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COMBINED EFFECTS OF RADIATION AND CHEMICAL REACTION ON HEAT AND MASS TRANSFER BY MHD STAGNATION-POINT FLOW OF A MICROPOLAR FLUID TOWARDS A STRETCHING SURFACE

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ABSTRACT. An analysis is performed to study the thermal-diffusion and diffusion-thermo effects on heat and mass transfer by magnetohydrodynamic (MHD) mixed convection stagnation point flow of a micropolar fluid towards a stretching surface in the presence of magnetic field, thermal radiation and homogenous chemical reaction effects. The Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. The resulting governing partial differential equations are transformed into a set of non-linear ordinary differential equations, which are solved using an implicit finite difference method with quasi-linearization technique. Various comparisons with previously published work are performed and the results are found to be in excellent agreement. Representative results for the velocity, microrotation, temperature and concentration profiles as well as the local skin-friction coefficient, local Nusselt and Sherwood numbers is presented graphically for various parametric conditions and discussed. It is found that the skin friction decreases with increasing chemical reaction, magnetic and material parameters. Also, the radiation parameter, Soret and Dufour effects tend to increase friction.

Keywords and phrases: Heat and Mass Transfer; Micropolar Fluid; Stagnation-Point Flow; Dufour and Soret Effects; Radiation; Chemical Reaction
2010 Mathematical Subject Classification: A80

1. INTRODUCTION

In recent years, the dynamics of micropolar fluids has been a popular area of research. As fluids consist of randomly oriented molecules and as each volume element of the fluid has translation as well as rotation motions, the analysis of physical problems in these fluids has revealed several interesting phenomena not found in Newtonian fluids. Micropolar convection flows have been analyzed by many authors following the

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seminal work of Eringen who introduced the micropolar fluid [1] as a special case of the micromorphic fluid [2], and excellent reviews about the applications of micropolar fluids have been written by Airman et al. [3,4]. Such fluids can accurately simulate the micro-rotational effects observed in colloidal solutions, blood, dielectric fluids, plasmas, liquid crystals etc. Micropolar convective flows find applications in the purification of crude oil, polymer technologies, centrifugal separation processes, cooling tower dynamics, chemical reaction engineering, metallurgical drawing of filaments and solar energy systems. Mansour et al. [5] studied the heat and mass transfer in magnetohydrodynamic flow of micropolar fluid on a circular cylinder with uniform heat and mass flux. They found that the micropolar fluids tends to reduce the friction and heat transfer rate as compared to Newtonian fluids. Mohamed and Abo-Dahab [6] analyzed the influence of chemical reaction on the heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium with heat generation and thermal radiation effects.

Zueco et al. [7] presented a study for unsteady MHD free convection of a micropolar fluid between two parallel porous vertical walls. Ishak et al. [8] discussed the MHD flow of a micropolar fluid towards a stagnation point on a vertical surface. They found that dual solutions exist for the assisting flow of micropolar fluids. Ishak et al. [9] examined the MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux. Modather et al. [10] studied the analytic approximate solutions for the heat and mass transfer of micropolar fluids in a saturated porous medium over an infinite moving permeable plate with chemical reaction effect. Pandey and Chaube [11] explained the peristaltic flow of a micropolar fluid through a porous medium in the presence of an external magnetic field. The problem of onset of Rayleigh-Bnard MHD convection in a micropolar fluid was studied by Alloui and Vasseur [12]. On other hand, Thermal diffusion, also called thermo-diffusion or Soret effect corresponds to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient [13]. The energy flux caused by a composition gradient is called Dufour or diffusion-thermo effect. These effects are considered as second order phenomena, on the basis that they are of smaller order of magnitude than the effects described by Fourier's and Fick's laws, but they may become significant in areas such geosciences or hydrology. Soret/Dufour diffusion effects have also garnered considerable interest in both Newtonian and non-Newtonian convective and heat and mass transfer. Such effects are significant when density differences exist in the flow regime. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary systems, often encountered in chemical process engineering and also in high-speed aerodynamics. Moreover, when chemical species are introduced at a surface in the fluid domain with different (lower) density than the surrounding fluid, both Soret (thermo-diffusion) and Dufour (diffusionthermal) effects can be influential. The effect of diffusion-thermal and thermal diffusion of heat and mass has been developed from the kinetic theory of gases by Chapman and Cowling [14] and Hirshfelder et al. [15]. They explained the phenomena and derived the necessary formulas to calculate the thermal diffusion coefficient and the thermal-diffusion factor for monatomic gases or for polyatomic gas mixtures. Kafoussias and Williams [16] studied the thermal-diffusion and the diffusion-thermo effects on the mixed free-forced convective and mass transfer steady laminar boundary layer flow over a vertical flat plate when the viscosity of the fluid is assumed to vary with temperature. Seddeek [17] investigated thermal-diffusion and diffusion-thermo effects on mixed free-forced convective flow and mass transfer over an accelerating surface with a heat source in the presence of suction and blowing under the case of variable viscosity. Alam et al. [18] studied the free convection and mass transfer flow past a continuously moving semi-infinite vertical porous plate in a porous medium with the Soret and Dufour's effects. EL-Aziz [19] considered the thermal-diffusion and diffusion-thermo effects on the heat and mass transfer characteristics of free convection past a continuously stretching permeable surface in the presence of magnetic field and radiation effects. Rahman [20] proposed the thermal diffusion and MHD effects on mixed convection and mass transfer of a viscous fluid flow through a porous medium with heat generation. Tsai and Huang [21] provided an extensive review of the heat and mass transfer for Soret and Dufour's effects on Hiemenz flow through porous medium onto a stretching surface. Hayat et al. [22] studied the heat and mass transfer on mixed convection boundary layer flow about a linearly stretching vertical surface in a porous medium filled with a viscoelastic fluid taking into account the thermal-diffusion and diffusion-thermo effects. EL-Kabeir et al. [23] presented a mathematical model to study the thermal-diffusion and diffusion-thermo effects radiation on coupled heat and mass transfer by MHD mixed convection stagnation-point flow of a non-Newtonian fluid towards a stretching surface in the presence of chemical reaction. Makinde and Olanrewaju [24] discussed the unsteady mixed convection with Dufour and Soret effects past a semi-infinite vertical plate moving through a binary mixture of chemically reacting fluid. The diffusion-thermo and thermal-diffusion effects on heat and mass transfer by MHD free convection along a vertical flat plate with streamwise temperature and the species concentration were considered numerically by Chamkha et al. [25]. Reddy et al. [26] studied the Soret effect on mixed convection flow in a nanofluid under convective boundary condition. The problem of unsteady heat and mass transfer by mixed convective flow over a rotating vertical cone in the presence the magnetic field, thermal-diffusion and diffusion-thermo and chemical reaction effects is considered by Chamkha and Rashad [27].

Mehmood et al. [28] investigated the stagnation point flow with heat transfer of a micropolar second grade fluid towards a stretching surface. They found that velocity at a point increases with increasing microrotation parameter , whereas heat transfer is increasing function of the elasticity parameter. Singh and Kumar [29] dealt with the flow and heat transfer characteristics during the melting process due to a stretching surface in micropolar fluid with heat absorption. They demonstrated that heat transfer rate decreases with melting parameter and heat absorption. Mahmood et al. [30] investigated hydromagnetic stagnation point flow and heat transfer of a micropolar fluid over a nonlinearly stretching/shrinking surface. They noticed that the flow and heat transfer rates can be controlled through the material parameter and magnetic force. El-Sayeda et al. [31] investigated MHD stagnation point flow and heat transfer of a micropolar fluid over a stretching sheet in the presence of radiation, heat generation and dissipations. Hussain et al. [32] studied MHD stagnation point flow of micropolar fluids towards a stretching sheet. They considered variable surface temperature and investigated the effects of various parameters on the dimensionless velocity and temperature.

In this paper, the effects of thermal radiation and chemical reaction on heat and mass transfer by MHD mixed convection stagnation point flow of a micropolar fluid towards a stretching surface in the presence of magnetic field, thermal-diffusion and diffusion-thermo effects is analyzed. The order of chemical reaction in this work is taken as a first-order reaction and the Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. The coupled nonlinear parabolic partial differential equations governing the flow and heat and mass transfer problem have been solved numerically using an implicit finite difference method with quasi-linearization technique. The effects of various governing parametric conditions on the velocity, microrotation, temperature and concentration profiles as well as the local skin-friction coefficient, local Nusselt number and the local Sherwood number have been presented graphically and discussed.

2. GOVERNING EQUATIONS

Consider the problem of coupled heat and mass transfer by MHD mixed convection stagnation point flow of an electrically-conducting, optically dense and micropolar fluid towards a stretching surface in the presence of radiation and chemical reaction, thermal-diffusion (Soret) and the diffusion-thermo (Dufour) effects. It is assumed that the stretching velocity is given by $U_w(x) = ax$, where $a(> 0)$ is constant and the velocity distribution in frictionless potential flow in the neighborhood of the stagnation point at $x = y = 0$ is given by $U(x) = cx$, where $c(> 0)$ is constant (see Fig. 1).

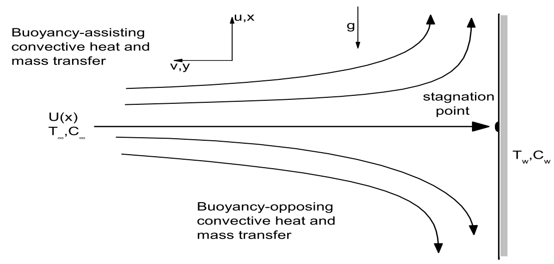

Fig. 1. Flow model and coordinate system.

The stretching surface is maintained at a constant temperature T_w and a constant concentration C_w and the ambient temperature and concentration far away from the surface T_∞ and C_∞ are assumed to be uniform. For $T_w > T_\infty$ and $C_w > C_\infty$, an upward flow is induced as a result of the thermal and concentration buoyancy effects. A uniform magnetic field is applied in the y-direction normal to the flow direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. In addition, the Hall effect and the electric field are assumed negligible. The small magnetic Reynolds number assumption uncouples the Navier-Stokes equations from Maxwell's equations. A first-order homogeneous chemical reaction is assumed to take place in the flow. The flow is assumed laminar, steady and all of the micropolar fluid properties are assumed to be constant except for the density variation in the buoyancy force term. By invoking all of the boundary layer, Boussineq and Rosseland diffusion approximations, the governing equations for this investigation can be written as;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\begin{aligned} & u\tfrac{\partial u}{\partial x} + \mathrm{v}\tfrac{\partial u}{\partial y} = U\tfrac{\mathrm{d}U}{\mathrm{d}x} + \left(\tfrac{\mu+k}{\rho}\right)\tfrac{\partial^2 u}{\partial y^2} + \tfrac{k}{\rho}\tfrac{\partial N}{\partial y} + g\beta(T - T_\infty) \\ & + g\beta^*(C - C_\infty) - \tfrac{\sigma B_0^2}{\rho}(u - U) \end{aligned} \tag{2}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma^*}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{Dk_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial y^2} - K_1(C - C_\infty), \tag{5}$$

and the relevant boundary conditions are given by

$$\begin{aligned} & u = u_w\ (x) = cx, \mathrm{v} = 0,\ N = 0,\ T = T_w,\ C = C_w \ \text{at} y = 0, \\ & u = U(x) = ax,\ N = 0,\ T = T_\infty,\ C = C_\infty \text{ at } y \to \infty \end{aligned} \tag{6}$$

where u, v, T, C are the fluid x-component of velocity, y-component of velocity, temperature, and concentration, respectively. g, r, α, D, β_T, and β_C are the gravitational acceleration, fluid density, thermal diffusivity, mass diffusivity, coefficient of thermal expansion, and coefficient of concentration of expansion, respectively. σ, B_0, q_r, and K_1 are the electrical conductivity, magnetic induction, mass diffusivity, radiative heat flux, and the dimensional of chemical reaction, respectively. N is the microrotation (or angular velocity), j microinertia density, γ^* is the spin gradient viscosity, k is the vortex viscosity (or the microrotation viscosity), C_p, T_m, k_T and C_s are the specific heat at constant pressure, mean fluid temperature, thermal diffusion ratio and concentration susceptibility.

In addition, the radiative heat flux q^r is described according to the Rosseland approximation such that:

$$\frac{\partial q^r}{\partial y} = -\frac{4\sigma_1}{3\chi}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ_1 and χ are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. As done by Raptis [33], the fluid-phase temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature. This is done by expanding T^4 in a CityplaceTaylor series about the free-stream temperature T_∞ and neglecting higher-order terms to yield

$$T^4 = 4T_\infty^4 T - 3T_\infty^4, \tag{8}$$

By using Eqs. (6) and (7) in the last term of Eq. (3), we obtain

$$\frac{\partial q^r}{\partial y} = -\frac{16\sigma_1 T_\infty^3}{3\chi}\frac{\partial^2 T}{\partial y^2}, \tag{9}$$

Following the work of many recent authors by assuming that γ^* is given by, see Rees and Bassom [34] or Rees and Pop [35],

$$\gamma^* = \left(\mu + \frac{k}{2}\right) j = \mu\left(1 + \frac{K}{2}\right) j, K = \frac{k}{\mu}, \tag{10}$$

Applying the following transformations;

$$\psi = (c\upsilon)^{1/2}\, x f(\eta),\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty},\ \eta = y\,(c/\upsilon)^{1/2},$$

$$N = cx\,(c/\upsilon)^{1/2}\, h(\eta), \tag{11}$$

where $u = \partial\psi/\partial y$and $v = -\partial\psi/\partial x$define the stream function ψ, the governing equations (2)-(5) become

$$(1+K)f''' + Kh' + ff'' - f'^2 - Mf' + M\frac{a}{c} + \frac{a^2}{c^2} + \lambda(\theta + \Lambda\phi) = 0, \tag{12}$$

$$(1 + K/2)h'' - KB(2h + f'') - f'h + fh' = 0, \tag{13}$$

$$\frac{1}{\Pr}\left(1+\frac{4R_d}{3}\right)\theta''+f\theta'+f'\theta+Df\phi''=0, \tag{14}$$

$$\frac{1}{Sc}\phi''+f\phi'+Sr\theta''-\gamma\phi=0, \tag{15}$$

and the transformed dimensionless boundary conditions become

$$\begin{aligned} &f(0)=0, f'(0)=1, h(0)=0, \theta(0)=1, \phi(0)=1,\\ &f'(\infty)\to\lambda_1, h(\infty)=0, \theta(\infty)\to 0, \phi(\infty)\to 0, \end{aligned} \tag{16}$$

where K is the material parameter (K=0 shows viscous fluid), M is the magnetic parameter, λ is the dimensionless mixed convection parameter, $\lambda_1$is the stretching parameter, Λ is the buoyancy parameter, Pr is the Prandtl number, Sc is the Schmidt number, R$_d$ is the radiation parameter, Df is the Dufour number, Sr is the Soret number and γ is the dimensionless chemical reaction parameter. These parameters are given by

$$\left.\begin{aligned} &\lambda=\tfrac{g\beta(T_w-T_\infty)x^3/\upsilon^2}{u_w^2x^2/\upsilon^2}=\tfrac{Gr_x}{Re_x^2}, \Lambda=\tfrac{\beta^*(C_w-C_\infty)}{\beta(T_w-T_\infty)}, \lambda_1=a/c,\\ &\Pr=\frac{\upsilon}{\alpha}, B=\frac{\upsilon}{jc}, Sc=\frac{\upsilon}{D}, R_d=\sigma_1T_\infty^3/3k\chi, M=\frac{\sigma B_0^2}{\rho c},\\ &Df=\frac{Dk_T(C_w-C_\infty)}{C_sC_p(T_w-T_\infty)\upsilon}, Sr=\frac{Dk_T(T_w-T_\infty)}{T_m(C_w-C_\infty)\upsilon}, \gamma=K_1/c \end{aligned}\right\} \tag{17}$$

with $Gr_x=g\beta(T_w-T_\infty)x^3/\upsilon^2$ being the local Grashof number,$Re_x = u_w x/\upsilon$ is the local Reynolds number and k is the thermal conductivity. It should be noted that$\lambda>0$ corresponds to an assisting flow (heated plate),$\lambda<0$ corresponds to an opposing flow (cooled plate) and$\lambda=0$ yields forced convection flow.

The skin-friction coefficient C$_f$ at the wall is given by:

$$C_f=\frac{1}{\frac{1}{2}\rho u_w^2}\left[(\mu+k)\frac{\partial u}{\partial y}+kN\right]_{y=0}=2Re_x^{1/2}\left[1+K\right]f''(\xi,0), \tag{18}$$

the local Nusselt number Nu_x is given by:

$$Nu_xRe_x^{-1/2}=-\left(1+\frac{4R_d}{3}\right)\theta'(0), \tag{19}$$

and the local Sherwood number Sh_x is given by:

$$Sh_xRe_x^{-1/2}=-\phi'(0). \tag{20}$$

3. NUMERICAL METHOD

MHD stagnation-point flow of a micropolar fluid over a stretching sheet is investigated numerically in the presence of thermal radiation, chemical reaction, Soret and Dufour effects. An implicit finite difference method with quasi-linearization technique is used to solve nonlinear coupled ordinary differential equations. For details, please see Bellman and Kalaba [36] and Inoyue and Tate [37]. The step size is taken as$\Delta\eta=0.001$and the convergence criteria was set to 10^{-6}. The asymptotic boundary conditions given by Eq. (16) were replaced by using a value of 18 for the similarity variable $\eta_{\max}$ as follows.

$$\eta_{\max} = 18, f'(18) = \lambda_1, \theta(18) = h(18) = \phi(18) = 0. \tag{21}$$

The choice of $\eta_{\max} = 18$ ensured that all numerical solutions approached the asymptotic values correctly.
The accuracy of the aforementioned numerical method was validated by direct comparisons with the numerical results reported earlier by and EL-Kabeir et al. [23] and Mahapatra et al. [38] in the absence of heat and mass transfer with various values of magnetic parameter, Mahapatra and Gupta [39], Nazar et al. [40], Ishak et al .[41] and Abbas et al. [42], in the absence of magnetic field, heat and mass transfer for different values of stretching parameter, Ishak et al. [41] and Abbas et al. [42] in the absence of magnetic field and mass transfer for different values of the Prandtl number. Tables 1-3 present the results of these various comparisons. It can be seen from this table that excellent agreement between the results exists. This favorable comparison lends confidence in the numerical results to be reported in the next section.

4. RESULTS AND DISCUSSIONS

The numerical results of skin friction and Nusselt numbers are compared with existing data in Tables 1-3 in the absence of heat and mass transfer. It is observed that the present results are in good agreement with the published data. We are, therefore, confidant that our results are accurate. The effects of different parameters on the dimensionless velocity for assisting, opposing and forced convection flows are shown in Figs. 2 and 3. In the absence of magnetic field, the overshoot velocity in the neighborhood of the surface is higher and it decreases to the stretching velocity with an increase in the magnetic field in case of assisting flow. This is due to the fact that magnetic field produces a force which tends to slow down the movement of the fluid. In the opposing flow, the fluid behavior is just opposite to assisting flow, as shown in Fig. 2(a). In case of forced convection, the velocity remains uniform and attains the stretching velocity. Figure 2(b) shows the effects of material and buoyancy parameters on the dimensionless velocity for assisting flows in the presence of magnetic field. Again, in assisting flows, the dimensionless velocity overshoots in the neighborhood of the surface. The maximum overshoot velocity is found to increase with the buoyancy parameter and decrease with the material parameter. The effects of Soret and Dufour parameters on the dimensionless velocity are depicted in Fig. 3(a) for assisting flows. It can be seen that these effects on the dimensionless velocity are not appreciable. The effects of radiation and stretching parameters are investigated in Fig. 3(b) for assisting flows. These effects are investigated in the presence of magnetic field. Inside the hydrodynamic boundary layer, the dimensionless velocity overshoots in the neighborhood of the surface with radiation. However, the effects of stretching parameter are not appreciable. It is noticed that the boundary layer thickness increases with an increase in the radiation parameter.

The variation of the dimensionless temperature with the transverse distance in the thermal boundary layer is shown in Figs. 4 and 5 for different values of controlling parameters. The effects of magnetic field on the dimensionless temperature for three different flows (assisting, opposing and forced convection) are shown in Fig. 4(a). No appreciable effects of higher values of magnetic parameter on the dimensionless temperature could be found. It is noticed that the thermal boundary layer thickness decreases from assisting flow to opposing flow. The effects of material and buoyancy parameters on the dimensionless temperature are shown in Fig. 4(b). It can be seen that both parameters have no effect on the thermal boundary layer thickness. The variation of the dimensionless temperature with transverse distance for different values of several controlling parameters is shown in Figs. 5(a) and 5(b). An increase in Dufour parameter increases the dimensionless temperature within the thermal boundary layer, as shown in Fig. 5(a). Similarly, the radiation parameter increases the thermal boundary layer thickness and the dimensionless temperature inside the thermal boundary layer. The effects of stretching parameter on the dimensionless temperature are found to be negligible for smaller values of radiation parameter.

The variation of the dimensionless concentration with transverse distance for several controlling parameters is shown in Figs. 6 and 7. The effects of magnetic field on the dimensionless concentration are displayed in Fig. 6(a) for assisting, opposing and forced convection flows. It can be seen that the magnetic field reduces the dimensionless concentration inside the concentration boundary layer. However, for higher values of magnetic field, no effect on the dimensionless concentration could be found. Similarly, the effects of the material and buoyancy parameters on the dimensionless concentration are found to be the same, as shown in Fig. 6(b). The effects of Soret and Dufour parameters on the dimensionless concentration are shown in Fig. 7(a) for assisting flow in the presence of magnetic field and radiation. It can be seen that the dimensionless concentration, inside the concentration boundary layer, increases with both Soret and Dufour parameters. Figure 7(b) depicts the effects of chemical reaction and stretching parameters on the dimensionless concentration in the presence of magnetic field and radiation. As the chemical reaction proceeds, the dimensionless concentration of the fluid decreases and as a result the concentration boundary layer thickness decreases. However, the dimensionless concentration increases due to increase in stretching velocity.

The effects of several parameters on the dimensionless angular velocity are shown in Figs. 8 and 9 for assisting, opposing and forced convection flows. For these flows, the behavior of angular velocity is shown in Fig. 8(a) in the presence of radiation. It can be seen that, for assisting and opposing flows, the dimensionless angular velocity increases/decreases with larger amplitude in the absence of magnetic field and this amplitude decreases with an increasing magnetic field. However, the dimensionless angular velocity remains uniform for forced convection flow. The effects of buoyancy and material parameters on the dimensionless angular velocity are displayed in Fig. 8(b) in the presence of other parameters. Both parameters decrease/increase the dimensionless angular velocity inside the boundary layer. The governing equation for the dimensionless angular velocity shows that it is independent of Soret, Dufour and radiation parameters. Therefore, no appreciable effect of these parameters could be found on the dimensionless angular velocity for assisting flow, as shown in Figs. 9(a) and 9(b). However, the effects of the stretching parameter on the dimensionless angular velocity can be observed clearly in Fig. 9(b).

The physical quantities of our interest include skin friction, heat and mass transfer rates for the MHD stagnation-point flow of a micropolar fluid over a stretching surface in the presence of radiation, chemical reaction, Soret and Dufour effects. The effects of several parameters on skin friction are investigated in Figs. 10(a) and (b). The variation of skin friction with magnetic and radiation parameters for two different values of material parameter is shown in Fig. 10(a). It is clear that the skin friction decreases with increasing magnetic and material parameters whereas it increases with radiation parameter. The Soret and Dufour effects tend to increase friction whereas chemical reaction reduces the friction. This could be observed in Fig. 10(b).

The variation of Nusselt numbers with different parameters is depicted in Figs. 11(a) and (b) for assisting flows. Figure 11(a) shows that the Nusselt numbers decrease with magnetic, radiation and material parameters for a stretching sheet. For a viscous fluid (K=0), the Nusselt numbers are found to be higher than micropolar fluids. Also, the Nusselt numbers are found to be higher in the absence of radiation and magnetic field. It is clear from Fig. 11(b) that the Nusselt numbers decrease with chemical reaction but increase with Soret and Dufour parameters for a stretching sheet.

Figures 12(a) and (b) show the variation of Sherwood numbers with different parameters for an assisting flow. The Sherwood numbers increase with magnetic, radiation, material, chemical reaction and Dufour parameters but decrease with Soret parameters.

Table 1. Comparison of$f''(0)$in the absence of heat and mass transfer when $\lambda_1 = 2$and $Rd = 0$.

M	EL-Kabeiret al.[23]	Mahapatraet al. [38]	Presentresults
0	2.017798	2.0175	2.0175
0.5	2.136279	2.1363	2.13632
1	2.249317	2.2491	2.2491
1.5	2.356882	2.3567	2.35667
2	2.459712	2.4597	2.45967
3	2.653975	2.654	2.65398
5	3.005763	3.0058	3.00577
10	3.744703	3.7447	3.74471
20	4.901177	4.9004	4.90037
1000	31.69113	31.6858	31.6859

Table 2. Comparison of$f''(0)$in the absence of heat and mass transfer for different values of λ_1 when $Rd = 0$.

λ_1	Mahapatra and Gupta [34]	Nazar et al. [40]	Ishak et al. [41]	Abbas et al. [42]	Present results
0.1	-0.9694	-0.9694	-0.9694	-0.9694	-0.96939
0.2	-0.9181	-0.9181	-0.9181	-0.9181	-0.91811
0.5	-0.6673	-0.6673	-0.6673	-0.6673	-0.66726
2	2.0175	2.0176	2.0175	2.0175	2.017503
3	4.7293	4.7296	4.7294	4.7293	4.729282

Table 3. Comparison of skin friction and Nusselt numbers in the absence of heat and mass transfer when $\lambda_1 = \Lambda = 1$ and $Rd = 0$.

Pr	$f''(0)$			$-\theta'(0)$		
	Abbas et al. [42]	Ishak et al. [41]	Present results	Abbas et al.[42]	Ishak et al. [41]	Present results
0.72	0.3645	0.3645	0.3644	1.0931	1.0931	1.0931
6.8	0.1804	0.1804	0.1804	3.2902	3.2902	3.2895
20	0.1175	0.1175	0.1175	5.623	5.623	5.6201
40	0.0874	0.0873	0.0872	7.9464	7.9463	7.9383
60	0.0729	0.0729	0.0728	9.7327	9.7327	9.7180
80	0.0641	0.064	0.0639	11.241	11.241	11.219
100	0.0577	0.0578	0.0577	12.572	12.572	12.541

5. CONCLUDING REMARKS

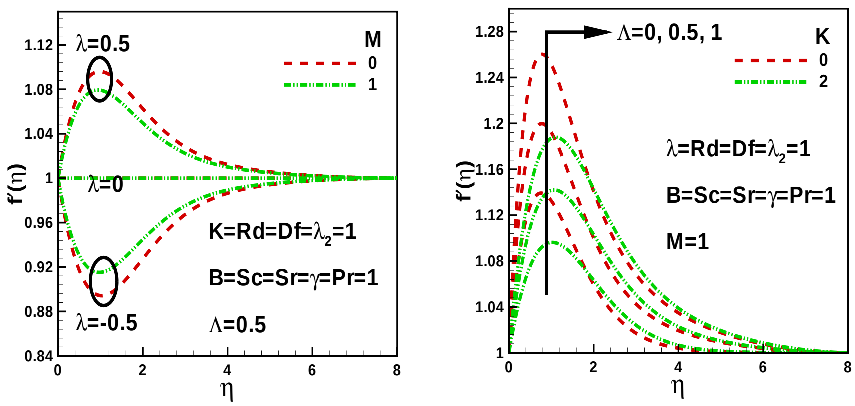

Fig. 2. Effect of magnetic field, material and buoyancy parameters on dimensionless velocity for different flows.

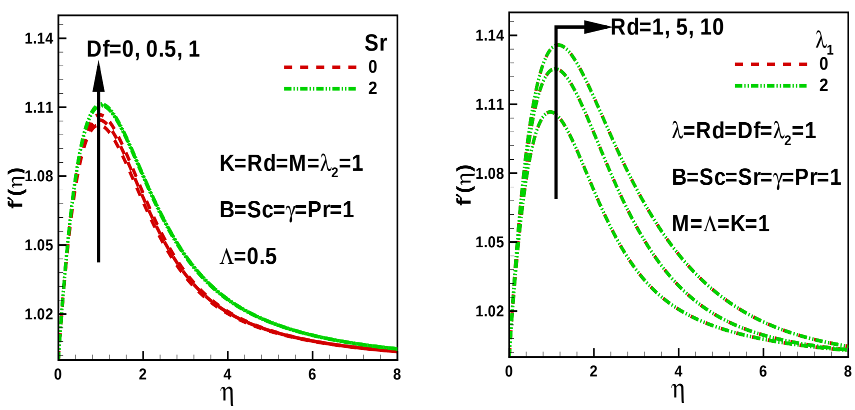

Fig. 3. Effects of Soret, Dufour, radiation and stretching parameters on dimensionless velocity for assisting flows

In this paper, the effects of thermal radiation and chemical reaction on coupled heat and mass transfer by MHD mixed convection stagnation point flow of a micropolar fluid towards a stretching surface in the presence of magnetic field, thermal radiation and homogenous chemical reaction effects have been analyzed. The governing boundary-layer equations were transformed into a similar form, and these equations were solved numerically using an implicit finite difference method with quasilinearization technique. Various comparisons with previously published work are performed and the results are found to be in excellent agreement. The effects of the thermal radiation, chemical reaction, material

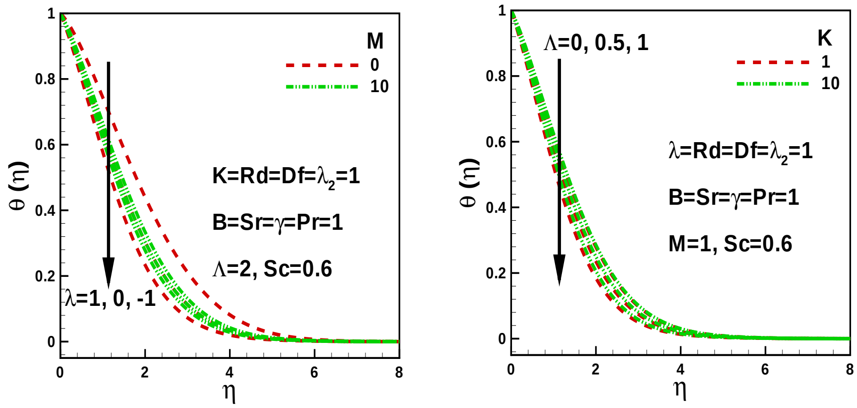

Fig. 4. Effect of magnetic field, material and buoyancy parameters on dimensionless temperature for different flows.

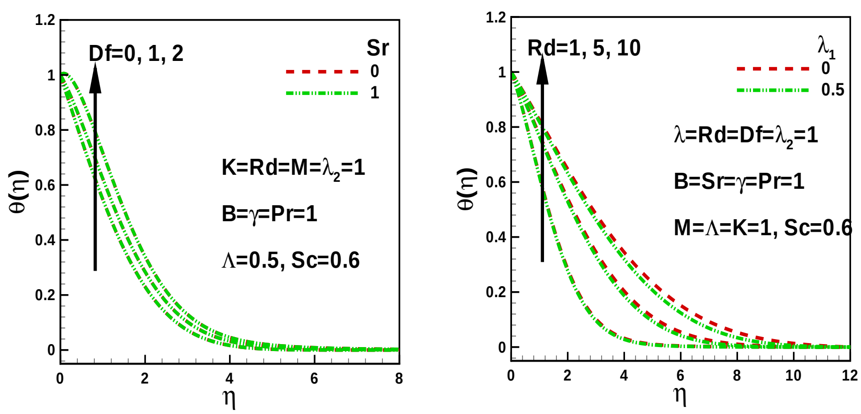

Fig. 5. Effects of Soret, Dufour, radiation and stretching parameters on dimensionless temperature for assisting flows.

parameter, Soret and Dufour parameters on the skin-friction coefficient as well as the Nusselt number and the Sherwood number were shown graphically and discussed. It is found that the

(1) skin friction decreases with increasing magnetic and material parameters whereas it increases with radiation parameter.
(2) Soret and Dufour effects tend to increase friction whereas chemical reaction reduces the friction.
(3) Nusselt numbers decrease with magnetic, radiation and material parameters for a stretching sheet.

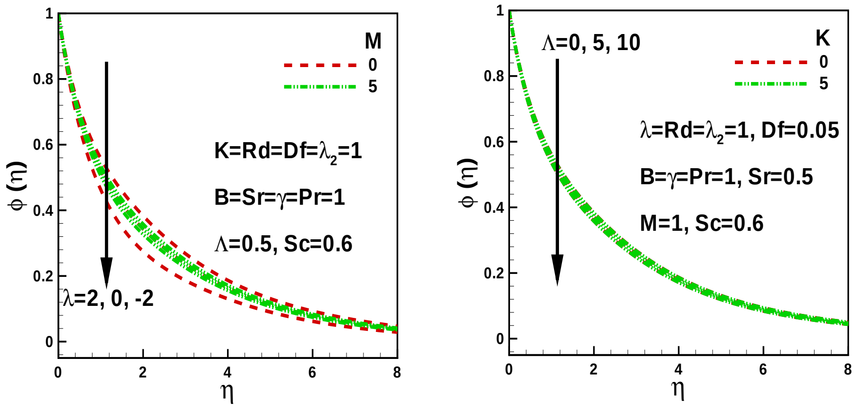

Fig. 6. Effect of magnetic field, material and buoyancy parameters on dimensionless concentration for different flows.

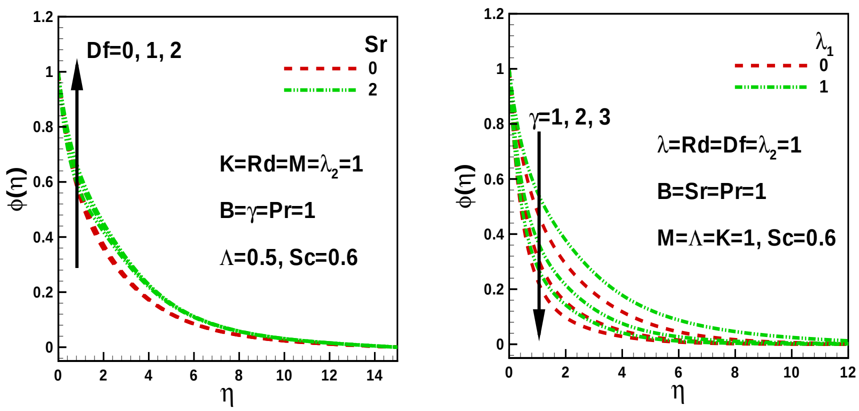

Fig. 7. Effects of Soret, Dufour, chemical reaction and stretching parameters on dimensionless concentration for assisting flows.

(4) Nusselt numbers for a viscous fluid (K=0) are found to be higher than micropolar fluids.
(5) Nusselt numbers are found to be higher in the absence of radiation and magnetic field.
(6) Nusselt numbers decrease with chemical reaction but increase with Soret and Dufour parameters for a stretching sheet.
(7) Sherwood numbers increase with magnetic, radiation, material, chemical reaction and Dufour parameters but decrease with Soret parameters.

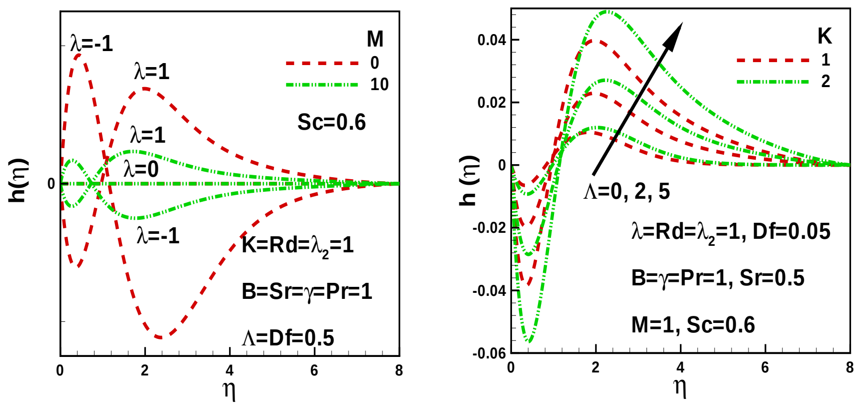

Fig. 8. Effect of magnetic field, material and buoyancy parameters on dimensionless angular velocity for different flows.

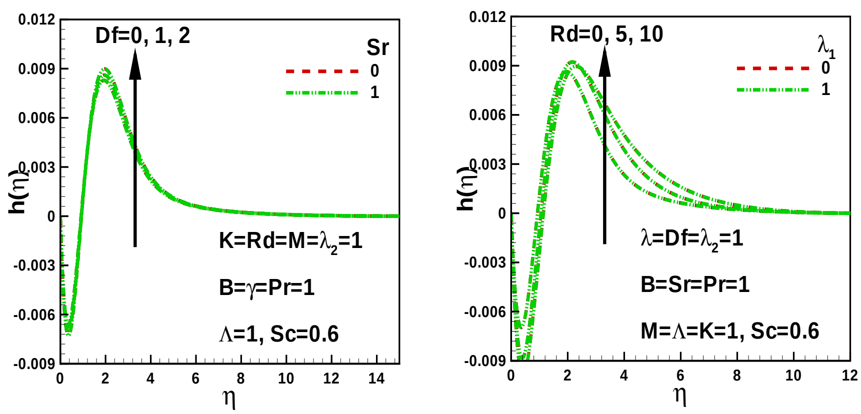

Fig. 9. Effects of Soret, Dufour, radiation and stretching parameters on dimensionless angular velocity for assisting flows.

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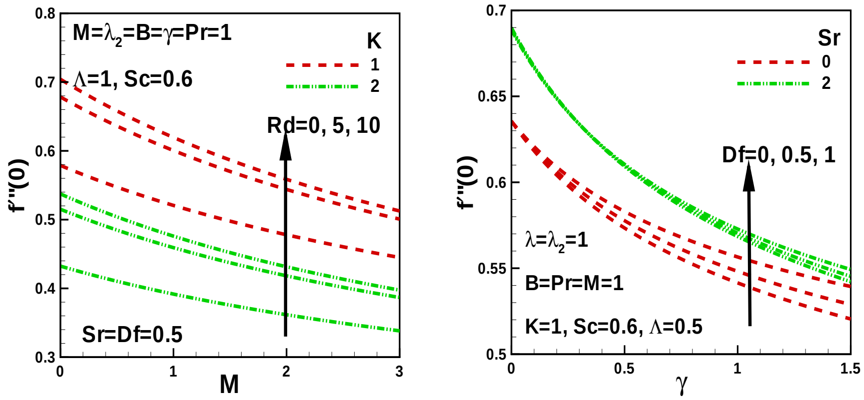

Fig. 10. Variation of skin friction with different parameters for assisting flows

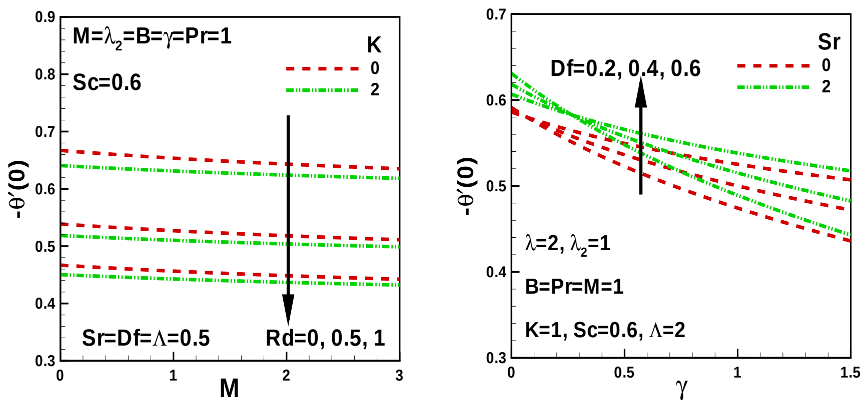

Fig. 11. Variation of Nusselt numbers with different parameters for assisting flows.

NOMENCLATURE

$U(x)$	Stretching velocity
$u(x)$	Potential flow velocity
C_w	Concentration at the surface
T_w	Wall temperature
T_∞	Ambient temperature
u, v	x- and y-components of velocity
g	Gravitational acceleration
ρ	fluid density

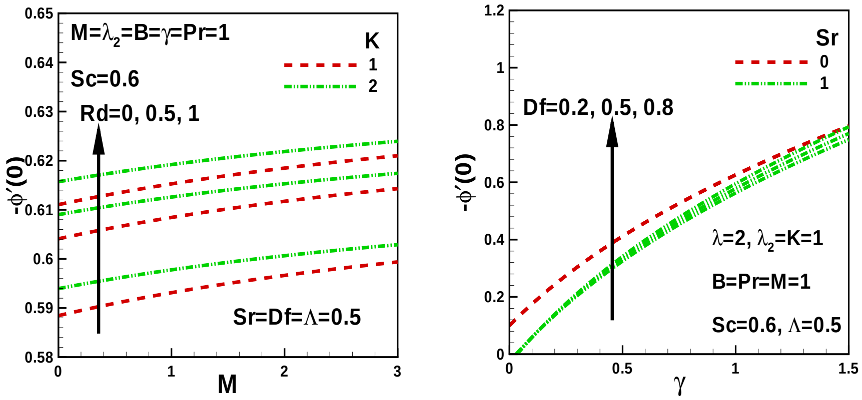

Fig. 12. Variation of Sherwood numbers with different parameters for assisting flows.

α	thermal diffusivity
D	mass diffusivity
β_T	coefficient of thermal expansion
β_C	coefficient of concentration expansion
σ	electrical conductivity
B_0	magnetic induction
q_r	radiative heat flux
K_1	dimensional of chemical reaction
N	Microrotation (or angular velocity)
j	microinertia density
γ^*	Spin gradient viscosity
k	Vortex viscosity (or the microrotation viscosity)
Cp	Specific heat at constant pressure
K	Material parameter
M	Magnetic parameter
λ	Dimensionless mixed convection parameter
λ_1	Stretching parameter
Λ	Buoyancy parameter
Pr	Prandtl number
Sc	Schmidt number
Rd	Radiation parameter
Df	Dufour number
Sr	Soret number
γ	dimensionless chemical reaction parameter

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