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EFFECT OF FREE CONVECTION ON THE FLOW AND HEAT TRANSFER BEHAVIOR OF EYRING POWELL FLUID OVER A STRATIFIED SHEET

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ABSTRACT. This study explores the significance of free convection in the transfer of heat within fluid flow, which holds significant relevance in the fields of hydrodynamics and engineering. An increase in free convection causes the fluid's velocity profile to rise and its temperature to decrease. The study focuses on the flow of Erying Powell fluid over a stratified sheet. The fluid is expected to have properties of viscosity, incompressibility, two-dimensionality, and laminar behavior. To analyze the flow, the governing system of partial differential equations (PDEs) that describes the flow is transformed into a system of nonlinear ordinary differential equations (ODEs) using an appropriate similarity transformation. These nonlinear equations are then solved using Maple software. The study examines the effects of various parameters on velocity and temperature profiles through graphical representations. Additionally, the flow's skin friction coefficient and Nusselt number are examined using graphs and tables. The findings conclusively demonstrate that increasing the free convection parameter leads to a significant reduction in skin friction, resulting in a notable enhancement in the velocity profile. This established relationship underscores the pivotal role of free convection in influencing the dynamic behavior of the Eyring Powell fluid flow. The findings from the study align well with the information available in the existing literature.

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1. INTRODUCTION

Fluid flow behavior in the vicinity of a stratified sheet is essential in various fields, such as environmental science, oceanography, and engineering applications that involve multi-phase or layered systems. Stratification in the sheet can have significant implications for transport phenomena, such as heat and mass transfer. Heat transfer is a complex mechanism by which internal energy is exchanged between different materials. It is an area of interest for researchers, physicists, engineers, and mathematicians. Investigating the impact of heat transfer in diverse materials and boundary layer flow over-stretching sheets is essential due to its vast range of applications in industrial units, biological phenomena, and engineering processes. These applications include metal extrusion, paper production, fermentation, and bubble absorption. ([1–6]).

The primary objective of studying heat transfer is to reduce heat loss during essential industrial processes. Heat loss can lead to increased production costs due to additional heat consumption and can potentially result in product failure. Heat transfer mechanisms occur through three modes: conduction, convection, and radiation [7].

A study investigated the flow of a power-law fluid in a square chamber driven by a moving lid. The chamber contained a discrete thermal source at the bottom surface. The researchers used the finite element technique to solve the governing equations for fluid flow (Navier-Stokes equations) and heat transfer. They performed a parametric simulation by varying the direction of the moving walls in three cases, as well as changing the Grashof and Reynolds numbers, power-law index (ranging from 0.6 to 1.4), and non-dimensional thermal source length (ranging from 0.25 to 1.00).

The results of the study were presented through isotherm and streamline plots, as well as the evaluation of various temperature parameters. The researchers found that the heat transfer increased as the size of the heat source decreased. The heat transfer performance of the smallest heat source was up to 76% higher than that of the largest heat source. When the sidewalls moved in opposite directions, the heat transmission efficiency was 132% and 52% higher compared to the cases where both walls moved downward and upward, respectively.

Additionally, the researchers observed changes in heat transfer performance with varying power-law index values. Specifically, the heat transfer performance increased by approximately 26.32% for one case and decreased by around 3.45% and 18.18% for the other two cases as the power-law index increased.

In a study conducted by Swaranjali et al [8] , two-dimensional simulations of laminar free convection were performed in a square enclosure filled with non-Newtonian fluid. A heated cylinder was present in the enclosure, which was both rotating and oscillating. The constant wall temperature created forced convection effects and natural convection. The researchers analyzed the dynamic behavior of flow patterns and temperature fields resulting from the rotation and oscillation of the hot cylinder within the square enclosure. They also compared the heat transmission characteristics between the rotating and oscillating cylinders. The primary focus of the study was to understand how the rotating and oscillating cylinder influences the thermo-fluid behavior of non-Newtonian fluids. The researchers aimed to quantify the heat transfer characteristics associated with these different cylinder movements. The study contributes to the understanding of the complex interactions between cylinder motion and heat transfer in non-Newtonian fluids within an enclosed space.

In a separate study, Iskandar et al [9] investigated the free convection and radiative flow of a non-Newtonian Reiner-Philippoff fluid near a permeable vertical plate. The fluid exhibited both shear thickening and shear thinning behaviors. The governing equations were formulated based on theoretical assumptions and then reduced to a set of ordinary differential equations (ODEs). The steady flow solutions were computed using Matlab software with the bvp4c solver. The researchers observed dual solutions, meaning there were two possible solutions for the flow behavior, and they analyzed the physical significance of these solutions using temporal stability analysis. The thermal development of the fluid was influenced by the presence of a magnetic field and suction effect. However, the high value of the radiation parameter had a negative impact on the thermal rate. The study also examined the impact of the Reiner-Philippoff fluid parameter on the skin friction coefficient and heat transfer rate. The results indicated that the maximum values of these parameters were observed for the shear-thickening fluid, followed by the Newtonian fluid, and finally the shear-thinning fluid.

In another study, Babar et al [10] studied the effect of energy transport and entropy generation in the mixed convective flow of a non-Newtonian fluid inside a square cavity subjected to a uniform, linear heat source on the bottom and side walls, while the top wall is adiabatic. The governing equations for the flow are formulated for a non-Newtonian bi-viscous fluid model using conservation laws. The Galerkin finite element method (FEM) is employed to solve the governing equations, and the penalty scheme is used to eliminate the pressure term.

Heatlines, based on the concept of Bejan, are utilized to identify the two-dimensional convective heat flow. They obtain numerical results for various flow control parameters, including bi-viscosity, Prandtl, Richardson, Reynolds, and Grashof numbers. The results reveal that the fluid velocity, energy transfer rate, and entropy generation increase with an increase in the bi-viscosity parameter. Conversely, the velocity and temperature decrease as the Hartmann number increases, while the entropy generation increases. Consequently, the minimum entropy generation is observed at specific values of the bi-viscosity, Hartmann, and Richardson numbers.

Muhammad et al [11]. presented a study on heat transfer in the mixed convective flow of an Eyring-Powell fluid over a stratified stretching sheet. The effects of different parameters on temperature and velocity profiles, as well as the skin friction coefficient and Nusselt number, are investigated and illustrated through graphs and numerical values. Hayat et al [12] investigated the impact of magnetohydrodynamics (MHD) and Joule heating on heat and mass transfer through a double stratified sheet. They analyzed the effects of different physio-chemical parameters on heat flow, both numerically and graphically. The findings indicated that an increase in temperature profiles resulted in a decrease in the mean absorption coefficient.

Hayat et al [13] conducted a study on the heat and fluid flow over a rotating disk, specifically analyzing the magnetohydrodynamics (MHD) flow of a nanofluid with double stratification effects. They employed analytical techniques and boundary layer approximation to calculate analytical solutions. The results revealed that the Prandtl number had a direct relationship with heat flow and an indirect relationship with the fluid temperature.

The analysis of fluid flow over a stretching or shrinking sheet is of significant interest to researchers, as it has diverse applications in various industries and processes. This includes industrial processes, condensation of liquid films, paper production, plastic wire and film drawing, crystal glowing, glass blowing, food industries, coatings, drug delivery systems, paints, ceramics, and manufacturing of rubber sheets, among others. Sakiadis [14, 15] was the first to investigate boundary layer flow on continuously moving flat and cylindrical surfaces. Dandapat and Gupta [16] examined heat transfer in viscoelastic fluids along a stretching sheet, analyzing boundary layer flow over continuous solid surfaces and contrasting them with moving surfaces of finite length. Erickson et al. [17] considered boundary layer flow with suction and injection, numerically solving the energy and diffusion equations. Crane

[18] discussed the flow generated by a stretching sheet. Anderson et al. [19] studied power-law fluid flow under the influence of magnetic forces over a linearly stretched surface. Hayat et al. [20] analyzed the flow of an Oldroyd-B fluid with heat generation/absorption effects. Sajid et al. [21] investigated fluid flow over a curved stretching surface using curvilinear coordinates. Overall, these studies contribute to the understanding of heat and fluid flow phenomena in various scenarios, offering insights into different flow patterns, boundary layer behavior, and the influence of factors such as magnetic forces, heat generation/absorption, and viscoelastic properties.

Researchers are currently interested in studying the effects of shear stress and fluid flow behavior of non-Newtonian fluids, driven by their industrial applications. Non-Newtonian fluids exhibit a diverse nature that cannot be described by a single reference equation, as they demonstrate a non-linear relationship between stress and strain. Among the various non-Newtonian fluid models, the Eyring-Powell fluid is preferred due to its foundation on a kinetic molecular model rather than empirical relations. The significance of different fluids in industries has prompted investigators to explore their usage and address challenges associated with heat transfer phenomena. Ibrahim et al.[22] investigated the magnetohydrodynamics (MHD) boundary layer flow of Eyring-Powell nanofluids, considering the Cattaneo-Christov heat flux model. The study aimed to establish a strong correlation between heat transfer, mass conservation, energy, and momentum. Eldabe et al. [23] focused on the magnetohydrodynamics effect on non-Newtonian, unsteady, and incompressible fluids under extreme stress conditions and external electric fields. They solved the mathematical model of Eyring-Powell fluid using computational tools and finite difference techniques for both first-order and second-order approximations. Malik et al. [24] investigated free convection flow over a stretching plate for MHD Eyring-Powell fluid. They numerically solved the problem using the shooting method and observed a significant decrease in heat and mass transfer with an increase in the Eyring-Powell material parameter. Ogunseye et al. [25] conducted a study on the thermal properties and characteristics of Eyring-Powell nanofluids, focusing on minimizing entropy loss. Prand et al. [26] recently discussed the boundary layer flow of Eyring-Powell fluid, considering thermal and physiochemical aspects of heat and fluid flow. Saleh et al. [27] conducted a comprehensive investigation on heat transfer and uniform, steady boundary layer flow of Eyring-Powell fluid with Newtonian heating. They employed the finite difference method to solve the nonlinear differential equations. In summary, researchers are actively exploring the behavior of non-Newtonian fluids, particularly the Eyring-Powell fluid model, to understand their shear stress and fluid flow characteristics. Studies focus on various aspects such as magnetohydrodynamics, heat transfer, entropy loss, and boundary layer flow, utilizing analytical and numerical methods to gain insights into these complex phenomena.

After conducting a thorough analysis of the aforementioned research, it is evident that there is a gap in the literature regarding the Eyring-Powell fluid model with free convection boundary layer flow over a stratified stretching sheet with heat generation/absorption. The model incorporates an energy equation to examine the temperature flow over the stratified stretching sheet. By employing the boundary layer approximation and implementing appropriate similarity transformations, the governing partial differential equations, along with the boundary conditions, are transformed to a dimensionless form. A numerical scheme based mainly on the shooting technique is used for the ODEs system. Graphs and tables are utilized to discuss the effects of various physical parameters. Additionally, numerical outcomes for the Nusselt number and skin friction are calculated. By the end of this article, we will be able to answer the following questions: What is the impact of the Eyring-Powell fluid parameter on the velocity profile? Does the free convection parameter have any effect on the fluid's velocity? How does the stratified sheet help increase the fluid's temperature? Is the impact of heat generation and heat absorption the same?

2. MATHEMATICAL FORMULATION

We have analyzed a flow situation where an incompressible Eyring Powell fluid moves steadily in a free convection manner in two dimensions. This fluid flow occurs through a stratified stretching sheet. The sheet is being stretched vertically along the x-axis at a constant speed of $\tilde{U}_w = ax$. The temperature of the fluid at the wall is denoted as $\tilde{T}_w$, while $\tilde{T}_\infty$ represents the temperature far away from the wall. The fluid's free stream velocity is zero 0. Additionally, we have considered the influence of heat generation absorption in the energy equation.

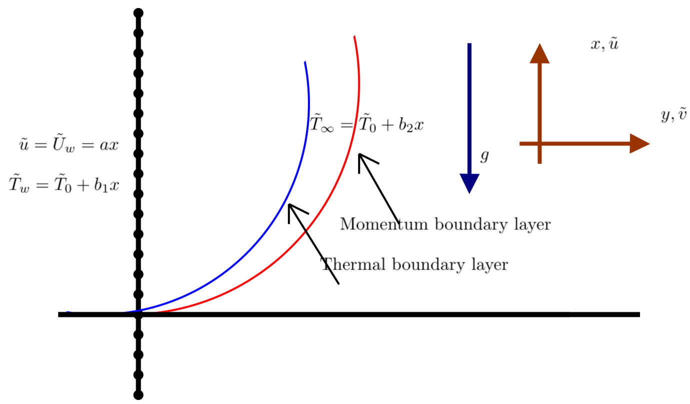

Fig. 1. Problem geometry

The effects of suction/injection and viscous dissipation have been disregarded in this particular problem. The application of the boundary layer approximation simplifies the continuity equation and momentum equation to the following form:

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{2.1}$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial x} + \tilde{v}\frac{\partial \tilde{u}}{\partial y} = \left(\nu + \frac{1}{\rho\beta C}\right)\frac{\partial^2 \tilde{u}}{\partial y^2} - \frac{1}{2\rho\beta C^3}\left(\frac{\partial \tilde{u}}{\partial y}\right)^2 \frac{\partial^2 \tilde{u}}{\partial y^2} + g\beta(\tilde{T} - \tilde{T}_\infty) \tag{2.2}$$

In the context of the boundary layer approximation, the energy equation can be expressed as follows, where β and $\tilde{C}$ are material parameters specific to the Eyring Powell fluid, μ represents the kinematic viscosity of the fluid, and g represents the gravitational acceleration.

$$\tilde{u}\frac{\partial \tilde{T}}{\partial x} + \tilde{v}\frac{\partial \tilde{T}}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 \tilde{T}}{\partial y^2} + \frac{Q}{\rho C_p}(\tilde{T} - \tilde{T}_\infty) \tag{2.3}$$

k represents thermal conductivity, C_p denotes the specific heat at constant pressure, and Q represents the coefficient of heat generation/absorption.In this particular flow problem, the boundary conditions are described as follows:

$$\begin{array}{llll} \tilde{u} = \tilde{U}_w = ax, & \tilde{v} = 0, & \tilde{T} = \tilde{T}_w = \tilde{T}_0 + b_1 x & \text{at } y = 0 \\ \tilde{u} \to 0 \Rightarrow & \tilde{T} \to \tilde{T}_\infty = \tilde{T}_0 + b_2 x, & y \to \infty & \text{as } y \to \infty \end{array} \tag{2.4}$$

In the given problem, the horizontal and vertical components of fluid velocity are represented by $\tilde{u}$ and $\tilde{v}$ respectively. The reference temperature of the stretching sheet is denoted by T_0, and positive dimensional constants b_1 and b_2 are introduced. By employing a similarity transformation, the system of partial differential equations (PDEs) along with their boundary conditions is transformed into a set of ordinary differential equations (ODEs). Notably, the continuity equation is automatically satisfied, while the momentum and energy equations are reduced to non-dimensional ODEs. The following stream function is defined

$$\tilde{u} = \frac{\partial \varphi}{\partial y}, \tilde{v} = -\frac{\partial \varphi}{\partial x} \tag{2.5}$$

now using the recommended similarity transformation as

$$\begin{aligned} \varphi &= \sqrt{a\nu}xf(\eta), \eta = \sqrt{\frac{a}{\nu}}y, \\ \theta(\eta) &= \frac{\tilde{T} - \tilde{T}_\infty}{\tilde{T}_w - \tilde{T}_0} \Rightarrow \tilde{T} = (\tilde{T}_w - \tilde{T}_0)(\theta)(\eta) + \tilde{T}_\infty \end{aligned} \tag{2.6}$$

where φ stream function, $f(\eta)$ is dimensionless stream function, and η presents similarity variable, also

$$\tilde{T} = b_1 x\theta(\eta) \text{ and } \tilde{T}_\infty = \tilde{T}_0 + b_2 x \tag{2.7}$$

So, the final dimensionless form of the present problem's mathematical model is

$$(1+\varepsilon)f''' - \varepsilon\sigma f''^2 f''' - f'^2 + ff'' + \lambda\theta = 0 \tag{2.8}$$

$$\theta'' + Pr\theta' - Pre_1 f' + Pr\gamma\theta = 0 \tag{2.9}$$

with boundary conditions

$$\begin{aligned} &f(0) = 0, f'(0) = 1, \theta(0) = 1 - e_1, \text{ at } \eta = 0, \\ &f(\eta) = 0, \theta(\infty) = 0, \text{ as } \eta \to \infty \end{aligned} \tag{2.10}$$

different parameters used in the above equations have the following formulations:

$$\varepsilon = \frac{1}{\rho v\beta C}, \sigma = \frac{aUw^2}{2C^2 v}, \lambda = \frac{Gr_c}{Re_x^2} = \frac{g\beta b_1}{a^2}, Pr = \frac{\mu C_p}{K}, e_1 = \frac{b_2}{b_1}, \gamma = \frac{Q}{\rho C_p a} \tag{2.11}$$

skin friction coefficient and Nusselt number is given as

$$C_{fx} = \frac{tw}{\rho U_w^2}, Nu_x = \frac{xq_w}{K(\tilde{T}_w - \tilde{T}_\infty)} \tag{2.12}$$

t_w is shear stress and $qw = -k\left(\frac{\partial \tilde{T}}{\partial y}\right)_{y=0}$ is heat flux at the surface. The dimensionless form of skin friction coefficient and Nusselt number is

$$\sqrt{Re_x C_f} = (1+\varepsilon)f''(0) - \frac{1}{3}\varepsilon\sigma f''^3(0), Nu_x/Re_x^{1/2} = -\theta'(0) \quad (2.13)$$

where Re_x represents local Reynolds number and is defined as

$$Re_x = \frac{x\tilde{U}_w}{\tilde{\nu}} = \frac{ax^2}{\tilde{\nu}} \quad (2.14)$$

3. METHOD OF SOLUTION

The transformed equation (2.8)-(2.9) with the boundary conditions (2.10) are solved numerically using the Maple script.

```
restart:
Eq[1] := (1 + epsilon)*diff(f(x), x $ 3) - epsilon*sigma*diff(f(x),
x $ 2)^2*diff(f(x), x $ 3) - diff(f(x), x)^2 + f(x)*diff(f(x),
x $ 2) + lambda*theta(x) = 0;
Eq[2] := diff(theta(x), x $ 2) + Pr*diff(theta(x), x)*f(x) -
Pr*theta(x)*diff(f(x), x) + Pr*gamma*theta(x) = 0;
BCs := [f(0), D(f)(0) - 1, D(f)(10), theta(0) - 1 + e[1], theta(10)];
pars := {Pr = 0.7, epsilon = 0.9, gamma = 0.1, lambda = 0.4,
sigma = 0.1, e[1] = 0.1};
for i to 2 do
eq[i] := subs(pars, Eq[i]);
end do;
eqs := eq[1], eq[2];
vars := f(x), theta(x);
bcs := op(subs(pars, BCs));
sol := dsolve({bcs, eqs}, {vars}, type = numeric);
```

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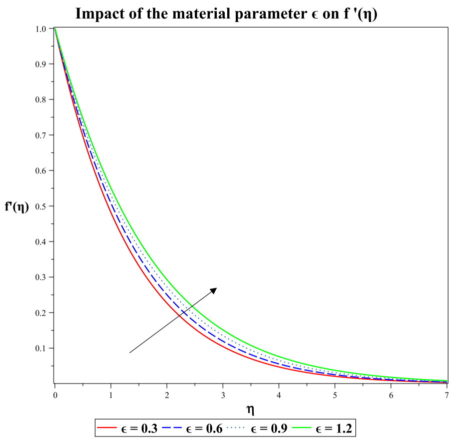
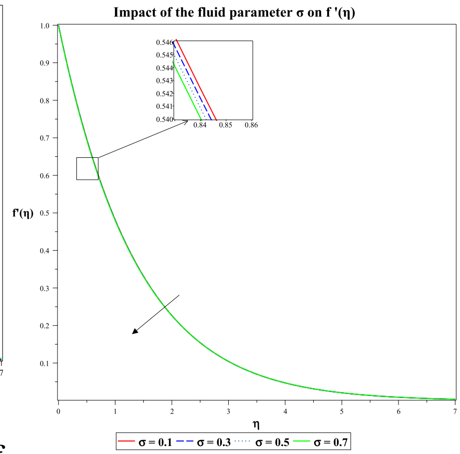
4. RESULTS AND DISCUSSION

Table 1. Impact of different parameters on Skin friction and Nusselt number

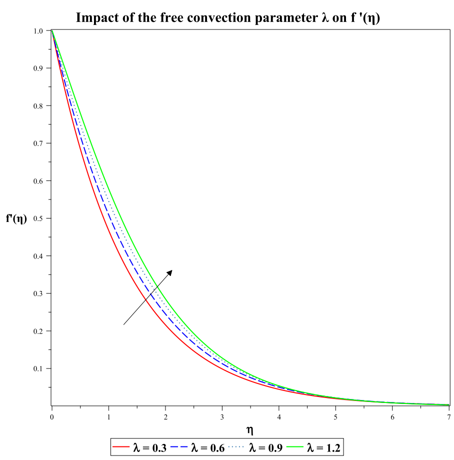
ε	σ	λ	Pr	e_1	γ	$(1+\varepsilon)f''(0)-\frac{1}{3}\varepsilon\sigma f''^3(0)$	$-\theta'(0)$
0.1	0.1	0.1	0.7	0.3	0.1	0.92535475	0.53032533
0.3	0.1	0.1	0.7	0.3	0.1	0.93901920	0.54484635
0.5	0.1	0.1	0.7	0.3	0.1	0.94809783	0.55661647
0.7	0.1	0.1	0.7	0.3	0.1	0.95447851	0.56635075
0.1	0.3	0.1	0.7	0.3	0.1	0.93723513	0.53004638
0.1	0.5	0.1	0.7	0.3	0.1	0.94943506	0.52978432
0.1	0.7	0.1	0.7	0.3	0.1	0.96197189	0.52949561
0.1	0.1	0.3	0.7	0.3	0.1	0.78328365	0.54658446
0.1	0.1	0.5	0.7	0.3	0.1	0.66082643	0.55912457
0.1	0.1	0.7	0.7	0.3	0.1	0.55351723	0.56987441
0.1	0.1	0.1	0.5	0.3	0.1	0.50870784	0.46831345
0.1	0.1	0.1	0.3	0.3	0.1	0.44771213	0.34889643
0.1	0.1	0.1	0.1	0.3	0.1	0.35455861	0.19127853
0.1	0.1	0.1	0.7	0.5	0.1	0.66082943	0.39941139
0.1	0.1	0.1	0.7	0.7	0.1	0.78328012	0.23426743
0.1	0.1	0.1	0.7	0.9	0.1	0.92536743	0.07576734
0.1	0.1	0.1	0.7	0.3	0.3	0.52100056	0.49324633
0.1	0.1	0.1	0.7	0.3	0.5	0.47344056	0.39522234
0.1	0.1	0.1	0.7	0.3	0.7	0.40548567	0.25874253



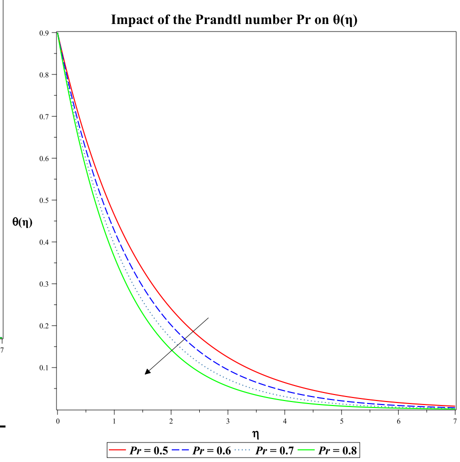
(A) Impact of the material parameter ε on $f'(\eta)$ for $\lambda = 0.4, \sigma = 0.1, Pr = 0.7, e_1 = 0.1, \gamma = 0.1$



(B) Impact of the fluid parameter σ on $f'(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, e_1 = 0.1, \gamma = 0.1$

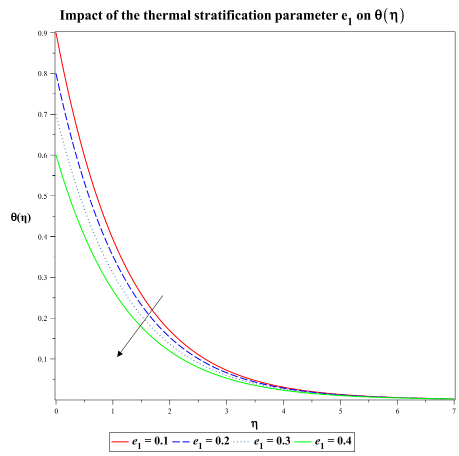


(C) Impact of the free convention parameter λ on $f'(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, e_1 = 0.1,$ $\sigma = 0.1$

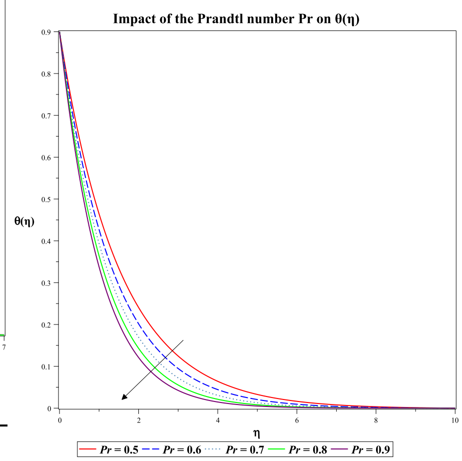


(D) Impact of the thermal stratification parameter e_1 on $f'(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, \sigma = 0.1, \gamma = 0.1$

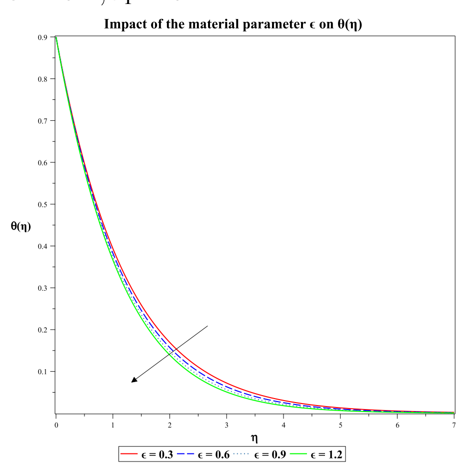
FIGURE 1



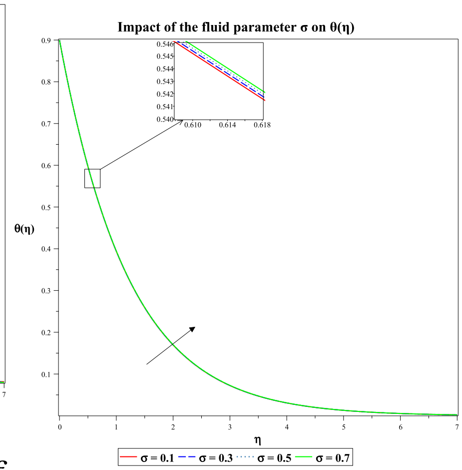
(A) Impact of the heat generation/absorption parameter γ on $f'(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, \sigma = 0.1, e_1 = 0.1$



(B) Impact of the Prandtl number Pr on $f'(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, e_1 = 0.1, \sigma = 0.1, \gamma = 0.1$

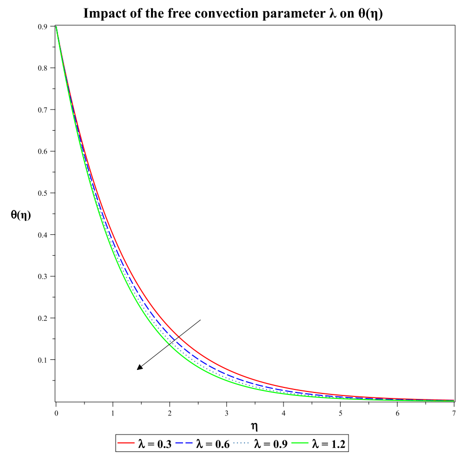


(C) Impact of the material parameter ε on $\theta(\eta)$ for $\lambda = 0.4, \sigma = 0.1, Pr = 0.7, e_1 = 0.1, \gamma = 0.1$

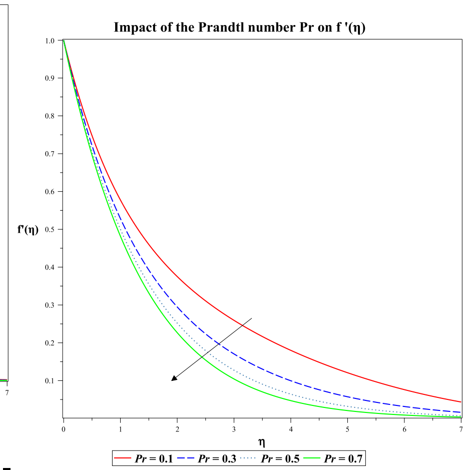


(D) Impact of the fluid parameter σ on $\theta(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, e_1 = 0.1, \gamma = 0.1$

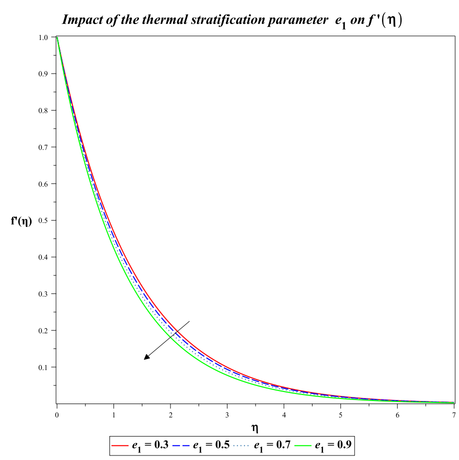
FIGURE 2



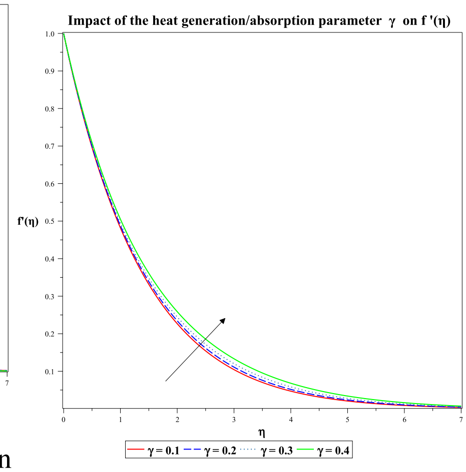
(A) Impact of the free convention parameter λ on $\theta(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, e_1 = 0.1, \sigma = 0.1$



(B) Impact of the Prandtl number Pr on $\theta(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, e_1 = 0.1, \sigma = 0.1, \gamma = 0.1$

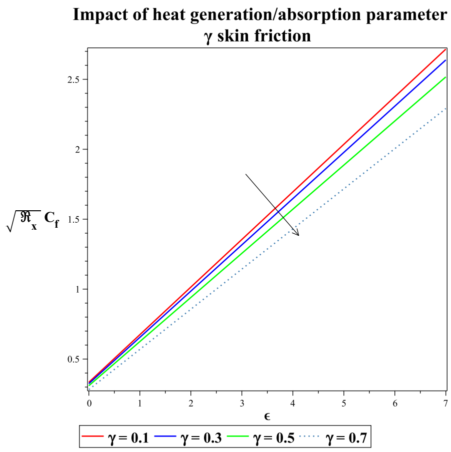


(C) Impact of the thermal stratification parameter e_1 on $\theta(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, \sigma = 0.1, \gamma = 0.1$

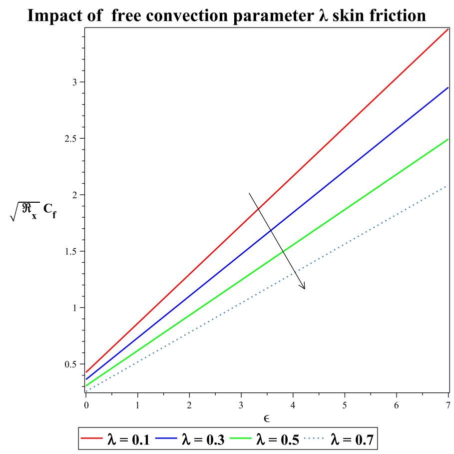


(D) Impact of the heat generation/absorption parameter γ on $\theta(\eta)$ for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, \sigma = 0.1, e_1 = 0.1$

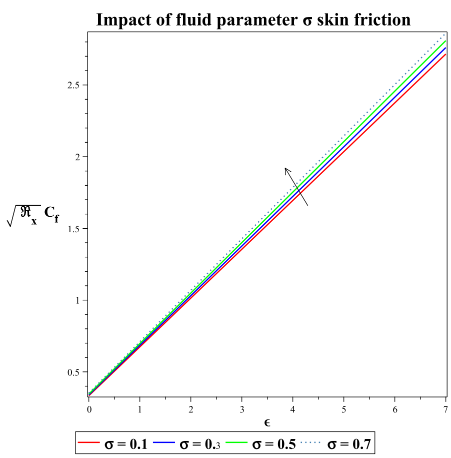
FIGURE 3



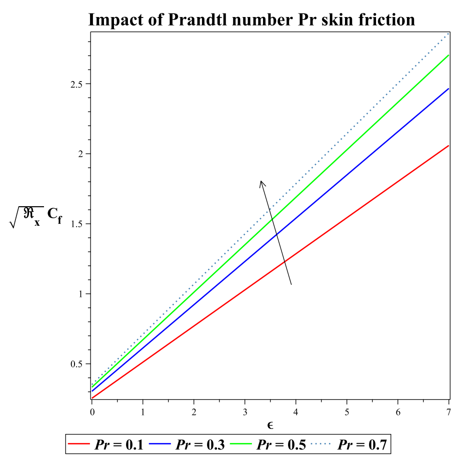
(A) Impact of the heat generation/absorption parameter γ on skin friction for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, \sigma = 0.1, e_1 = 0.1$



(B) Impact of the free convention parameter λ on skin friction for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, e_1 = 0.1, \sigma = 0.1$



(C) Impact of the fluid parameter σ on skin friction for $\lambda = 0.4, \varepsilon = 0.3, Pr = 0.7, e_1 = 0.1, \gamma = 0.1$



(D) Impact of the Prandtl number Pr on skin friction for $\lambda = 0.4, \varepsilon = 0.3, e_1 = 0.1, \sigma = 0.1, \gamma = 0.1$

FIGURE 4

The transformed governing equation considers several significant parameters in the following sequence: $\varepsilon, \sigma, \lambda$, representing the material parameter, fluid parameter, and free convection parameter, respectively. Additionally, Pr, e_1, and γ denote the Prandtl number, thermal stratification parameter, and heat generation/absorption parameter, respectively. The study investigates the influence of these physical parameters on the distribution of velocity and temperature. Graphical figures were utilized to elucidate the meaning and significance of each physical parameter in this research. The detailed results of this study can be enumerated as follows.

4.1 Velocity Distribution: The velocity distribution, also referred to as the velocity profile, plays a crucial role in determining the rate of fluid flow within a particular material. It is widely recognized that the behavior of the fluid flow rate is entirely governed by the characteristics of the velocity distribution. Thus, studying the variations in the velocity distribution is essential for understanding the changes in the fluid flow rate. Figure (1a) illustrates how the velocity profile of the fluid increases as the material parameter ε increases. This increment in the velocity profile can be attributed to a decrease in flow resistance. The material parameter ε is inversely related to the viscosity of the non-Newtonian fluid, meaning that as ε increases, the viscosity decreases. Consequently, the reduced viscosity leads to a lower flow resistance, resulting in an increase in the velocity profile of the fluid. Figure (1b) demonstrates the impact of the material parameter σ on the velocity profile of the fluid. It reveals that as σ increases, the velocity profile decreases. Figure (1c) portrays the influence of the free convection parameter λ. It reveals that an increase in the free convection parameter leads to a rise in the velocity of the fluid. This increase in velocity is attributed to the thermal buoyancy force. As the buoyancy force becomes stronger, it causes an enlargement of the momentum boundary layer thickness. Figure (1d) illustrates the impact of the thermal stratification parameter e_1 on the velocity profile. The graph indicates that an increase in the thermal stratification parameter leads to a reduction in the fluid motion. This effect can be attributed to a decrease in the convective potential between the surface of the sheet and the ambient temperature. Figure (2a) displays the influence of the heat generation/absorption parameter γ on the velocity profile. It indicates that an increase in γ intensifies the fluid motion over the stretching sheet. Figure (2b) depicts the impact of the Prandtl number Pr on the fluid motion. It is observed that an increase in the Prandtl number leads to a reduction in the velocity profile.

4.2 Temperature Distribution: The variation in fluid temperature plays a crucial role in understanding fluid behavior, and it is essential to examine the influence of fluid parameters on the temperature distribution over a stratified sheet. Figure (2c) illustrates the impact of the material parameter ε on the fluid temperature. As mentioned previously, an increase in ε leads to a decrease in the fluid viscosity, which consequently contributes to a reduction in temperature. Figure (2d) demonstrates the impact of the fluid parameter σ on the temperature distribution of the fluid. The graph clearly indicates that as σ increases, the temperature of the fluid also increases. Figure (3a) illustrates the influence of the free convection parameter λ on the fluid temperature. It is evident from the graph that an increase in λ leads to a reduction in the thermal buoyancy force, resulting in a decrease in the fluid temperature. In Figure (3b), the effect of the Prandtl number Pr on the temperature profile is displayed. The Prandtl number is inversely related to the thermal conductivity k and indicates the energy diffusion capability of the fluid. A higher Prandtl number implies weaker energy diffusion. Therefore, as the Prandtl number increases, the fluid temperature decreases, leading to a thinner thermal boundary layer. The temperature profile of the moving fluid decreases with an increasing stratification parameter, as shown in Figure 3c. The stratification parameter e_1 affects the temperature difference between the ambient fluid and the fluid on the surface. As e_1 increases, the temperature distribution decreases. Figure (3d) presents the effect of the heat generation and absorption parameter γ on the temperature profile. Increasing γ leads to an elevation in the fluid temperature.
4.3 Skin friction and Nusselt number profile: The skin friction coefficient is a dimensionless measure of the resistance to fluid flow over a solid surface, often referred to as drag. It plays a significant role in aerodynamics and fluid dynamics, indicating the level of drag experienced by an object moving through a fluid. A higher skin friction coefficient implies greater drag and increased resistance to flow. On the other hand, the Nusselt number quantifies the enhancement of heat transfer resulting from convective effects compared to conductive heat transfer. A higher Nusselt number signifies more efficient convective heat transfer. Figure (4a) demonstrates the influence of the heat generation and absorption parameter γ on the skin friction coefficient. As γ increases, the skin friction coefficient decreases, as evidenced in Figure (2a), where the velocity profile increases with higher γ. Figure (4b) showcases the impact of the free convection parameter λ on skin friction, indicating a

decrease in the skin friction coefficient for increasing values of λ. This observation aligns with the increased velocity resulting from reduced resistance in fluid motion due to λ. In Figure (4c), and Figure (4d) an increase in the skin friction coefficient is illustrated for increasing values of the fluid parameter σ, Prandtl number Pr, and thermal stratification parameter e_1, respectively.

Table 1 presents the effects of different fluid parameters on both the skin friction coefficient and the Nusselt number. It provides a comprehensive overview of the impact of these parameters on these important measures.

5. CONCLUSION

In conclusion, this study highlights the significance of free convection on boundary layer flow of an Eyring Powell fluid over a stratified sheet. The findings show that changes in various fluid parameters significantly affect the fluid velocity and temperature profiles. The results presented in Table 1 provide valuable insights into the influence of specific parameters. Overall, this study helps to enhance our understanding of the behavior of Eyring Powell fluids in free convection boundary layer flows over stratified sheets. Based on the obtained findings, it can be concluded that.

(1) When the free convection parameter λ is heightened, the fluid velocity increases while the temperature decreases.
(2) The augmentation of the Eyring-Powell fluid material parameter ε leads to an elevation in both fluid velocity and temperature as it flows over a stretching sheet.
(3) The magnitude of the thermal stratification parameter $e_1$aids in controlling the fluid's velocity and temperature.
(4) By increasing the values of the heat generation parameter γ, both the velocity and temperature profiles experience an increment.
(5) An escalation in the Eyring-Powell material parameter σ results in an increase in temperature but a decrease in the velocity profile.
(6) An increase in the Prandtl number Pr induces a reduction in both fluid velocity and temperature.

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