

**THERMAL DIFFUSION, RADIATION AND INCLINED
MAGNETIC FIELD EFFECTS ON OSCILLATORY FLOW
IN AN ASYMMETRIC CHANNEL IN PRESENCE OF
HEAT SOURCE AND CHEMICAL REACTION**

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ABSTRACT. Thermal diffusion and radiation effects on the MHD oscillatory flow in presence of chemical reaction and heat source are analyzed in the present investigation. An inclined magnetic field of uniform strength is applied to the channel. An oscillatory pressure gradient across the ends of the channel makes the flow unsteady. Regular perturbation method is adopted to solve the governing equations analytically. The influence of various apposite parameters on the flow, temperature and concentration characteristics are explained numerically and illustrated graphically. The conclusion given is based on these plots. .

Keywords and phrases: Micropolar fluid, Inclined Magnetic field, Non-uniform channel, Heat generation, Radiation, Thermal diffusion.

2010 Mathematical Subject Classification: 76-XX, 76Zxx

1. INTRODUCTION

The phenomenon of heat and mass transfer by inclined MHD oscillatory flow has received considerable attention in the last few decades because of its wide range of applications in many engineering and scientific fields such as power engineering, petroleum production, food preservation, polymer technology and electro-static precipitation. A significant role is played by unsteady oscillatory free convective flows in chemical engineering, in turbo machinery and in aerospace technology. Such flows arise due to either unsteady motion or boundary temperature. Moreover, unsteadiness may also be due to oscillatory free stream velocity or temperature. The main reason behind the MHD oscillatory flow in a channel is that, if heat source is associated to a heat sink via a fluid and

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the fluid is oscillated, the convective motion will bring about sharp spikes in the velocity profile which in turn will enhance the heat transport over pure conduction due to both radial and axial gradients. This research has many applications including removing heat from outer space modules, reactors, closed cabins, NASAs long-term manned and unmanned missions. Many parameters can be varied to maximize the convective heat transport, such as pulse amplitude and frequency, pipe radius and length, as well as the transporting fluid and the geometry of the transport zone. It has been established both experimentally and analytically that large quantities of heat is transported axially provided the fluid is oscillated at high frequency with large tidal displacement. It is also established that under laminar conditions, the radial variation in velocity and temperature produces an effective axial transport of heat, which is several orders of magnitude larger than the absence of oscillations. Many researchers have investigated the MHD oscillatory flows under different conditions. Pal and Biswas [1] analyzed the magnetohydrodynamics oscillatory flow on convective radiative heat and mass transfer of micropolar fluid in presence of chemical reaction by using the method of perturbation. Mishra and Adhikary [2] studied the effect of chemical reaction on heat and mass transfer of MHD oscillatory channel flow. Moreover, Muthuray and Srinivas [3] studied the heat transfer in MHD oscillatory flow in an asymmetric wavy channel with porous medium. Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source has been studied by Das et al. [4]. Makinde and Mhone [5] studied the heat transfer to MHD oscillatory flow in a channel filled with a porous medium. They investigated the combined effect of a transverse magnetic field and radiative heat transfer to MHD oscillatory flow. Adensya and Makinde [6] investigated the effect of slip on the hydromagnetic pulsatile flow through a porous channel. The effects of dissipation, heat sink and chemical reaction on MHD free convecting oscillatory flow past a porous plate has been discussed by Vijaya and Viswanadh [7]. An oscillatory MHD convective flow in a vertical channel filled with porous medium in presence of Hall and thermal radiation effects have been analyzed by Das et al. [37].

The study of MHD flow deals with the interface of electrically conducting fluids and magnetic fields. If a magnetic field is applied to a moving and electrically conducting fluid, it will induce electric as well as magnetic fields. These fields interact and hence produce

a body force per unit volume, known as the Lorentz force, which has a significant impact to oppose the movement of the liquid [8]. Sandeep and Sugunamma [9] examined the inclined magnetic field and radiation effects on unsteady hydromagnetic free convection flow past a vertical plate in a porous medium. Recently, Raju et al. [10] studied the effects of radiation, inclined magnetic field and cross diffusion on the flow over a stretching surface. Radiative heat transfer and thermal conductivity effects on an electrically conducting fluid over a stretching surface in presence of a uniform inclined magnetic field in a porous medium has been observed by Salawa and Dada [11].

The investigation of heat transfer with chemical reaction is of great importance to engineers and scientists due to its enormous applications in science and engineering such as designing of chemical processing equipments, formation and dispersion of fog, food processing and cooling of tower, flow in a desert cooler etc. Chamkha et al.[12] investigated the Soret, Dufour and chemical reaction effects on heat and mass transfer in stagnation point flow of a polar fluid towards a stretching surface in porous media. Abd El-Aziz [13] studied the effect of time dependent chemical reaction on stagnation point flow and heat transfer over a stretching sheet in a nanofluid. Das et al.[14] studied the effect of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Investigations have been made on the effects of chemical reaction on MHD fluid flows under different conditions [15]-[23].

In many transport processes, flow is driven due to differences in densities which is caused by temperature gradient. Mass fluxes are caused because of temperature gradients and it is called Soret effect (Thermal Diffusion Effect). The Soret effect is of great importance in isotope separation and mixture of light and medium molecular weight gases. The effect of thermal diffusion on two dimensional viscous flow through porous channel of weak permeability with slowly expanding or contracting walls has been studied by Srinivas et al.[24]. Hayat et al.[25] investigated the effects of thermal-diffusion and diffusion-thermo on axi-symmetric flow of a second grade fluid. Moreover, thermal diffusion effects are of significant importance in engineering applications like propulsion systems, reactors etc. By applying Chandrasekhar discrete transfer method (DTM), Talukdar et al.[26] investigated the radiative-convective flow in a porous

channel. Krupa Lakshmi et al. [27] numerically studied the two-phase flow, heat and mass transfer of a dusty fluid with Soret and Dufour's effect over a stretching surface in the presence of a magnetic field, thermal radiation and chemical reaction. Unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate with constant heat flux has been investigated by Ogulu and Makinde [28]. Makinde and Moitsheki [29] adopted ADM (Adomian decomposition method) and VIM (Hes variational iteration method) to compute a nonperturbative solution for thermal radiation effect on natural convection boundary layer flow past a vertical plate embedded in a saturated porous medium. Olanrewaju and Makinde [30] examined the effects of thermal diffusion and diffusion thermo on chemically reacting MHD boundary layer flow of heat and mass transfer past a moving vertical plate with suction/injection. Mohamed et al.[31] studied the chemical reaction and thermal radiation effects on heat and mass transfer in MHD micropolar fluid over a vertical moving porous plate with heat generation. Hardy et al.[32] studied the radiation effects on viscous flow of a nanofluid and heat transfer over non-linearly stretching sheet. Studies related to the combined effects of heat and mass transfer are of considerable interest in physiological flows and industrial process.

All the investigations mentioned above ignored the thermal diffusion, heat source and inclined magnetic field effects on an oscillatory flow. Hence, the main objective of the present study is to analyze the combined effect of chemical reaction, thermal diffusion, thermal radiation, heat source and inclined magnetic field on the oscillatory flow in an asymmetric porous and wavy channel. The thermal diffusion, inclined magnetic field and chemical reaction effects on the oscillatory flow are of particular prominence in the present study. Analytical expressions of axial velocity, temperature, concentration and rate of heat and mass transfer are obtained. The graphical results are analyzed and illustrated quantitatively.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The unsteady, incompressible, viscous, electrically conducting and chemically reacting optically thin fluid in an asymmetric wavy channel in presence of an inclined magnetic field, thermal diffusion and heat source is considered. The geometry of the problem is shown

in Fig.(1) and is expressed mathematically as

$$\begin{aligned} H_1 &= d_1 + a_1 \cos\left(\frac{2\pi x}{\lambda}\right) \\ H_2 &= -d_1 - b_1 \cos\left(\frac{2\pi x}{\lambda} + \phi\right) \end{aligned} \quad (1)$$

Where a_1 and b_1 are the amplitudes of wave, $d_1 + d_2$ is the width of the channel, λ is the wave length. The phase difference ϕ varies in the range $0 \leq \phi \leq \pi$ and $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ with waves in phase, and further a_1, b_1, d_1, d_2 and ϕ satisfies the condition

$$a_1^2 + b_1^2 + 2a_1b_1 \cos \phi \leq (d_1 + d_2)^2 \quad (2)$$

The walls of the channel are maintained at temperatures T_1 and T_2 respectively. An oscillatory pressure gradient across the ends of the channel and asymmetric wavy channel makes the fluid oscillatory. An inclined magnetic field of uniform strength B_0 is applied at an angle of θ to the y-direction.

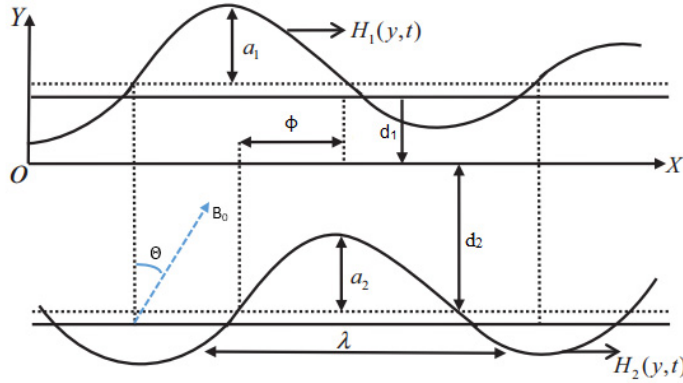


FIGURE 1. Geometry of the problem

The momentum, heat transfer and mass transfer equations are considered in the form

$$\begin{aligned} \frac{\partial u}{\partial t} = & -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_2) + g\beta_C(C - C_2) - \frac{\nu}{k^*} u \\ & - \frac{\sigma}{\rho} \cos^2 \theta u \end{aligned} \quad (3)$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q_H}{\rho c_p} (T - T_2) \quad (4)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - K_r^* (C - C_2) + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (5)$$

Where u and v are the velocity components in the x and y directions, respectively, ρ is the density of the fluid, p is the pressure, μ is the viscosity constant of the classical fluid dynamics, σ is the electrical conductivity, B_0 is the strength of the magnetic field, θ is inclination angle of the magnetic field parameter, g is the acceleration due to gravity, T is temperature, C is the concentration, c_p is specific heat at constant pressure, q_r is the radiation heat flux, K is thermal conductivity, Q_H is heat generation, D is the coefficient of mass diffusivity and K_T is the thermal diffusion ratio.

The corresponding boundary conditions are given below

$$u = 0, \quad T = T_1, \quad C = C_1 \quad \text{on } y = H_1$$

$$u = 0, \quad T = T_2, \quad C = C_2 \quad \text{on } y = H_2 \quad (6)$$

The expression for radiative heat flux (see Ref.[33]) is given as

$$\frac{\partial q_r}{\partial y} = -4\alpha^2(T - T_2) \quad (7)$$

where $\alpha^2 = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$, $K_{\lambda w}$ is the absorption coefficient and $e_{b\lambda}$ is Plank's Constant [34]

Now we introduce the non-dimensional variables and parameters as follows

$$\begin{aligned} x' &= \frac{x}{\lambda}, y' = \frac{y}{d}, u' = \frac{u}{U}, t' = \frac{Ut}{d}, p' = \frac{pd^2}{U\lambda\mu}, M = \sqrt{\frac{\sigma}{\mu}} dB_0, \\ R_e &= \frac{\rho Ud}{\mu}, a = \frac{a_1}{d_1}, b = \frac{b_1}{d_1}, h_1 = \frac{H_1}{d_1}, h_2 = \frac{H_2}{d_2}, d = \frac{d_2}{d_1}, \\ \theta &= \frac{T - T_2}{T_1 - T_2}, \Phi = \frac{C - C_2}{C_1 - C_2}, P_e = \frac{Ud\rho c_p}{K}, S_r = \frac{DK_T(T_1 - T_2)}{UT_md(C_1 - C_2)}, \\ K_1 &= \frac{K^*}{d^2}, S_c = \frac{Ud}{D}, Q = \frac{Q_H d^2}{K}, R = \frac{4\alpha^2 d^2}{K}, \\ G_r &= \frac{g\beta_T(T_1 - T_2)d^2}{\nu U}, G_c = \frac{g\beta_C(C_1 - C_2)d^2}{\nu U} \quad (8) \end{aligned}$$

Where M is the Hartman number, R_e is the Reynolds number, K_1 is the porosity parameter, P_e is the Peclet number, S_c is the Schmidt number, S_r is the Soret number, Q is the heat generation parameter and N_r is the thermal radiation parameter, T_m is the

mean temperature, G_r is Thermal Grashof number and G_c is Solutal Grashof number.

Using the non-dimensional variables and parameters from Eq.(8) in Eqs. (3)-(5) and neglecting the prime symbols, we get the modified equations as

$$R_e \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_c \Phi - \left(M^2 \cos^2 \Theta + \frac{1}{K_1} \right) u \quad (9)$$

$$P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (R + Q) \theta \quad (10)$$

$$\frac{\partial \Phi}{\partial t} = \frac{1}{S_c} \frac{\partial^2 \Phi}{\partial y^2} - K_r \Phi + S_r \frac{\partial^2 \theta}{\partial y^2} \quad (11)$$

The corresponding boundary conditions in the non-dimensional form are expressed as

$$\begin{aligned} u = 0, \quad \theta = 1, \quad \Phi = 1 \quad \text{on } y = h_1 \\ u = 0, \quad \theta = 0, \quad \Phi = 0 \quad \text{on } y = h_2 \end{aligned} \quad (12)$$

3. ANALYTICAL APPROXIMATE SOLUTIONS

The dimensionless form of the system of partial differential equations (9)-(11) can be obtained by considering the pressure gradient, velocity, temperature and concentration for purely oscillatory flow as (see Refs. [35, 36, 37])

$$\begin{aligned} \frac{\partial p}{\partial x} &= \lambda e^{i\omega t} \\ u(y, t) &= u_0(y) e^{i\omega t} \\ \theta(y, t) &= \theta_0(y) e^{i\omega t} \\ \Phi(y, t) &= \Phi_0(y) e^{i\omega t} \end{aligned} \quad (13)$$

By substituting Eq.(13) into Eqs.(9)-(11), we obtain the following set of equations

$$\frac{\partial^2 u_0}{\partial y^2} - n^2 u_0 = -\lambda - G_r \theta_0 - G_c \Phi_0 \quad (14)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + m^2 \theta_0 = 0 \quad (15)$$

$$\frac{\partial^2 \Phi_0}{\partial y^2} - l^2 \Phi_0 + S_c S_r \frac{\partial^2 \theta_0}{\partial y^2} = 0 \quad (16)$$

where $n^2 = M^2 \cos^2 \Theta + \frac{1}{K_1} + \iota \omega R_e$, $m^2 = Q + R - \iota \omega P_e$ and $l^2 = S_c(K_r + \iota \omega)$

The boundary conditions becomes

$$\begin{aligned} u_0 &= 0, \quad \theta_0 = 1, \quad \Phi_0 = 1 \text{ on } y = h_1 \\ u_0 &= 0, \quad \theta_0 = 0, \quad \Phi_0 = 0 \text{ on } y = h_2 \end{aligned} \quad (17)$$

Upon solving Eqs(14)-(16) by using the boundary conditions from Eq.(17), we get the expressions for velocity, temperature and concentration as given below

$$u(y, t) = \left\{ \frac{\lambda}{n^2} + (C_5 e^{ny} + C_6 e^{-ny}) - \frac{(C_3 e^{ly} + C_4 e^{-ly}) G_c}{A_2} + \frac{(C_1 \cos(my) + C_2 \sin(my)) A_7}{A_1} \right\} e^{\iota \omega t} \quad (18)$$

$$\theta(y, t) = (C_1 \cos(my) + C_2 \sin(my)) e^{\iota \omega t} \quad (19)$$

$$\begin{aligned} \Phi(y, t) &= (C_3 e^{ly} + C_4 e^{-ly}) e^{\iota \omega t} \\ &\quad - \frac{m^2 S_c S_r (C_1 \cos(my) + C_2 \sin(my))}{l^2 + m^2} e^{\iota \omega t} \end{aligned} \quad (20)$$

where

$$\begin{aligned} C_1 &= -\frac{\sin(mh_2)}{\sin[m(h_1 - h_2)]} \\ C_2 &= \frac{\cos(mh_2)}{\sin[m(h_1 - h_2)]} \\ C_3 &= \frac{e^{lh_1}}{e^{2lh_1} - e^{2lh_2}} + \frac{m^2 S_c S_r C_1 (e^{lh_1} \cos(mh_1) - e^{lh_2} \cos(mh_2))}{(e^{2lh_1} - e^{2lh_2})(l^2 + m^2)} \\ &\quad + \frac{m^2 S_c S_r C_2 (e^{lh_1} \sin(mh_1) - e^{lh_2} \sin(mh_2))}{(e^{2lh_1} - e^{2lh_2})(l^2 + m^2)} \\ C_4 &= -C_3 e^{2lh_2} + \frac{m^2 S_c S_r e^{lh_2} (C_1 \cos(mh_2) + C_2 \sin(mh_2))}{l^2 + m^2} \\ C_5 &= \frac{A_3 G_c e^{-(l+n)h_1}}{A_2 A_4} + \frac{A_5 A_1 - n^2 A_6 A_7}{n^2 A_1 A_4} \\ C_6 &= -\frac{\lambda}{n^2} e^{nh_2} - C_5 e^{2nh_2} + G_c A_8 - e^{nh_2} A_9 \end{aligned}$$

$$A_1 = (l^2 + m^2)(m^2 + n^2)$$

$$A_2 = (l + n)(l - n)$$

$$A_3 = C_3(e^{(2l+n)h_1} - e^{lh_1+(l+n)h_2}) + C_4(e^{nh_1} - e^{lh_1+(-l+n)h_2})$$

$$A_4 = e^{nh_1} - e^{-nh_1+2nh_2}$$

$$A_5 = n^2 + \lambda(-1 + e^{n(h_2-h_1)})$$

$$A_6 = C_1(\cos(mh_1) - e^{n(h_2-h_1)}\cos(mh_2)) + C_2(\sin(mh_1) - e^{n(h_2-h_1)}\sin(mh_2))$$

$$A_7 = (l^2 + m^2)G_r - m^2G_cS_cS_r$$

$$A_8 = \frac{e^{nh_2}(C_3e^{lh_2} + C_4e^{-lh_2})}{A_2}$$

$$A_9 = \frac{(C_1\cos(mh_2) + C_2\sin(mh_2))A_7}{A_1}$$

The expression for skin friction coefficient across the channel walls is given by

$$\begin{aligned} \tau &= \mu \left(\frac{\partial u}{\partial y} \right)_{y=h_1, h_2} \\ &= n(C_5e^{ny} - C_6e^{-ny})e^{\omega t} - \frac{l(C_3e^{ly} - C_4e^{-ly})G_c}{A_2}e^{\omega t} \\ &\quad + \frac{m(-C_1\sin(my) + C_2\cos(my))A_7}{A_1}e^{\omega t} \end{aligned} \quad (21)$$

The expression for the rate of heat transfer across the walls of the channel is given by

$$N_u = - \left(\frac{\partial \theta}{\partial y} \right)_{y=h_1, h_2} = m(C_1\sin(my) - C_2\cos(my))e^{\omega t} \quad (22)$$

The expression for the rate of mass flow across the walls of the channel is given by

$$\begin{aligned} S_h &= - \left(\frac{\partial \Phi}{\partial y} \right)_{y=h_1, h_2} = -l(C_3e^{ly} - C_4e^{-ly})e^{\omega t} \\ &\quad - \frac{m^3S_cS_r(C_1\sin(my) - C_2\cos(my))}{l^2 + m^2}e^{\omega t} \end{aligned} \quad (23)$$

It is worth mentioning that in the absence of chemical reaction parameter, Schmidt number, modified Grashof number, and heat source parameter ($K_r, S_c, G_c, Q \rightarrow 0$) the present problem reduces to that of Ref. [3].

4. RESULTS AND DISCUSSION

The formulation of the effect of thermal diffusion, thermal radiation, chemical reaction and heat source/sink on an oscillatory flow through an asymmetric wavy channel in presence of an inclined magnetic field has been performed in the preceding section. The influence of various physical parameters of interest on velocity, temperature, concentration as well as skin friction τ , Nusselt number N_u and Sherwood number S_h are analyzed. The values of various parameters considered are $\omega = 0.1, \lambda = 0.1, t = 0.1, \pi = 3.14, x = 0.1, a = 0.2, b = 1.2, d = 2.0$ and $Re = 0.1$ while other parameters are varied over a range and are given in the caption of the figures.

Figs. 2 and 3 illustrate the effect of various embedded parameters on the velocity of the fluid. In Fig.2(a)-(d), it is observed that the velocity decreases by increasing the magnetic field parameter, Peclet number, Schmidt number and chemical reaction parameter. The increase in the magnetic field parameter decreases the velocity along the surface. The transverse magnetic field gives rise to a resistive force called as the Lorentz Force. This force slows down the fluid motion. These results are same as noted in Refs. [5, 3]. This result has an essential role in large number of industrial applications, particularly in favor to solidification processes such as casting and semiconductor single crystal growth applications. In these claims, as the liquids experience solidification, fluid flow and turbulence occur in the solidifying liquid pool and have critical conclusions on the product quality control. The practice of magnetic fields has effectively been applied to monitoring melt convection in solidification systems. Fig.2(d) shows that velocity decreases with the increase of chemical reaction parameter K_r . The destructive reaction $K_r > 0$ diminishes the concentration field which makes the buoyancy effects weaker due to concentration gradients. Accordingly, the motion of the fluid is retarded. Fig.2(e)-(f) illustrates the variation of velocity for different values of heat source and heat sink parameters. The velocity field increases by increasing the values of Q at $\phi = 0$. The positive sign of Q indicates the heat generation (heat source) while the negative sign of Q indicates heat absorption (heat sink). The effect of heat source $Q > 0$ on the velocity field

is much more predominantly related to the heat sink parameter $Q < 0$. These results are clearly supported from the physical point of view.

From Fig.3(a), it is observed that the velocity increases with increase in the thermal radiation parameter R . The reason for increase in the velocity is the weakening of bonds which are holding the fluid particles together. By increasing thermal radiation parameter, the bonds between the fluid particles are broken easily and hence the fluid velocity increases. Thus, it is observed that the radiation should be decreased so that the cooling process can be done at a faster rate. Fig.(b) reveals that the increase in the inclination angle of the magnetic field parameter increases the fluid velocity. Also from Fig.3(c), it is illustrated that as the permeability parameter K_1 increases, the velocity also increases. By increasing the value of K_1 , the porosity degree of the porous medium also increases which makes the fluid to flow freely within the channel. Fig.3(d)-(e) illustrates that the fluid velocity increases by increasing the values of G_r and G_c . Hence our observation in respect of these parameters agree with [38] in the absence of porous medium. Thus, the above result indicates that heavier species with lower thermal conductivity reduces the fluid flow.

Fig.4 illustrates the variation of temperature profile for different values of physical parameters of interest like Thermal radiation parameter R , Heat generation ($Q > 0$) or heat absorption ($Q < 0$) and the Peclet number P_e . Fig.4(a) illustrates that the temperature profile shows an increase when Thermal radiation parameter R is increased. From Fig.4(b)-(c), it is observed that the temperature profile increases by increasing the value of Q . The effect of heat source parameter $Q > 0$ on temperature profile is very much significantly related to the heat sink parameter $Q < 0$. These results are clearly supported from the physical point of view. Fig.4(d) depicts that the temperature profile decreases by increasing the values of Peclet number P_e .

Fig.5 illustrates the variation of concentration profile with respect to y for different values of physical parameters of interest. It is observed from Fig.5(a)-(d) that the concentration profile decreases by increasing the values of Thermal radiation parameter R , heat source parameter Q , Schmidt number S_c and chemical reaction parameter K_r . It is clear from the figures that the velocity and concentration profiles decrease with the increase of K_r . This

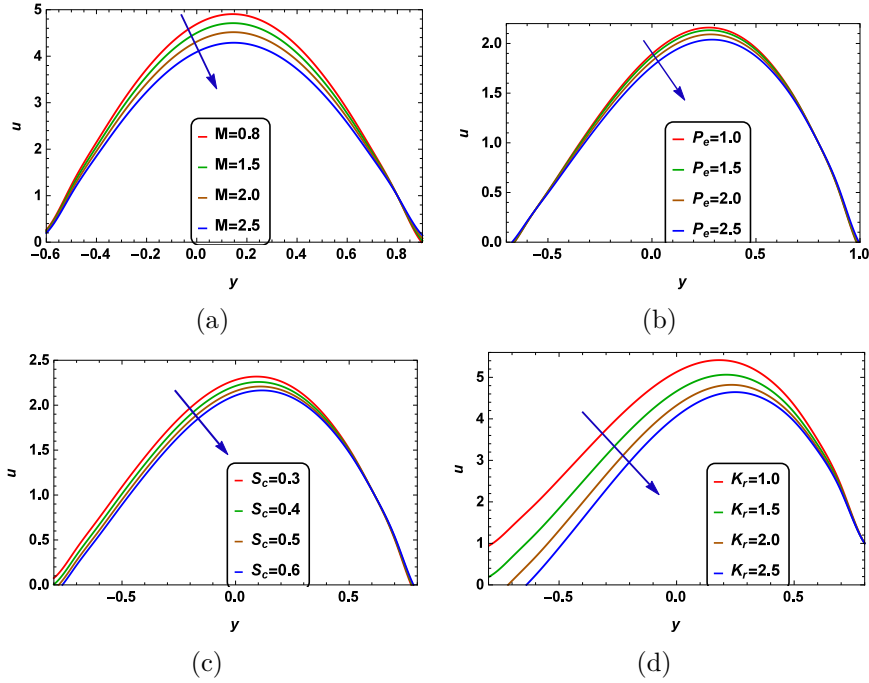
illustrates that the buoyancy effects (due to concentration and temperature difference) are important in the channel. Furthermore it is observed that the fluid motion is retarded on account of chemical reaction. This shows that the destructive reaction $K_r > 0$ leads to fall in the concentration field which in turn weakens the buoyancy effects due to concentration gradients. Therefore, the flow field is retarded. In addition to this, in the generative reaction, i.e. for $K_r < 0$, the reverse effect is observed. This existence has a superior agreement with the physical realities. It is observed that the influence of increasing values of S_c results in a decreasing concentration profiles through the boundary layer. Actually, the rise in the value of Schmidt number means reduction of molecular diffusion. Hence, the concentration of the species is progressive for smaller values of S_c and lesser for higher values of S_c . This result is in good agreement with Das et al. [4]. From Fig.5(e), it is observed that the effect of Soret number S_r on the concentration profile is totally opposite as compared to the Thermal radiation parameter and Schmidt number. The concentration profile increases by increasing the values of Soret number S_r . The concentration distributions are in good agreement with the outcomes found in case of Rout et al. [23]. From Fig.4 and Fig.5, it is illustrated that the temperature and concentration profiles show opposite behavior when compared with each other.

Fig.6 depicts the variation of Local Skin Friction τ for different values of magnetic field parameter, chemical reaction parameter, Schmidt number and Soret number. Fig.6(a) displays the variation of the local skin friction against G_r for different values of magnetic field parameter M at $y = h_1$ and $y = h_2$. It is depicted that the local skin friction coefficient τ at $y = h_1$ increases with increase in the magnetic field parameter M while at $y = h_2$, it shows opposite trend. Similar effects have been observed by increasing chemical reaction K_r and Schmidt number S_c on local skin friction coefficient τ . From Fig.6(d). it is displayed that the local skin friction coefficient τ decreases by increasing the magnitude of Soret number S_r .

Fig.7 and Fig.8 display the variation of heat transfer coefficient against Q for different values of R and P_e respectively. Fig.7 displays that the rate of heat transfer coefficient N_u against Q increases by increasing R at $y = h_1$, but the reverse trend is observed at $y = h_2$. Fig.8 depicts that the effect of Peclet number P_e on the

rate of heat transfer coefficient has opposite behavior when compared with R .

Fig.9 illustrates the variation of the rate of mass transfer S_h against Q for different values of chemical reaction parameter, thermal radiation parameter, Schmidt number and Soret number at $y = h_1$ or $y = h_2$. From Fig.9(a), it is observed that the mass transfer coefficient S_h decreases by increasing the chemical reaction parameter K_r at $y = h_1$. Fig.9(b)-(c) illustrates that the mass transfer coefficient increases by increasing R and S_c at $y = h_2$ but at $y = h_1$, opposite behavior is observed. Fig.9(d) displays that the Sherwood number S_h increases by increasing Soret number S_r at $y = h_2$. It is clear that the equations (4) and (5) are independent of M . So, M has no effect on N_u and S_h .



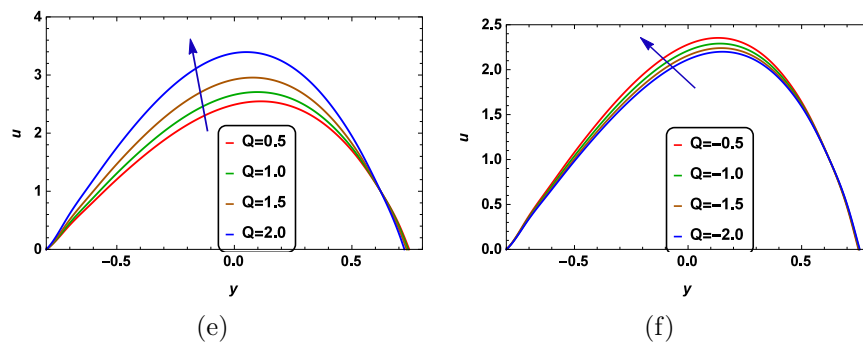
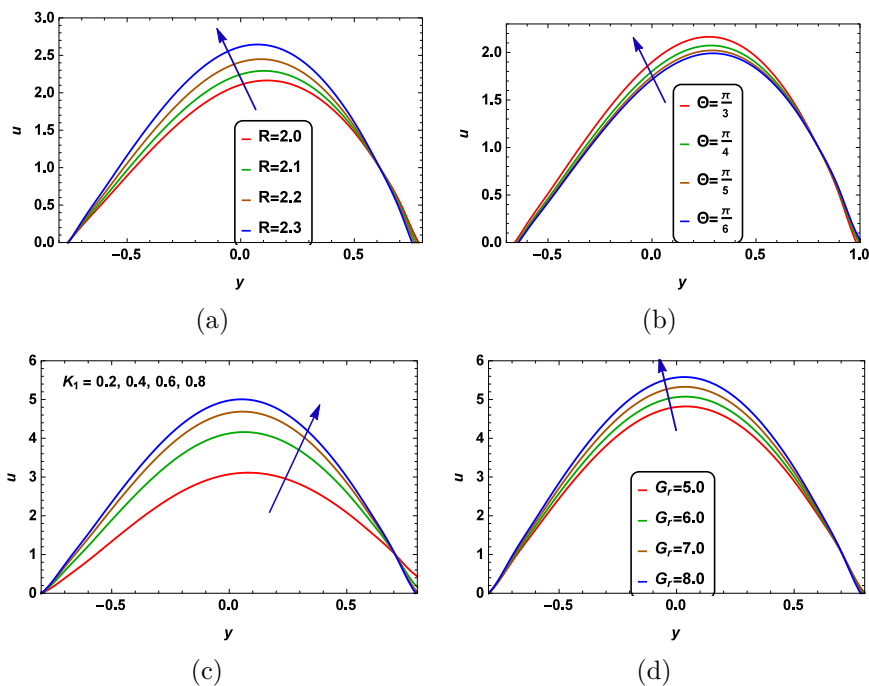


FIGURE 2. Variation of M, P_e, S_c, K_r and Q on the axial velocity (u) with respect to y with $G_r = 5, G_c = 5, S_r = 0.1, K_1 = 0.2, \phi = \frac{\pi}{2}$



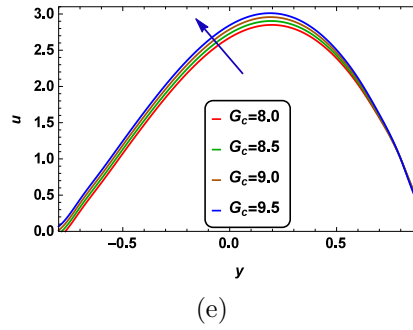


FIGURE 3. Variation of the velocity (u) with respect to y for different values of Θ , K_1 , G_r and G_c

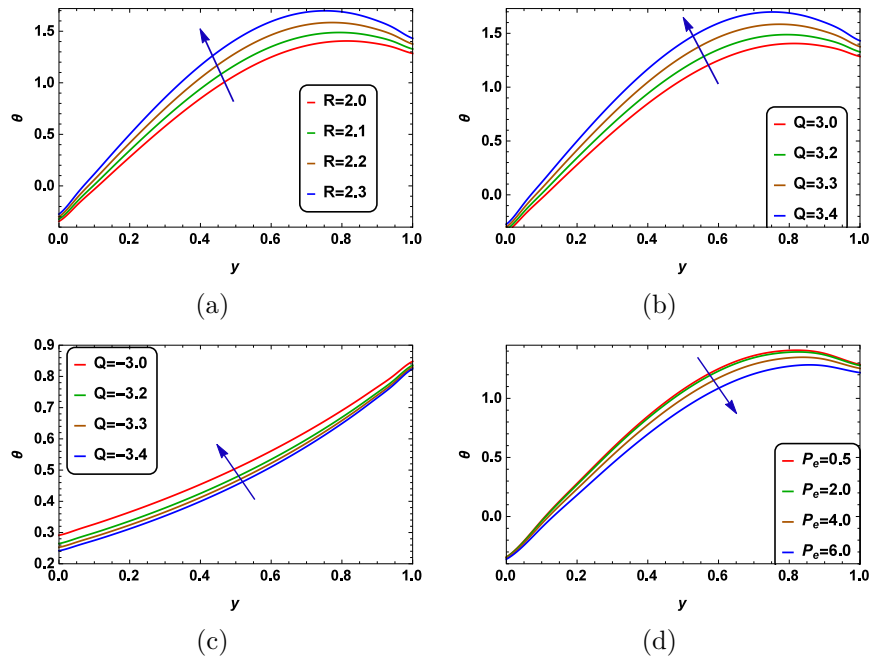


FIGURE 4. Variation of the Temperature distribution (θ) with respect to y for different values of N_r , P_r and Q

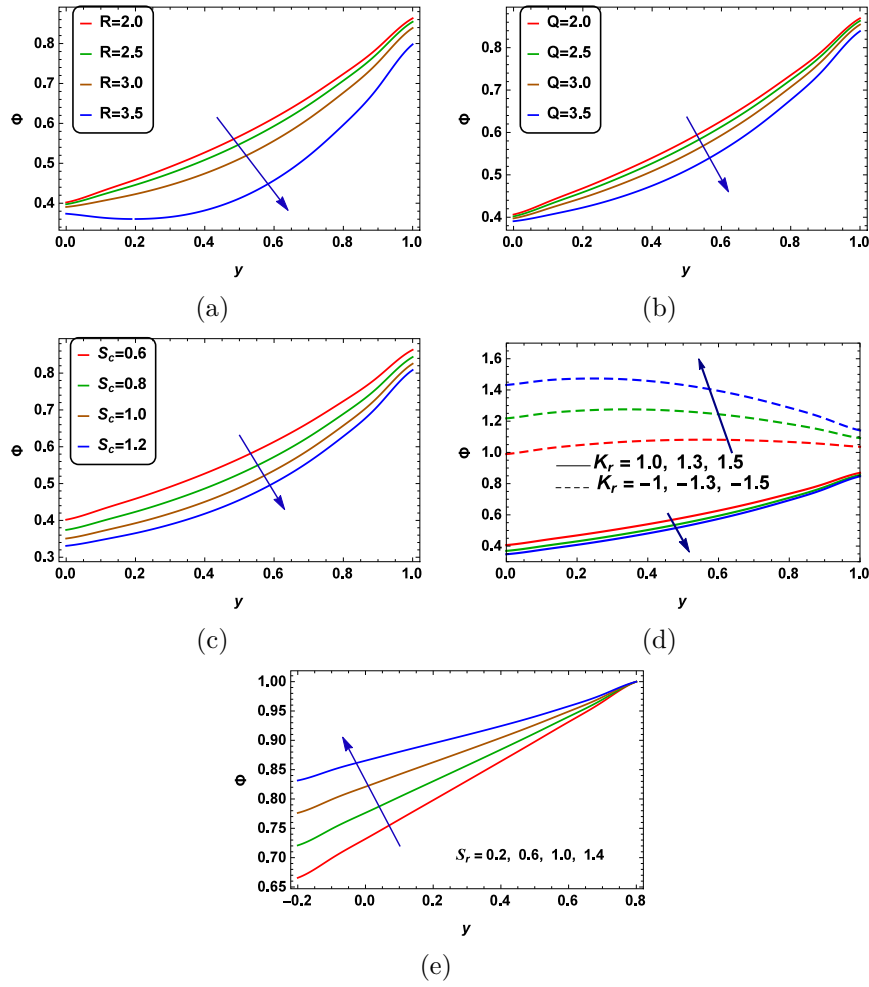


FIGURE 5. Variation of the concentration profile (Φ) with respect to y for different values of R, Q, S_c, K_r and S_r .

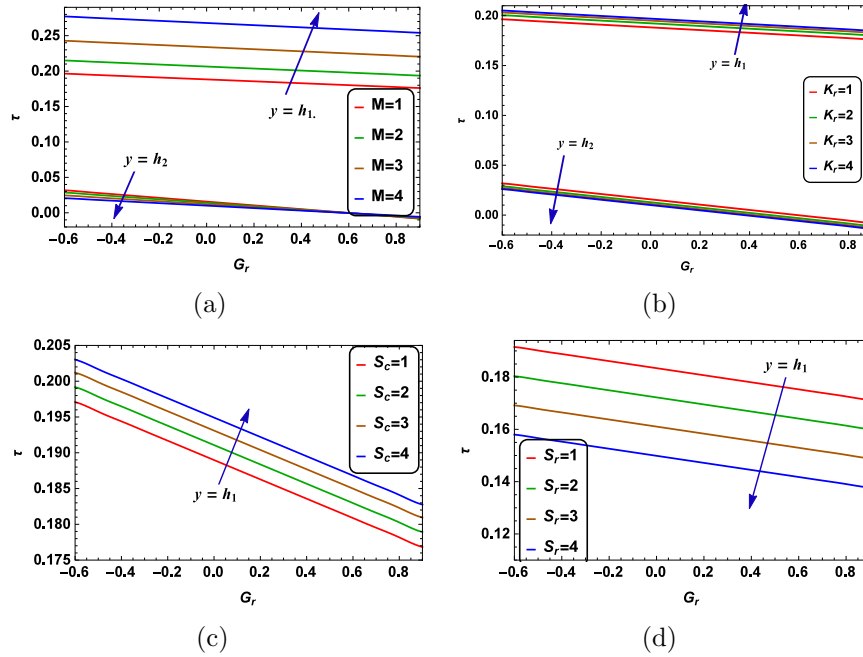


FIGURE 6. Variation of the skin friction coefficient (τ) with respect to y for different values of M , K_r , S_c and S_r

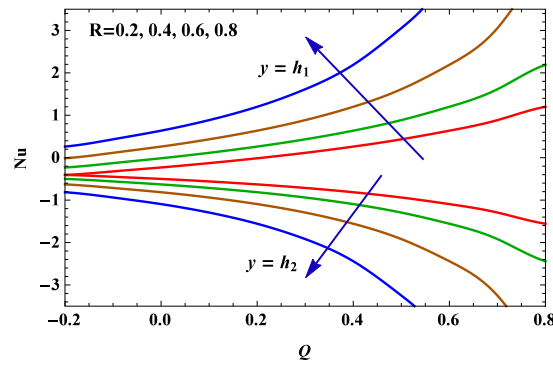


FIGURE 7. Distribution of rate of heat transfer Nu for different value of R

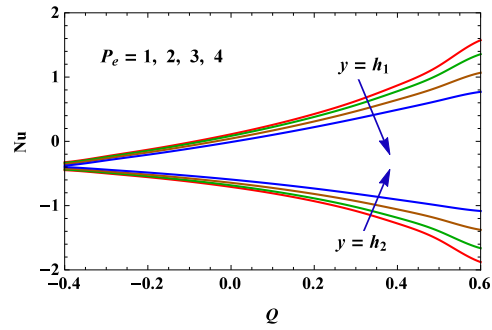


FIGURE 8. Distribution of rate of heat transfer N_u for different value of P_e

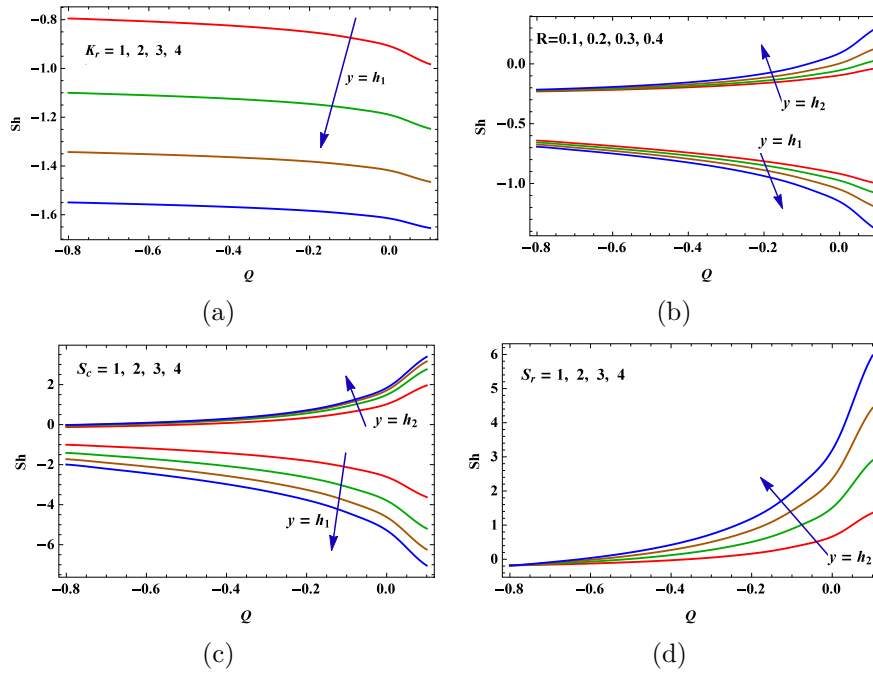


FIGURE 9. Variation of the rate of mass transfer (S_h) for different values of K_r , R , S_c and S_r

CONCLUSION

In this investigation, we have observed the effects of Thermal diffusion, radiation and heat source/sink on the oscillatory flow in a wavy asymmetric channel in presence of an inclined magnetic field and chemical reaction. Regular perturbation method has been employed to solve the coupled and non-linear governing equations analytically. Results are calculated and described graphically. The main findings in this investigation are summarized as follows:

- By increasing the magnetic field parameter, the fluid velocity decreases. Similar trend is followed by the Peclet number, Schmidt number and Chemical reaction parameter on the fluid velocity.
- Thermal radiation parameter, Permeability parameter and Heat source/sink parameter display similar behavior on the fluid velocity, while these parameters show exactly reverse behavior on fluid flow when compared with the magnetic field parameter.
- The increase in the inclination angle of the magnetic field parameter enhances the fluid velocity.
- Effect of Thermal radiation parameter on temperature profile is similar to that of heat source/sink but opposite when compared with the Peclet number.
- Radiation parameter enhances the fluid velocity as well as the fluid temperature.
- The behavior of temperature profile is opposite to the concentration profile.
- By increasing the chemical reaction parameter, the fluid motion slows down and causes a drop in the concentration field which in turn reduce the buoyancy effects because of concentration gradients.
- The skin friction coefficient increases when the magnetic field parameter is increased at the wall $y = h_2$ of the channel, while at $y = h_1$, opposite behavior is observed. The effects of Schmidt number and Soret number on the skin friction coefficient are totally opposite.
- Nusselt number increases by increasing thermal radiation parameter at $y = h_1$, while reverse trend is observed at $y = h_2$. It is also noted that the effect of Peclet number on Nusselt number is opposite when compared with thermal radiation parameter.

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