*, Applied Mathematical Modelling **33**, 3226-3236, 2009.

[5] C-Y Cheng, *The effect of temperature-dependent viscosity on the natural convection heat transfer from a horizontal isothermal cylinder of an elliptic cross-section*, Int. Commun. Heat Mass Transfer **33**, 1021-1028, 2006.

[6] Z. Uddin and M. Kumar, *Effect of temperature dependent properties on mhd free convection flow and heat transfer near the lower stagnation point of a porous isothermal cylinder*, Mathematical Physics and Computer Modelling **13** (4), 15-20, 2009.

[7] S. Ahmad, M. A. Norihan, N. Roslinda and P. Ioan, *Mixed convection boundary layer flow past an isothermal horizontal circular cylinder with temperature-dependent viscosity*, Int. J. Thermal Sciences **48**, 1943-1948, 2009.

[8] M. M. Molla, S. C. Saha and M. A. I. Khan, *Mhd natural convection flow from an isothermal horizontal circular cylinder under consideration of temperature dependent viscosity*, Int. J. Computer-Aided Engineering and Software **29** (8), 875-88, 2012.

[9] Z. Uddin and M. Kumar, *Mhd heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in porous domain with the presence of radiation*, Jordan Journal of Mechanical and Industrial **5** (5), 419-423, 2011.

[10] M. M. Alam, M. A. Alim and M. K. Chowdhury, *Effect of pressure stress work and viscous dissipation in natural convection flow along a vertical flat plate with heat conduction*, Journal of Naval Architecture and Marine Engineering **3**, 69-76, 2006.

[11] V. R. Prasad, R. Bhuvanavijaya, S. A. Gaffar, E. K. Reddy and O. A. Beg, *Flow and heat transfer of Jeffreys non-Newtonian fluid from horizontal circular cylinder*, Journal Of Thermophysics And Heat Transfer **28** (4), 764-770, 2014.

[12] T. M. E. Nabil, E. M. A. Elbashbeshy, I. K. Youssef and A. M. Sedki, *The Effects of temperature dependent viscosity and viscous dissipation on mhd convection flow from an isothermal horizontal circular cylinder in the presence of stress work and heat generation*, European Scientific Journal **10** (36), 81-103, 2014.

[13] H. Schlichting and K. Gersten *Boundary layer theory, 9th edition*, Springer- Verlag, Berlin Heidelberg, 2017

[14] T. E. Akinbobola and S. S. Okoya, *The flow of second-grade fluid over a stretching sheet with variable thermal conductivity and viscosity in the presence of heat source/sink*, J. Nig. Math. Soc. **34** (3), 331-342, 2015.

[15] S. I. Opadiran and S. S. Okoya, *Importance of convective boundary layer flows with inhomogeneous material properties under linear and quadratic Boussinesq approximations around a horizontal cylinder*, Heliyon **7** (5) 1-13, 2021

[16] E. O. Fatunmbi, S. S. Okoya and O. D. Makinde, *Convective heat transfer analysis of hydromagnetic micropolar fluid flow past an inclined nonlinear stretching sheet with variable thermo-Physical properties*, Diffusion Foundations **26**, 63-77, 2020.

[17] N. H. M. A. Azim and M. K. Chowdhury, *Mhd-conjugate free convection from an isothermal horizontal circular cylinder with joule heating and heat generation*, J. Comp. Meth. Phy. **2013**, 1-11, 2013.

[18] M. M. Molla, M. A. Hossain and M. C. Paul, *Natural convection flow from an isothermal horizontal circular cylinder in presence of heat generation*, Int. J. Eng. Sci. **44**, 949-958, 2006.

[19] N. H. M. A. Azim, *Magnetohydrodynamic conjugate free convection flow from an isothermal horizontal circular cylinder* Ph.D. thesis, Bangladesh University of Engineering and Technology, Dhaka, 2014.

[20] M. M. Molla, M. A. Hossain and R. S. R. Gorla, *Natural convection flow from an isothermal horizontal circular cylinder with temperature dependent viscosity*, Heat and Mass Transfer **41**, 594-598, 2005.

[21] J. H. Merkin, *Free convection boundary layer on an isothermal horizontal cylinder*, ASME/ AIChe Heat Transfer Conference, St. Louis, USA, 1976.

[22] R. Nazar, N. Amin and I. Pop, *Free convection boundary layer on an isothermal horizontal circular cylinder in a micropolar fluid*, Proceedings of Twelfth Int Heat Transfer Conference, Paris, Elsevier, **2**, 525-530, 2002.

[23] A. R. M. Kasim, M. A. Admon and S. Sharidan, *Free convection boundary layer flow of a viscoelastic fluid in the presence of heat generation*, Int. J. Mathematical, Computational, Physical, Electrical and Computer Engineering **5** (3), 227-234, 2011.

[24] A. Adeniyan, F. Mabood and S. S. Okoya, *Effect of heat radiating and generating second-grade mixed convection flow over a vertical slender cylinder with variable physical properties*, International Communications in Heat and Mass Transfer, **121**, 1-11, 2001.

[25] A. R. M. Kasim, N. F. Mohammad and S. Shafie, *Effect of heat generation on free convection boundary layer flow of a viscoelastic fluid past a horizontal circular cylinder with constant surface heat flux*, The 5th International Conference on Research and Education in Mathematics, AIP Conf. Proc. **1450**, 286-292, 2012.

[26] B. Mahanthesh, I. L. Animasaun, R. G. Mohammad and I. M. Alarifi, *Quadratic convective transport of dusty Casson and dusty Carreau fluids past a stretched surface with nonlinear thermal radiation, convective condition and non-uniform heat source/sink*, Physica A **535** (122471), 1-16, 2019.

[27] B. Mahanthesh, J. Mackolil, M. Radhika and A. W. Siddabasappa, *Significance of quadratic thermal radiation and quadratic convection on boundary layer two-phase flow of a dusty nanoliquid past a vertical plate*, International Communications in Heat and Mass Transfer **120** (6), 1-12, 2020.

[28] C-Y Cheng, *Natural convection boundary layer on a horizontal elliptical cylinder with constant heat flux and internal heat generation*, Int. Commun. Heat Mass Transfer **36**, 1025-1029, 2009.

[29] E. O. Fatunmbi and S. S. Okoya, *Heat transfer in boundary layer magneto-micropolar fluids with temperature-dependent material properties over a stretching sheet*, Advances in Materials Science and Engineering **2020**, 1-11, 2020.

[30] J. C. Umavathi, A. J. Chamkha and S. Mohiuddin, *Combined effect of variable viscosity and thermal conductivity on free convection flow of a viscous fluid in a vertical channel*, Int. J. Numer. Meth. Heat Fluid Flow **26** (1), 18-39, 2016.

[31] K. J. Bassant and J. G. Bello, *Mhd free convection in a vertical slit microchannel with superhydrophobic slip and temperature jump: nonlinear Boussinesq approximation approach* SN Applied Sciences **1** (603), 1-10, 2019.

[32] K. Thriveni and B. Mahanthesh, *Optimization and sensitivity analysis of heat transport of hybrid Nano liquid in an annulus with quadratic Boussinesq approximation and quadratic thermal Radiation*, Eur. Phys. J. Plus. **135** (459), 1-22, 2020a.

[33] K. Thriveni and B. Mahanthesh, *Sensitivity analysis ofnonlinear radiated heat transport ofhybrid nanoliquid inanannulus subjected tothenonlinear Boussinesq approximation*, Journal of Thermal Analysis and Calorimetry, 1-21, 2020b.

[34] K. Rakesh, S. Shilpa, *Interaction of magnetic field and nonlinear convection in the stagnation point flow over a shrinking sheet*, Journal of Engineering **2016**, 1-8, 2016.

[35] K. Vajravelu and K. S. Sastri, *Fully developed laminar free convection flow between two parallel vertical walls-I*, Int. J. Heat Mass Transfer **20** (6), 655-660, 1977.

[36] N. C. Roy and R. S. R. Gorla, *Effects of radiation and magnetic field on mixed convection flow of non-Newtonian power-law fluids across a cylinder in the presence of chemical reaction, heat and mass transfer* **55**, 341-351, 2019.

[37] N. C. Roy, M. A. Hossain and R. S. R. Gorla, *Free and forced convection flow past a cylinder with slip boundary conditions*, Commun. Numerical Analysis **1**, 1-20, 2019.

[38] N. C. Roy, T. Rahman and M. A. Hossain, *Boundary-layer characteristics of compressible flow past a heated cylinder with viscous dissipation*, J. Thermophysics and Heat Transfer **33** (1), 1-13, 2018.

[39] N. Zainuddin, I. Hashim, H. Saleh and R. Roslan, *Effects of radiation on free convection from a heated horizontal circular cylinder in the presence of heat generation*, Sains Malaysiana **45** (2), 315-321, 2016.

[40] N. Zainuddin, I. Hashim, M. Ismoen and R. Roslan, *The effect of radiation on free convection from a heated horizontal circular cylinder*, Applied Mechanics and Materials **773-774**, 378-386, 2015.

[41] P. C. Sinha, *Fully developed laminar free convection flow between vertical parallel plates*, Chem. Eng. Sci. **24**, 33-36, 1969.

[42] S. I. Opadiran, *Natural convection boundary layer flow with variable viscosity, thermal conductivity and heat generation about a horizontal circular cylinder*, M.Sc. thesis, Obafemi Awolowo University, Ile-Ife, Nigeria, 2021 (Unpublished).

[43] S. L. Goren, *On free convection in water at 4 C*, Chem. Eng. Sci. **21** (67), 515-518, 1966.

[44] S. M. Zokri, N. S. Arifin, M. K. A. Mohamed, A. R. M. Kasim, N. F. Mohammad and M. Z. Salleh, *Influence of viscous dissipation on the flow and heat transfer of a Jeffrey fluid towards horizontal circular cylinder with free convection: A numerical study*, Malaysian Journal of Fundamental and Applied Sciences **14**, (1), 40-47, 2018.

[45] S. Rath and S. K. Dah, *Numerical study of laminar and turbulent natural convection from a stack of solid horizontal cylinders*, Int. J. Thermal Sciences **148**, 106-147, 2020.

[46] T. Kunnegowda, B. Mahanthesh, G. Lorenzini and I. L. Animasaun, *Significance of induced magnetic field and exponential space dependent heat source on the quadratic convective flow of Casson fluid in a micro-channel via HPM*, Math. Model. Eng. Probl. **6** (3), 369-384, 2019.

[47] V. R. Prasad, R. Bhuvanavijaya and B.Mallikarjuna, *Natural convection on heat transfer flow of non-Newtonian second grade fluid over horizontal circular cylinder with thermal radiation*, Journal of Naval Architecture and Marine Engineering **13**, 63-78, 2016.

[48] V. P. Ramachandra, R. A. Subba and R. A. Anwar, *Computational analysis of viscous dissipation and joule-heating effects on non-Darcy mhd natural convection flow from a horizontal cylinder in porous media with internal heat generation*, Theoret. Appl. Mech., **41**, (1), 3770, 2014.

DEPARTMENT OF MATHEMATICS, OBAFEMI AWOLOWO UNIVERSITY, ILE - IFE, NIGERIA

E-mail address: `sundayiyiola@gmail.com`

DEPARTMENT OF MATHEMATICS, OBAFEMI AWOLOWO UNIVERSITY, ILE - IFE, NIGERIA

E-mail addresses: `sokoya@oauife.edu.ng, ssokoya@gmail.com`