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ON INDEPENDENCE POLYNOMIALS OF QUOTIENT BASED GRAPHS FOR FINITE ABELIAN GROUPS

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ABSTRACT. In this paper we investigated the independence polynomial of quotient based graph $\wp_H(G)$ of an abelian group G relative to all it's subgroups since a subgroup H of an abelian group is normal . The graph $\wp_H(G)$ is a graph with condition of adjacency where two distinct elements $x, y \in G$ are adjacent in the graph iff $xy \notin H$. A formula for computing the size and independence polynomial of the graph are obtained.

Keywords and Phrases : Independent Set, Independence Polynomial, Quotient Based Graph, Abelian Group, Size of Graph.
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1. INTRODUCTION

There are many works on graphs of algebraic structures , researchers are now interested in interpreting the properties of algebraic structures using the properties of graphs. As can be seen in the work of Beck [5] where he worked on the graph of a commutative ring with unity in such away that two distinct elements x and y are adjacent in the graph if $xy = 0$. In 1999 Anderson and Livingston [3] introduced the Zero divisor graph of a commutative ring ; the graph is such that two vertices x and y are adjacent if they are non-zero zero divisors of the ring and $xy = 0$. Anderson and Badawi [2] worked on the total graph of a commutative ring R where $x, y \in R$ are adjacent in the graph if $x + y \in \mathbb{Z}(R)$ with $\mathbb{Z}(R)$ the zero divisor of R. The non-commuting graph of a

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group is a graph whose vertex set is the non-central elements of the group with two elements x and y been adjacent in the graph if $xy \neq yx$, the non-commuting graph was investigated by Abdollahi et. al [1] in 2006. Erfanian and Tolue [7] came up with the conjugate graphs of finite groups in 2012. The work of Sarmin et. al [15] in 2016 was on conjugate graph and conjugate class graph of some finite metacyclic -2 groups. They viewed the conjugate graph as a graph whose set of vertices is the non-central elements of the group with two vertices adjacent if they are conjugate of each other while the conjugate class graph has adjacent vertices if they are not coprime. Parker [14] in 2013 investigated the commuting graph of soluble groups with trivial center. And recently in 2022 Kiri and Suleiman [11] investigated the quotient based graphs for some finite non-abelian groups, where the work has to do with the graph of a group G relative to its normal subgroup H, with two distinct elements $x, y \in G$ adjacent in the graph iff $xy \notin H$.

In this paper we investigated the quotient based graphs of a finite abelian group G relative to all its subgroups H since all the subgroups are normal. We equally looked at the independence polynomial of the graph . Hoede and Li [10] first came up with the idea of independent sets and independence polynomials of graphs in 1999 when they worked on clique polynomials and independent sets polynomials of graphs. Other works on independence polynomials include the independence polynomials of conjugate graphs and non-commuting graphs of groups of small order by Najmuddin et. al [12] in 2017 , they considered dihedral groups of order 6,8 and the quaternion. Najmuddin et. al [13] in 2017 also worked on independence polynomials of nth-central graphs of dihedral groups of order 6 and 8 . Ferrin [8] looked at independence polynomials in 2014. Kiri and Suleiman [16] in 2020 worked on independence polynomials of inverse commuting graphs of dihedral group, they came up with a pattern for obtaining the polynomials without necessary using the Hoede and Li's [10] approach.

The paper has five sections ; starting with introduction, section two has to do with definitions and some existing results that are relevant for the work. Section three shows the results on size of the graphs while four has to do with the polynomial and the last section concludes the work.

2. PRELIMINARY

Definition 2.1. Graph [6]: The pair φ(V,E) is called a graph with V the vertex set and E edge set of the graph. An element $e \in E$ is an edge where $e = \{x, y\}$ and $x, y \in V$.
The cardinality of V is called the order of the graph while the size of the graph is given by the cardinality of E. A graph is called undirected if the edges have no orientation whereas a simple graph has no multiple edges. And the degree of a vertex is the number of edges adjacent to it.

Definition 2.2. Quotient Based Graph [11]: Quotient based graph denoted as $\varphi_H(G)$ is a graph of a group G relative to its normal subgroup H with two distinct elements $x, y \in G$ been adjacent iff $xy \notin H$.
The graph is a simple connected graph which is undirected.

Definition 2.3. Independence Polynomial [10]: An independence polynomial is a polynomial given by $I(\varphi,x)=\sum_{i=0}^{m} b_i x^i$ with b_i the number of independent sets of cardinality i, where independent sets refers to the set of pairwise non-adjacent vertices in a graph φ under consideration and m is the maximum cardinality of the sets and m is known as independence number.

Lemma 2.4. *Hand Shaking Lemma* [4]*: The sum of the degree of a graph is twice the size of the graph.*
Note that the sum of all the degrees of vertices in a graph divided by 2 gives the size of the graph.

Theorem 2.5. *Lagrange's Theorem* [9]*: Let H be a subgroup of a finite group G, the order of H divides the order of G.*
The index of a subgroup H in G given by $|G|/|H| = [G : H]$ is the number of distinct left(or right) cosets of H in G.

3. RESULTS

We begin by looking at the definition below

Definition 3.1. Lone Coset : A lone coset P is an element of a quotient group G by its normal subgroup H such that $P \neq H$ and $\forall\ x \in P$ the inverse $x^{-1} \in P$.

Lemma 3.2. *Let G be a finite abelian group, the quotient group of G by H (G/H) of even order has a lone coset .*

Proof. It is clear that elements of G/H are sets consisting of H and other cosets, so $e \in H$ and $\forall\ x \in H,\ x^{-1} \in H$ since it satisfies the closure property.
The remaining cosets do not satisfy the closure property as such $\forall\ x \in K$ (K a coset) $\exists$ another coset say L with $x^{-1} \in L$ since $x, x^{-1} \in G$. $\Rightarrow L$ and K are related pair of cosets. The index of H in G is even so removing all the pair of related cosets leaves H and a lone coset say P and $\forall\ x \in P$ its inverse x^{-1} belongs to P since it does not belong to any other coset. □

Remark 3.3. Note that the graph $\wp_H(G)$ is a graph of a group G relative to its normal subgroup H, for finite abelian groups any subgroup is a suitable choice for H.

Example 3.4. The example below shows each group G with a normal subgroup H and corresponding lone coset P.

i. $G = \mathbb{Z}_{12}$ and $H = \{0, 6\}$, so the lone coset $P = \{3, 9\}$
ii. Given $\mathbb{Z}_{24}$, if $H = \{0, 8, 16\}$ the lone coset $P = \{4, 12, 20\}$
iii. If $G = \mathbb{Z}_{24}$ and $H = \{0, 6, 12, 18\}$ then $P = \{3, 9, 15, 21\}$
iv. For the group of units U_{13} if $H = \{1, 12\}$ we get a lone $P = \{5, 8\}$

3.1. **Quotient Based Graph for Finite Abelian Groups.** In this case the graph is going to be relative to all subgroups of an abelian group G since all its subgroups are normal .The theorems below are on the size of the quotient based graphs .

Theorem 3.5. *The size of a quotient based graph $\wp_H(G)$ for a simple abelian group G is given by* $\begin{cases} \frac{n^2-2n+1}{2}, & |G| = n - odd \\ \frac{n^2-2n+2}{2}, & |G| = n - even. \end{cases}$

Proof. Since G is simple the subgroup is a trivial subgroup which is normal , given $|G| = n$ the index of H in G is n. The group G/H consist of H and $(n-1)$ other cosets each a singleton.

Case one : $n - odd$
By definition of the graph $\forall\ x \in G,\ x$ is adjacent to every $y \in G$ except if $xy \in H$, since each coset is a singleton x is adjacent to every y excluding $y = x^{-1}$. $\Rightarrow$ the identity element is adjacent to $(n-1)$ elements resulting to $(n-1)edges$, and $\forall\ x \in G, x \neq e$; x is adjacent to $(n-2)$ elements since its not adjacent to itself and its inverse. This gives $(n-1)(n-2)$ edges between the remaining elements of the group.

Adding all the edges and applying handshake lemma give the size as

$$Size = \frac{(n-1)+(n-1)(n-2)}{2} = \frac{n^2-2n+1}{2}$$

Case two : $n - even$

When n is even, in the $(n-1)$ cosets $\exists$ a lone coset as shown in lemma 3.1 with a single element which is self-inverse and also adjacent to $(n-1)$ elements just like the identity element resulting into $2(n-1)$ edges. For the remaining $(n-2)$ cosets ,each element in the set is adjacent to $(n-2)$ elements excluding its inverse leading to $(n-2)^2$edges. Applying lemma 2.4 gives the result

$$\frac{2(n-1)+(n-2)^2}{2} = \frac{n^2-2n+2}{2}$$

□

Theorem 3.6. *The quotient based graph of a finite group G of order n relative to any of its subgroup H of order α is a graph* $\varphi_H(G)$ *of size* $\begin{cases} \frac{n^2-(\alpha+1)n+\alpha}{2}, & \frac{n}{\alpha} - odd \\ \frac{n^2-(\alpha+1)n+2\alpha}{2}, & \frac{n}{\alpha} - even. \end{cases}$

Proof. The Lagrange's theorem gives the index of H in G i.e $\frac{n}{\alpha}$, its clear from the definition of the graph that the vertex set V($\varphi_H(G)$) is partitioned into $\frac{n}{\alpha}$ comprising of H and other cosets.

Case one: $n - odd$

The index $\frac{n}{\alpha}$ of H in G is always odd since n and α are odd ; comprising of H and even pairs of related cosets. $|H| = \alpha$ and elements of H are not adjacent to each other by definition 2.2 but adjacent to elements of $(\frac{n}{\alpha}-1)$ cosets leading to $\alpha(\frac{n}{\alpha}-1)$ edges since each coset equally has α elements.

$$\Rightarrow (n-\alpha)\ edges \tag{*}$$

for a single element of H

then for all elements of H we have

$$\alpha(n-\alpha)\ edges \tag{**}$$

For the even cosets ,$\forall\ x \in K$ (say coset K) x is adjacent to elements of all cosets including H except a particular related coset say L with $x^{-1} \in L$, this gives $\alpha(\frac{n}{\alpha}-1)-1$ edges since it is not adjacent to itself.

$\Rightarrow (n-\alpha-1)$ *edges* and for all elements of one coset we have

$$\alpha(n-\alpha-1)\ edges \tag{* * *}$$

For all the cosets the edges becomes

$$\alpha(n-\alpha-1)(\frac{n}{\alpha}-1) = n^2-(2\alpha+1)n+\alpha+\alpha^2 \ edges \qquad (****)$$

Applying handshake lemma to (**) and (****) gives the result.

Case two: $n - even$

Subcase 1: When the index $\frac{n}{\alpha}$ is odd the result is similar to case one above.

Subcase 2 : When we have an even index comprising of H , lone coset say P as seen in lemma 3.1 and other related pairs of cosets.The total edges between elements of H and the lone coset P will be

$$2\alpha(n-\alpha) \qquad (*)$$

So we are left with $(\frac{n}{\alpha}-2)$ cosets, $\forall\ x \in coset\ K$, x is adjacent to $[\alpha(\frac{n}{\alpha}-1)-1]$ elements since it is not adjacent to itself and this gives

$$(n-\alpha-1)\ edges \qquad (**)$$

for all elements of K we multiply (**) by α to get

$$\alpha(n-\alpha-1)\ edges \qquad (***)$$

To get the total edges for all elements of the cosets (i.e without H and P) we multiply (***) by the number of cosets to get ;

$$\alpha(n-\alpha-1)(\frac{n}{\alpha}-2) = n^2-(3\alpha+1)n+2\alpha+2\alpha^2 \ edges \qquad (****)$$

We apply handshake lemma to (*) and (****) get the result. □

Example 3.7. Let us consider some examples with simple groups.

i. For $\mathbb{Z}_7$ the subgroup $H = \{0\}$, and to get the size we apply theorem 3.3 for $n - odd\ size = \frac{n^2-2n+1}{2} = \frac{7^2-14+1}{2} = 18$.
ii. For $G = U_{10}$ and $H = \{1\}$ the $Size = \frac{n^2-2n+2}{2} = \frac{4^2-Z(4)+2}{2} = 5$.

Example 3.8. This example has to do with groups that are not simple and we apply theorem 3.4 to get the size of their quotient based graphs

i. Given $U_{14} = \{1,3,5,9,11,13\}$ let $H = \{1,13\}$ then cosets $K = \{3,11\}$, $L = \{5,9\}$ so $\alpha = 2$, $n = 6$ and the index is odd. The graph $\varphi_H(U_{14})$ has $size = \frac{6^2-(3)6+2}{2} = 10$.
ii. For $\mathbb{Z}_{16}$ if $H = \{0,4,8,12\}$ $\alpha = 4$ and the index $\frac{n}{\alpha} = 4$ meaning we have a lone coset $P = \{2,6,10,14\}$. The $size = \frac{16^2-(5)16+2\times 4}{2} = 92$.

4. Independence Polynomials

In this section we apply Hoede and Li's [10] concept to compute the independence polynomials of the quotient based graphs for finite abelian groups. They came up with the formula below ;

$I(\varphi_H(G),x)= \sum_{i=0}^{m} b_i x^i$

The b_i are the independent sets of cardinality i (pairwise non-adjacent vertices) we could get from the graphs under consideration.

Theorem 4.1. *The independence polynomial of the quotient based graphs $\varphi_H(G)$ for finite abelian group G is a polynomial of degree α given by*
$1 + nx + (\omega\alpha\ {}^{\alpha}C_1 + q\ {}^{\alpha}C_2)x^2 + q\ {}^{\alpha}C_3 x^3 + q\ {}^{\alpha}C_4 x^4 + \ ...\ + q\ {}^{\alpha}C_{\alpha} x^{\alpha}$, *with $|H| = \alpha$, ω the number of related pair of cosets and q cosets whose elements are not adjacent.*

Proof. Using Hoede and Li's [10] formula entails getting the coefficients b_k s which are the independent sets, they were able to show that $b_0 = 1$ and $b_i = |\mathrm{V}(\varphi_H(G))| = n$ for $i = 1$. $\Rightarrow$ each vertex is an independent set of its own. Leaving us with b_i for $i > 1$. So it suffices to show how to generate the b_i , $i > 1$.
The vertex set of the graph comprises of elements of H and all the cosets. We know that elements of H are not adjacent to each other likewise those of the lone coset say P while that of an ordinary coset are adjacent, the maximum set of pairwise non-adjacent vertices is obtained from H (or/and P) which implies that the independence number $= \alpha$ and $I(\varphi_H(G),x)= \sum_{i=0}^{\alpha} b_i x^i$.
To get b_i for $i > 1$, it is clear to see that we can get an independent set of cardinality 1 in a coset since all the elements are adjacent except it is a lone coset or H, but pairing elements of a related coset can only lead to an independent set of cardinality 2 while for H and lone coset P we could generate independent sets of cardinality 2 up to α as all elements are pairwise non-adjacent.
For $i = 2$ elements of H and all cosets are involved. When the index $\frac{n}{\alpha}$ is odd there is no lone coset , so selecting two elements from H will be done in

$$ {}^{\alpha}C_2 \ ways \qquad \text{(i.)}$$

while for related pairs of cosets say K and L you can only get independent sets by selecting an element from K and pairing it with each element of L which can be done in

$$\alpha\ {}^{\alpha}C_1\ ways \tag{ii.}$$

The total independent sets of order two for the graph will be sum of (i) and $(ii.)$

$$\Rightarrow\ [\alpha\ {}^{\alpha}C_1 + {}^{\alpha}C_2] \tag{iii.}$$

When the index is even recall there is a subgroup H , a lone coset say P, a number of related cosets.
Let ω be the number of pair of related cosets.
Let q represent the set whose elements are not adjacent i.e H and P.
Multiplying $(i.)$ above by q gives the number of ways two elements can be selected in H and P, which lead to equation

$$q\ {}^{\alpha}C_2\ ways \tag{iv.}$$

Multiplying $(ii.)$ by ω gives the number of ways two elements can be selected in all related cosets. So

$$\omega\ \alpha\ {}^{\alpha}C_1\ ways \tag{v.}$$

Adding $(iv.)$ and $(v.)$ gives the coefficient b_2;

$$[\omega\ \alpha\ {}^{\alpha}C_1 + q\ {}^{\alpha}C_2],\ where\ \omega \geq 1\ and\ q = \{1\ or\ 2\}.$$

For $i > 2$ you could also generate such independent sets within H and lone coset only, since any three vertices selected from two ordinary cosets will have two adjacent vertices. $i - vertices$ could be selected from α in $q\ {}^{\alpha}C_i$ ways since the selection is done within same set H(and L in some cases). This results to;

$$I(\varphi_H(G), x) = \sum_{i=0}^{\alpha} b_i x^i$$
$$= b_0x^0 + b_1x^1 + b_2x^2 + ... + b_\alpha x^\alpha$$
$$= 1 + nx + [\omega\ \alpha\ {}^{\alpha}C_1 + q\ {}^{\alpha}C_2]x^2 + q\ {}^{\alpha}C_3x^3 + ... + q\ {}^{\alpha}C_\alpha x^\alpha.$$

□

Example 4.2. We computed the independence polynomials for quotient based graphs for some groups

i. The vertex set of the graph $(\varphi_H(\mathbb{Z}_{15})$ is partitioned into the subgroup $H = \{0, 5, 10\}$ and related pairs of cosets $K = \{1, 6, 11\}$, $L = \{2, 7, 12\}$, $M = \{3, 8, 13\}$ and $N\{4, 9, 14\}$. This shows that $\alpha = 3$, $n = 15$, $\omega = 2$ and $q = 1$ since no lone coset. We apply theorem 4.1 to get;
$I(\varphi_H(\mathbb{Z}_{15}),x) = 1 + 15x + 21x^2 + x^3$.

ii. For the group U_{11} its quotient based graph has vertex set made from these sets $H = \{1, 10\}$, $K = \{2, 9\}$, $L = \{3, 8\}$, $M = \{4, 7\}$, and $N = \{5, 6\}$, with K related to N since elements of K are inverses of N, likewise L and M, making $\omega = 2$, $q = 1$, $n = 10$,$\alpha = 2$. From theorem 4.1 we get the polynomial to be ;
$I(\varphi_H(U_{11}),x) = 1 + 10x + 9x^2$.

iii. Given $G = \mathbb{Z}_{16}$ and $H = \{0, 4, 8, 12\}$ the index of H in G is 4, apart from H we have three cosets say K,L ,P. K and L related pair while P a lone coset , this implies we use $n = 16$, $\alpha = 4$, $\omega = 1$ and $q = 2$ in theorem 4.1. So;
$I(\varphi_H(\mathbb{Z}_{16}),x) = 1 + 16x + 28x^2 + 8x^3 + 2x^4$.

For the case when the trivial subgroup is used in constructing the graph , theorem 4.1 would not be valid as one cannot select two items from 1 ($i.e\ {}^{\alpha}C_2,\ where\ \alpha = 1$). The following theorem takes care of the independence polynomial for the quotient based graphs relative to a trivial subgroup.

Theorem 4.3. *The independence polynomial for a quotient based graph of a finite abelian group G of order n relative to a trivial group is given by*

$$1 + nx + \frac{(n-q)}{2}x^2, q = \begin{cases} 1, & n\text{-}odd \\ 2, & n\text{-}even \end{cases}$$

Proof. When the trivial subgroup $H = \{e\}$ is used the vertex set of the graph is partitioned into H and $(n-1)$ cosets each of cardinality 1. By definition 2.2 each $x \in G$ is adjacent to every element of G except its inverse (x^{-1}) making e adjacent to every element of the group. Definition 2.3 gave the formula $I(\varphi,x)= \sum_{i=0}^{m} b_i x^i$ with $b_0 = 1$ and $b_1 = n$, in this case the maximum cardinality of the pairwise non-adjacent vertices will be $m = 2$ since any three vertices you pick two must be adjacent making the polynomial to be of degree 2. We just figure out the coefficient b_2.
For $n - odd$: Apart from the identity element we have $(n - 1)$ elements in the vertex set of the graph where each x you pick has its inverse x^{-1} in G resulting into $\frac{(n-1)}{2}$ pairs of independent sets since x and x^{-1} are pairwise non-adjacent. Given $b_2 = \frac{(n-1)}{2}$.
For $n - even$: When n is even $\exists$ an element whose inverse is itself , removing the identity element and this self inverse element means

we are left with $\frac{(n-2)}{2}$ pair of elements of the form $\{x, x^{-1}\}$ which are also non-adjacent to each other in the graph, hence

$$b_2 = \frac{(n-2)}{2}$$

$\square$

Below is a corollary to theorem 4.1

Corollary 4.4. *The independence polynomial of a quotient based graph $I(\varphi_H(G)$ of a finite abelian group G of order n relative to it's subgroup H of order $\frac{n}{2}$ is*

$$1 + nx + q\ {}^{\alpha}C_2 x^2 + q\ {}^{\alpha}C_3 x^3 + q\ {}^{\alpha}C_4 x^4\ \ldots\ q\ {}^{\alpha}C_{\alpha} x^{\alpha}.$$

Proof. The proof follows from the proof of theorem 4.1 with $\omega = 0$, since the index of H in G is 2 with each set having elements that are not adjacent i.e H and a lone coset P. Implying $q = 2$. $\square$

Example 4.5. We look at examples involving theorem 4.3 and corollary 4.4

i. The group $\mathbb{Z}_5$ has the subgroup $H = \{0\}$ so the independence polynomial for the graph $I(\varphi_e(\mathbb{Z}_5), x) = 1 + nx + \frac{(n-q)}{2}x^2 = 1 + 5x + 2x^2$.
ii. The independence polynomial for the $\varphi_H(\mathbb{Z}_{12})$ with $H = \{0, 2, 4, 6, 8, 10\}$ is
$I(\varphi_e(\mathbb{Z}_5), x) = 1 + 12x + 30x^2 + 40x^3 + 30x^4 + 12x^5 + 2x^6$.

5. Conclusion

The formulae for finding the size of the quotient based graph $\varphi_H(G)$ for finite a abelian group G relative to its subgroup H were given in this paper for both trivial and non-trivial subgroups. And the independence polynomials $I(\varphi_H(G, x)$ for the graphs were also shown to be polynomials of degree equal to the order of the subgroup when the subgroup is not trivial and of degree 2 for trivial subgroup.

6. ACKNOWLEDGEMENT

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